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B. J. Hopkinson

THE
ELEMENTS
OF
Physical and Geometrical
ASTRONOMY.

By DAVID GREGORY, M. D.
Late *Savilian* Professor of ASTRONOMY at *Oxford*, and Fellow of the
Royal Society.

Done into *English*, with Additions and Corrections.

To which is annex'd,
Dr. HALLEY's Synopsis of the
Astronomy of *Comets*.

The Whole newly revised and compared with
the *Latin*, and corrected throughout by
Mr. STONE, F. R. S.

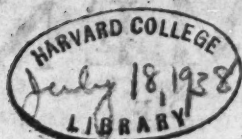
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To which is annexed
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Astronomy of Comets

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In two volumes

Vol. I.

Printed for D. Milner, in the
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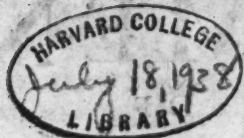
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BOOK IV.

*Concerning the Theory of the
Secondary Planets.*

HAVING in our First Book laid down the true and genuine Foundations of Physical Astronomy; in the Second Book explain'd the *Phænomena* of the first Motion or Appearances of the *Primum Mobile*, which arise from the Motion of the Earth, about its own Axis; and given the Theory of the Primary Planets in the Third Book; that is, how their Orbits are describ'd, and how their Places are determin'd, if the Observer be suppos'd in the Sun, or in any one of 'em; and also how to

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466 *The* ELEMENTS Book IV.

define the Magnitude, Density, and mutual Distances of the Primary Planets, and other such Things: That we may go on in Order, we must, in this Fourth Book, give the Theory of the Secondary Planets, which go round the Primary ones. Altho' the Motion of every Primary Planet be simple enough, as being (like a projectile Motion) compounded of an uniform progressive Motion in a Right-line Tangent to its Orbit, and another Motion of Descent by which (as attracted) it tends towards the Sun, their immense Distance from one another rendring the Effect of their mutual Action, or Gravity on each other, insensible; yet the Motion of the Secondary Planets will be quite otherwise. For tho' every one of 'em tends mostly to its respective Primary Planet; yet it is urg'd towards the Sun by the same accelerating Gravity as its Primary, when at the same Distance; by a lesser Gravity, when at a greater Distance; and by a greater Gravity, when at a less Distance. From this different Tendency to two Bodies, namely, to the Sun and to its Primary Planet, the Motion of a Secondary Planet becomes very complex, and has such Unequalities as the Astronomers hop'd to account for by their Hypotheses, sooner than the Philosophers cou'd explain them from Physical Causes, till Sir *Isaac Newton* was so happy as to do it.

SECTION

SECTION I.

Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is concentric to its Primary Planet.

PROPOSITION I.

I*F a Primary Planet, revolving about the Sun, carries with it a Satellite, this Satellite will be so mov'd about the Primary Planet, as to be continually accelerated from its Quadrature with the Sun to its next Conjunction or Opposition; but from the Syzygy to the Quadrature it will be retarded: Therefore the Satellite will move swifter near the Syzygies, and slower near the Quadratures.*

Because it has been shewn in *Book I.* that in these three Bodies, viz. The Sun, a Primary Planet, and its Satellite, the Attractions of any two on the third were to one another reciprocally, as the Squares of the Distances from it; it is manifest that the Case of this Proposition is the same with that of *Prop. 60. Book I.* Therefore if, as there [in *Fig. 1.*] *S* denotes the Sun, *T* a Primary Planet revolving in the Orb *ETE*, *L* a Satellite describing the Orbit *BCGD* round the Primary Planet, in which the Points *B* and *G* denote the Syzygies with the Sun, and *C* and *D* the Quadratures: If *SK*, the middle Distance of a Satellite from the Sun, be put from the accelerating Attraction of that Satellite from the Sun in that middle Distance, and *L* be suppos'd any Place of the Satellite in its Orbit, in *LS*, or the same produc'd, take *SA*, which make to *SK* in a duplicate Ratio of *SK* to *SL*; and this *SA* will denote

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468 *The* ELEMENTS Book IV.

the Attraction of the Satellite L towards S . Join TL , and draw AM parallel to it meeting ST in M . It is well known that the Attraction SA may be resolv'd into the Attractions AM and MS , in the Directions of these Lines; of which that, which is express'd by NS , is reduc'd to the Attraction MN , taking away the Attraction NS , common to the Satellite and its Primary Planet; as it has been amply shewn in the said 60 *Prop.* of *B.I.* The Satellite therefore is thus acted upon by a triple Attraction: The first and principal is that by which T attracts L ; the second is as AM in the Direction AM , that is, in the Direction LT parallel to AM . Whence it happens, that the Force compounded of these two is also directed towards T ; and if the Body L was acted upon only by a Force compounded of those two, it wou'd then describe Areas about T proportionable to the Times, by *Prop.* 11. *B. I.* But the Satellite in L is also urg'd by a third Force, which is as MN , and its Direction is from M towards N ; that is, from L towards F , (supposing LF parallel to MN ;) which as it does not tend to T , so neither does the whole Force compounded of these three, (*viz.* that by which L is truly urg'd,) tend to T . Therefore the Satellite L will not describe Areas about T proportionable to the Times; but this Force, which is express'd by MN , will disturb the Description of Areas proportionable to the Times; for in CG the Quadrant of the Semicircle CGD , it will accelerate the Motion of the Body L about T , when it moves from C to G ; but after the Conjunction at G in the Quadrant GD , when the Direction of the Right-line LF from L to F is forward, the Force acting along it retards the Motion. When the Satellite is come to the Quadrature in

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Book IV. of ASTRONOMY. 469

in D , MN vanishes, (because then SK , SL , and also SA are equal) and therefore the Effect of the Force express'd by it is nothing; therefore now the Satellite L acted on by the other Forces describes, by the Radius that carries it, Areas proportionable to the Times. But when L goes thro' the Quadrant DB , SM is less than SN , and the Force disturbing the Description of the Area by the Radius TL that carries the Satellite (and hindring it from being proportionable to the Time which is as MN) has its Direction from M to N , or from L to F , (that is *in consequentia*) and therefore accelerates the Motion of the Satellite in the Quadrant: But after the Opposition in B , LF tends *in antecedentia*, and retards the Motion, till in the Quadrature at C , MN vanishes again, and therefore its Effect ceases. Again, since the Force MN hindring the Radius TL from describing an Area proportionable to the Time (whilst the Satellite is going from C to G , or from D to B) is continually increas'd, and is greatest in G and B , and from thence is continually diminish'd till the Satellite comes to D or C , where this Force vanishes; it is plain that the Motion of the Satellite, so disturb'd, is the swiftest (*cæteris paribus*) in the Syzygies B and G , and slowest in the Quadratures C and D . Q. E. D.

PROPOSITION II.

THings being as before, the Orbit which the Satellite L describes round its Primary Planet T , is more curve in C and D , the Quadratures with the Sun than in the Syzygies G and B .

For because the Satellite L moves swifter in the Syzygies than in the Quadratures, it will less deflect from a Right-line (which it wou'd describe by an impress'd Force only) in the former

470 *The* ELEMENTS Book IV.

than in the latter; that is, the Orbit of the Satellite is less curve in the Conjunction and Opposition, than in the Quadratures. But this greater Curvity is also owing to another Cause: The Force MN (of which we spoke in the foregoing Proposition) in the Conjunction and Opposition at G and B , is contrary to the Force by which the Body T attracts L , and therefore diminishes it; whence the Body L is less push'd towards T at G and B , than at C and D ; and therefore will less deflect from a Right Line in the first, than in the last Case. Therefore the Satellite's Way is more curve in the Quadratures C and D , than in the Syzygies B and G .

COROLLARY.

From what has been said it follows, That the Satellite L (*cæteris paribus*) recedes farther from the Primary Planet T in the Quadratures, than in the Syzygies. [*Fig. 2.*] Therefore the Orbit of the Satellite, about its Primary Planet, is in a Manner elliptical, having the Primary in its Center, the greater Axis being between the Quadratures with the Sun, and the lesser between the Syzygies; and so the Orbit is more curve in the first, than in the last Case. And the Encrease of the Celerity of the Satellite, near the Syzygies, will, in this Orbit, be different from what it wou'd have been in the circular Orbit of the foregoing Proposition, because the Direction of the Force MN , accelerating, or retarding the Motion of the Satellite, is different in this, from what it is in that.

What has been said here of the Figure of the Orbit of a Satellite, about its Primary Planet, is only true, if, excluding the extrinsecal Force of the Sun, its Orbit is concentrical with the Primary Planet. For if it be excentrical, it may happen, by reason of that Excentricity, that the Satellite
L shall

Book IV. of ASTRONOMY. 471

L shall be farther from the Primary Planet *T*, and seen from it shall move slower in the Syzygies, than in the Quadratures; namely, when the Satellite has its greatest Apfis in its Conjunction or Opposition with the Sun. But here also what was before demonstrated will take Place (*cæteris paribus*); for even then, by reason of the said Force, *M N* the Satellite will move faster, and come nearer to its Primary Planet, than it would have done without it.

This aforesaid elliptic Figure of the Orbit of a Satellite was observ'd in the Moon by that most ingenious Astronomer Dr. *Halley*, who in the Year 1679, first of any Body, publish'd it in his Treatise concerning the Emendation of the Moon's Theory. But the History of the Academy of Sciences in *France*, publish'd in 1698, says, that Mr. *Picard* had made the same Observation long before, namely in 1668.

SECTION II.

Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is excentric to its Primary Planet.

IF the Orb of the lesser Body *L* [*Fig 3.*] be not a Circle concentric to the great Body *T*, but an Ellipsis, in one of whose *Foci* the Body *T* is plac'd, the Forces of the greatest Body *S* will so disturb that Body *L*, that instead of a constant and unmov'd Ellipsis, which (by *Prop. 39. B. I.*) it ought to describe, it will describe an Ellipsis whose Excentricity will continually be chang'd; and that Orbit of the Body *L*, so disturb'd, will

be the same, as if it shou'd describe its Ellipsis, at the same Time that the Axis of the Ellipsis, (and the Ellipsis itself) is, by an angular Motion, carried forward or backward upon the Center of the Primary Planet, the Plane of the Orbit continuing the same; that is, in the astronomical Phrase, The Motion of a Satellite, that describes an excentric Orbit, is so disturb'd by the Sun, that whilst the Satellite goes thro' its Orbit, the Apses of its Orbit are sometimes carried in *consequentia*, and sometimes in *antecedentia*. To prove this, the Thing must be farther fetch'd, and some *Lemma's* laid down.

PROPOSITION III.

HOW to find the Law of the centripetal Force, by whose Action a Body may be mov'd in any Trajectory revolving about the Center of the Forces, whilst another Body is mov'd in a similar and equal Trajectory, which is at rest.

Let ALP (Fig. 3.) be the Trajectory at rest, which the Body T , tending to the Center of the Forces, describes, going from A , thro' L , towards P . It is requir'd to find the Law of the centripetal Force, tending to T , by Means of which, another Body shall be carried in the Perimeter of the Figure alp , similar and equal to the first, whilst at the same Time, the said Figure alp revolves about T , backwards or forwards, in such Manner, that whilst the first Body goes thro' any Arc, as AL , in the Orbit ALP , which is at rest; the last shall describe a similar and equal Arc al , in the revolving Orb alp .

Let Tl , keeping always equal to TL , move about the Center T , in the Plain ALT , and always make the Angle ATl , proportionable to the Angle ATL ; that is, in such Manner that this Angle ATL may be to its Correspondent ATL ,

as

Book IV. of ASTRONOMY. 473

as any other Angle ATl , to any other correspondent one ATL . Therefore the Area, which Tl (thus carried in an angular Motion about T) describes, will be proportional to that which TL describes. Let $AlEGH$ be a Curve, which the Point l , mov'd, as has been said, touches: Now, because the Body, which is urg'd by a centripetal Force, that tends towards T , is revolv'd along with L , the Area $ATLA$ (by *Prop. 11. B. I.*) will be proportionable to the Time; but it is also in an unmoveable Plain, namely, that in which the Body L revolves about T ; that is, the Motion of the Point l is such, that it is always found in the unmoveable Plain drawn thro' T , and makes the Areas $ATlA$ (describ'd by a Radius drawn thro' T) proportionable to the Times. Therefore (by *Prop. 12. B. I.*) it is manifest that a Body being acted upon by a due Quantity of centripetal Force, may revolve together with the Point l , in the Curve $AlEGH$, which the Point l describes. But this Curve $AlEGH$ is given, as being generated by the given Line $ALPA$ in a constant and given Manner; therefore let the Law of the centripetal Force, by which a Body may revolve in the said Curve, be found (by *Cor. Prop. 37. B. I.*) Let the Angle ATa be made equal to the Angle LTL , and the Line Ta to TA ; and then the Angle aTl will be equal to the Angle ATL . Now let the Figure alT be similar and equal to ALT ; therefore the Point l , is in the Perimeter of a Figure, similar and equal to $ALPA$ revolving about T . Therefore a Body carried along with l , and describing the unmov'd Figure $AlEGH$, describes also the Perimeter $alpa$ of an Orb similar to $ALPA$, and revolving about T ; so that the Arc al is describ'd in the revolving Orb $alpa$, at the same Time that this Proposition is imper-

474 *The* ELEMENTS Book IV.

imperfect, the Arc AL , similar and equal to it, is describ'd by another Body in the Orb $ALPA$, which is at rest. Therefore the Law of the Centripetal Force requir'd is found. QEF .

PROPOSITION IV. LEMMA.

The Difference of the Centripetal Forces, by which two Bodies may be equally mov'd, the one in a quiescent, and the other in a revolving Orb (which is shewn in the foregoing Proposition) is in a triplicate Proportion of the common Altitude inversely.

All Things being as laid down in the foregoing Prop. I say that the Difference of the Centripetal Forces, by which the Bodies L and l [Fig. 4.] are acted upon, are reciprocally in a triplicate Ratio of the common Height TL or Tl ; or that the Difference of the Forces in the Points L and l is, to the Difference of the Forces in any other Points of the quiescent and revolving Orbs, where the Bodies will be found at the same Moment, as the Cube of the Height or Distance of those Points from T the Center of the Forces, to the Cube of the Right-line LT or lT .

Let the very small equal Arcs LB , and lb , be taken, which therefore will be describ'd by the Bodies L and l in very small Times. From B and b draw BE , be perpendicular to TL , Tl , which will be equal, as also LE and le . In the Line be (produc'd towards b , if Occasion require) take the Point C , so that Ce may be to be , as the Angle ATl to the Angle ATL . Now because each Motion of the Bodies in the Places L and l is distinguish'd into Two, of which One is along the Lines LT , lT , and the Other transverse to the first, along Lines perpendicular to LT , lT ; the Motions towards the Center will be equal, because, by Supposition, $LT : lT :: BT,$

Book IV. of ASTRONOMY. 475

$BT: bT$, and therefore their small Increments and Decrements are equal. And the transverse Motion of the Body l will be to the transverse Motion of the Body L , as the angular Motion of the Line Tl to the angular Motion of the Line TL ; that is, as the Angle ATl to the Angle ATL , or as $Ceto be$ or BE . And therefore in the same Time that the Body L , by each of its two Motions comes to B , the Body l will be found in the Place C . And this will happen where the Bodies L and l are equally mov'd along the Directions LT , lT ; that is, when they are acted upon by equal Centripetal Forces: But when the Point D (the Place of the Body describing the revolving Orbit) is found (by the foregoing Proposition) by taking the Angle lTD in that said constant Ratio to lTB (*viz.* the Ratio of the Angle ATL to ATl) and the Right-line TD equal to Tb or TB , (the Figure alp and the Line lT being in this Case put for the Figure ALP and TA in the other;) the Difference of the Centripetal Forces of the Body in L or l , and that of the Body in D , is as its Effect CD ; namely, the Interval of the Places thro' which the Body, carry'd in the revolving Orb, must by the Action of the Orb be transferr'd in that given Space of a very small Time. Therefore we must compare this CD , or the Difference of Centripetal Forces equal to it (by which the Body is urg'd in the quiescent Orb, and by which it is urg'd in the revolving Orb, in that Situation of a moveable Orbit, and that Distance Tl of the Body from the Center) with the like Difference in the other Case, where the same, or a like Construction is suppos'd as here, and the same Things are likewise suppos'd to have been done: namely, let a Circle cutting Te in n , Ce in F ,
and

476 *The* ELEMENTS Book IV.

and CD in G be drawn with the Distance Tb , or TD , round the Center T ; and let the new Triangle lbT be such, that its Area may be swept thro' by the Radius Tb , in the same Time that this Area lbT is gone thro' by this Radius Tb : Whence (forasmuch as in equal Triangles the Bases are reciprocally proportionable to their Heights) this Line eb is to the other eb as another Tl to this Tl , that is, eb is reciprocally as the Correspondent Tl ; wherefore also Ce (always having the same Ratio to be) will be reciprocally as Tl : Therefore CF the Aggregate of 'em, and the Interval cb will be reciprocally proportional to the same Tl ; and therefore the Rectangle under them, *viz.* $FC \times Cb$ is reciprocally proportionable to the Square of the said Tl . Wherefore (by *Prop. 35. or 36. El. 3.*)

$$CD \times CG = FC \times Cb, \text{ that is, } CD = \frac{FC \times Cb}{CG}$$

Therefore CD , in this Case, is to the analogous CD , in any other Case, directly as this $FC \times Cb$ to that other $FC \times Cb$, and reciprocally as this Line CG to any other CG at the same Time. But it has been shewn above, that $FC \times Cb$ is reciprocally as the Square of the Right Line Tl ; and CG is as DG its nearly Equal, or as its Half TD , or Tb , or Tl , very little different from it: Therefore CD , in this Situation of a revolving Orbit is to CD produc'd in the same Manner in any other Situation of the same Orbit (that is, the Difference of the centripetal Forces, by which one Body in a quiescent Orbit, and another Body in the same Orbit revolving, may be equally mov'd in the Height TL , or Tl , is to the like Difference of Forces in any other Height) reciprocally, as the Square of the first Height, to the Square of the last, and, reciprocally, as the first Height to the last Height at the same Time; that

Book IV. of ASTRONOMY. 477

that is, reciprocally, as the Cube of the first Height, to the Cube of the last Height; that is, (speaking in short of one Height, or Altitude, when we are to be understood of two taken together) the said Difference of centripetal Forces is in a triplicate Ratio of the common Height inversely. *Q. E. D.*

C O R O L L A R Y I.

A Body moving from its upper, to its lower Apfis, in an Orb revolving *in consequentia*, is urg'd by a greater centripetal Force, than if its Orb was at rest. For because (in the foregoing Figure) in that Case the Angle ATl is greater than ATL , eC also will be greater than eb ; as being in the same Ratio as the foregoing Angle, therefore the Point C is out of the Periphery of the Circle BbD . But the Body, mov'd in the revolving Orbit, is found in the Point D ; and it has been above shewn, if the Body l be mov'd with a centripetal Force equal to L , that it will come to the Point C ; therefore it will not be so much urg'd towards the Center T . [*Fig. 5.*]

Again, the like will happen if (Things being as before) the revolving Orb be mov'd *in antecedentia* with a greater Celerity, than the Double of that Celerity, in which the Line TL is mov'd *in consequentia*, as in *Fig. 1.* Because, in this Case, aTA is more than twice greater than ATL , or aTl ; ATl will be greater than ATL : Therefore (as before) eC will be greater than eb , and the Point C will be without the Circle bDB ; and, for the same Reason as before, the centripetal Force of the Body, moving in the revolving Orbit alp , is even now greater than the centripetal Force of the Body, moving in the quiescent Orb ALP .

But

478 *The* ELEMENTS Book IV.

But if this Orb alp goes back with a Celerity less than twice that, by which the Line TL goes forwards as in *Fig. 3.* the Body l , descending towards its lower Apfis, will be urg'd by a less Centripetal Force than L in the quiescent Orb. For because the Angle ATa is not the Double of ATL or aTl ; ATl will be less than aTl , (that is, ATL) and therefore ec less than eb ; and for that Reason the Point C , to which l wou'd come, if it was urg'd with the same Centripetal Force as L , will be within the Circumference of the Circle BbD . But a Body, moving in a revolving Orb, has been before shewn to be found in the Point D , more remote from the Center than C is; therefore the Body l , describing the Orb alp , is urg'd with a less Centripetal Force, than the Body L , which is carried in the quiescent Orb ALP . And all Things happen contrary to a Body ascending from its lower, to its upper Apfis; and so on every Time.

COROLLARY 2.

Hence also the Difference of the Centripetal Forces in a moveable and unmoveable Orbit (as in B and b , *Fig. 6.*) is to the Centripetal Force, by which a Body may be mov'd in a Circular Motion from N to B (in the same Time that the Body L in the unmoveable Orbit describes the Arc LB) as $RC \times Cb$ to eb . For the said Forces are as their Effects CD and Hb , produc'd in the same small Portion of Time; but the nascent Line Hb is equal to ne , and CD has been shewn equal to $RC \times Cb$

$\frac{CG}{RC \times Cb}$: Wherefore the said Forces are as $\frac{RC \times Cb}{CG}$ and ne , that is, as $RC \times Cb$, and $CG \times ne$. But the Right-line CG differs little from the Right-line Me , and (by *Prop. 35. El. 3.*) the

Book IV. of ASTRONOMY. 479

the Rectangle under Me and en is equal to the Square of eb : Therefore the said Centripetal Forces are as $RC \times Cb$, and eb^2 . Now if the known Quantities F and G are taken in that Ratio to one another, as the Angle ATL has to ATl , that is of be to Ce ; CR will be as $G + F$, and Cb as $G - F$: Therefore the Difference of the Centripetal Forces, by which Bodies are urg'd in a movable and unmovable Orbit in B and b , is to the Centripetal Force by which a Body may be mov'd in a Circular Motion from N to B , in the same Time as L describes the Arc LB , as $G + F \times G - F$ to F^2 ; that is, as $G^2 - F^2$ to F^2 .

LEMMA.

If a Body tending to a Focus or Center T , describes the Ellipsis ABP , whose Apses are A and P ; the Centripetal Force, by which the Body is urg'd in the Apse A , is to the Centripetal Force of another Body revolving in the Circle AEH , at the same Distance TA , with the same Velocity, as the Diameter of the Circle to the latus rectum of the Ellipsis. [Fig. 7.]

Thro' A draw the Right-line AG perpendicular to AT , which will touch the Circle as well as the Ellipsis. Thro' the Point G very near A , draw GEB towards T , cutting the Circle in E and the Ellipsis in B . Because the nascent Arcs AB, AE , are very nearly equal, Bodies, setting out together with equal Velocities from A , will come at the same Time to B and E ; and their Centripetal Forces are as their Effects GB, GE , produc'd in the same small Time. Besides, from the Nature of the Circle, $EI^2 = AI \times IH$, or $AI \times Diam.$ And by the Nature of the Ellipsis, $BD^2 =$ to the latus rectum $\times AD$ (for the Defect, or Quantity, by which BD^2 is less than that Rectangle, vanishes in this Case as soon as BD is nascent;) but

480 *The* ELEMENTS Book IV

but the small nascent Lines $E I$ and $B D$ are equal: Therefore $A I \times$ Diatn. of the Circle = $A D \times$ *latus rectum* of the Ellipsis: Therefore $A D$ is to $A I$, or $G B$ to $G E$, or the centripetal Force in the Ellipsis to the centripetal Force in the Circle, as the Diameter of the Circle to the *latus rectum* of the Ellipsis. Q. E. D.

PROPOSITION V. LEMMA.

If the Orb ALP be an Ellipsis, whose *latus rectum* is $2R$, having its Focus at T , its upper Apfis at A , and lower at P ; and there be another Ellipsis alp , equal and similar to it, revolving in the Manner aforesaid, round the Focus T , and the given Quantities, G and F , have the same Ratio as the Angles ATl and ATL ; if the Height of the revolving Body, or the Distance from the Center be A : The centripetal Force, by which the Body may revolve in the moveable Ellipsis will be, as $\frac{F^q}{A^q} + \frac{R \times G^q - R \times F^q}{A \text{ cub.}}$; and, on the contrary, when such a centripetal Force acts on a Body, it will be carried in a moveable Ellipsis.

Since the Force, by which the Body can revolve in the unmoveable Ellipsis ALP , is (by Prop. 39. B.I.) reciprocally, as the Square of the Distance from the Center, or of the Height A ; let that Force be express'd by $\frac{F^q}{A^q}$, and the Force

in the Apfis A will be $\frac{F^q}{AT^q}$. But the Force by which the Body may at the Distance TA , revolve in a Circle with the Velocity, that the Body revolving in an Ellipsis has in A , is (by the foregoing Lemma) to the Force by which the Body revolving in an Ellipsis, is acted upon in A , as the half *latus rectum* of the Ellipsis to the Semi-diameter

Book IV. of ASTRONOMY. 481

Diameter of the Circle, or as R to AT ; therefore its Value is $\frac{R \times F^q}{AT \text{ cub.}}$. Now because (by *Corol. 2. Prop. 4.*) the Difference of the Forces in the

Points A and a is to the aforesaid Force in the Circle, (which is equal to $\frac{R \times F^q}{AT \text{ cub.}}$ as $G^q - F^q$ to

F^q : that Difference of Forces will be equal to $\frac{R \times G^q - R \times F^q}{AT \text{ cub.}}$. And (by *Prop. 4.*) the Difference

of the Forces in any other Points of the same Height (namely, where the Height A is undetermin'd) is to the Difference of the Forces in the Apfis A (where the Height is TA) as the Cube of TA to $A \text{ cub.}$; therefore in any Height its Value will be $\frac{R \times G^q - R \times F^q}{A \text{ cub.}}$. If therefore

to the Force $\frac{F^q}{A^q}$, by which a Body may be revolv'd in an unmoveable Ellipsis ALP , be added the said Difference $\frac{R \times G^q - R \times F^q}{A \text{ cub.}}$; the whole

compounded Force will be $\frac{F^q}{A^q} + \frac{R \times G^q - R \times F^q}{A \text{ cub.}}$;

by which a Body may, in the moveable Ellipsis alp be revolv'd in the same Time. Q. E. D.

COROLLARY I.

Hence, if to the centripetal Force, (which is in a duplicate reciprocal Ratio of the Distance from the Center, by which a Body revolves in an unmov'd Ellipsis) be superadded to any other Force which is reciprocally as the Cube of the Distance; the Body will describe an Ellipsis whose Apfes are carried *in consequentia*. For, because

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the Difference, which is as $\frac{R \times G^3 - R \times F^3}{A \text{ Cub.}}$

is in this Case positive, G will exceed L ; that is, the Angle ATL does exceed the Angle ATL ; whence (by *Corol. 1. Prop. 4.*) the Apfes are either carried, *in consequentia*, or *in antecedentia*, with a Celerity greater than twice that by which the Body L is carried, *in consequentia*, in the unmoveable Orb; but because this last does not happen in any of the cœlestial Bodies, we abstract from it in this Case. But, if from the Force (which is, reciprocally, in a duplicate Ratio of the Distance) be taken the Force which is reciprocally in a triplicate Ratio of it, the Remainder will be the Force, by whose Action a Body will describe an Ellipsis whose Apfes go backward; for the Difference of the Forces $\frac{R \times G^3 - R \times F^3}{A \text{ Cub.}}$ is in this

Case negative; and therefore G is less than F , or the Angle ATL than ATL .

COROLLARY 2. [Fig. IX.]

Hence it follows also, that if in an Orb which differs very little from a Circle, the centripetal Force, as you recede from the Center, decreases more (or becomes less) than in a duplicate Ratio of the Distance encreased; the Apfes of the Orb will be mov'd, *in consequentia*. For, let the unmoveable Ellipsis ALP be the Orb describ'd by a Body, which is urg'd by a Force tending to the Center T , whose upper Apfis A is twice as far distant from the Center T as the lower P ; not that such an Orb comes near to a Circle, but we put this Proportion only by Way of Example, and to make the Case plainer. Let us suppose the centripetal Force by which

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Book IV. of ASTRONOMY. 483

the Body P is urg'd, to consist of 400 Parts, and the Force by which it is urg'd in A will be 100, and so proportionably in the intermediate Places; that is, that Force, by whose Action the Body wou'd describe the unmovable Ellipsis ALP , is in a duplicate reciprocal Ratio of the Distance from the Center. Now if besides this Force, the Body in P is urg'd by another Force, which is for *Ex.* as 64, and in A by a Force which is as 8, and proportionably in the intermediate Places; namely, if that superadded Force be inversly as the Cube of the Distance from the Center; in that Case (by the foregoing Corol.) the Apses will be carried *in consequentia*. But the whole Force in P , or that made up of the Two, is as 464, and the Force in A as 108, and proportionably in the intermediate Places; that is, as you recede from the Center the centripetal Force decreases faster than in a Duplicate Ratio of the Distance encreas'd; for if it was only diminish'd in that duplicate Ratio of the Distance, the Force in A wou'd be as 116. And tho' it be requir'd that a new Force, superadded to that First, which is inversly as the Square of the Distance, be every where precisely as the Cube of the said Distance inversly, that an Elliptic Orbit may be revolv'd, *in consequentia*, about the Center of the Forces: yet if this Ellipsis differs but little from a Circle, and such a Force wholly acts upon the Body, as in receding from the Center is diminish'd more than the Square of the Distance from the Center is increas'd; the Apses of such an Orb will be mov'd, *in consequentia*. For because in an Orb that is almost circular, the Distances from the Center differ but very little; that Force, (the adding of which occasions the whole Force by which the Body is urg'd

484 *The* ELEMENTS Book IV.

to be diminish'd more than the Square of the Distance from the Center is encreas'd) is diminish'd in a Ratio, which differs not much from a triplicate inverse Ratio of the Distance.

It follows also, that if, in an Orb nearly circular, the centripetal Force decreases less than in a duplicate Ratio of the Distance encreas'd, (that is, be greater than in the said Ratio) the Apses of the Orb must move *in antecedentia*. For the same Force remaining in *P* and *A*, by which a Body describes an unmoveable Ellipsis, if from it you take away a Force reciprocally proportionable to the Cube of the Distance from the Center; that is, if from the Force in *P* you take a Force that is as 64, and from the Force in *A* a Force as 8, and after the same Manner in the intermediate Places; there will every where remain a Force, which (by the foregoing *Corol.*) will make a Body describe an Ellipsis whose Apses are mov'd *in antecedentia*. But the Force remaining in *P* is as 336, and the Force remaining in *A* as 92, and after the same Manner in the intermediate Places; therefore, as you recede from the Center, the centripetal Force, is less diminish'd (or is greater) than in a duplicate Ratio of the Distance encreas'd: For the Force diminish'd in a duplicate Ratio of the said Distance, wou'd in *A* consist of 84 Parts. Whence, by the Reasons before given, if a Body in an Orb nearly circular be urg'd by a Force, which, as you recede from the Center, is diminish'd less than in a duplicate Ratio of the Distance encreas'd; the Apses of the Orb will be carried *in antecedentia*. Which also appears by *Corol. 1. Prop. 4.*

Book IV. of ASTRONOMY. 485

PROPOSITION VI. LEMMA.

IN Elliptic Orbits, nearly circular, to determine the Motion of the Apses, from the given Law of the centripetal Force; and, on the contrary, from the given Motion of the Apses, to find the Law of the centripetal Force, supposing the centripetal Force to be as some Power of the Altitude.

If from any given Law of centripetal Force, the Motion of the Apses be requir'd, the Problem will be solv'd by making the Orb, which a Body, revolving in a moveable Ellipsis, describes in an unmov'd Plane, to come near the Figure of the Orb, whose Apses are requir'd, and finding the Apses of an Orb so describ'd. Now the Orbs come nearest to that Figure, when the centripetal Forces, by which they are describ'd, are proportional at equal Distances from the Center.

Let the Point *A* (in any of the foregoing Figures) be the upper Apse, *T* the Center of the Forces. Let *AT*, the greatest Height, be call'd *T*, and any other undeterminate Altitude *A*, and their Difference *X*. Then the Force by which the Body is urg'd in an Ellipsis, which revolves about its Focus *T*, is, (by what foregoing) as $\frac{F^q}{A^q} +$

$$\frac{R \times G^q - R \times F^q}{A^{\text{Cub.}}}, \text{ or as } \frac{A \times F^q + R \times G^q - R \times F^q}{A^{\text{Cub.}}},$$

and lastly, if instead of *A* you substitute *T - X*, as $\frac{T \times F^q - X \times F^q + R \times G^q - R \times F^q}{A^{\text{Cub.}}}$. Let

the Value of the centripetal Force, whose Law is given, be reduc'd to a similar Fraction; that is, such a one whose Denominator is the same, *ACub.* and the Numerators (by comparing the homo-

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486 *The* ELEMENTS Book IV.

logous Terms) are to be taken analogous; from the Comparison of which will be made out the Ratio of G to F , or the requir'd Motion of the Apfes. Let the Law of the centripetal Force be given, as, for Example, let it be as a Power of the Distance from the Center, whose Exponent

p is given, or as A^p , that is as $\frac{A^p + 3}{A^3}$. Instead

of $p + 3$ given, put n , and instead of A in the Numerator $T - X$; and the centripetal Force will

be as $\frac{T - X^n}{A^3}$. Let $T - X^n$, the Numerator of

this Fraction, be reduc'd to an infinite Series,

namely, to $T^n - n \times T^{n-1} X + n \times \frac{n-1}{2} \times T^{n-2} X^2 - \&c.$

Let the Terms of this Numerator be compar'd with the Terms of the foregoing Numerator, viz. $T \times F^q - X \times F^q + R \times G^q - R \times F^q$; that is, the given Quantity with the given, and the not given (or those in which the undetermin'd Quantity X is found) with the not given; and $T \times F^q + R \times G^q - R \times F^q$ will be to T^n as $-F^q \times X$

to $-n \times T^{n-1} X + n \times \frac{n-1}{2} \times T^{n-2} X^2, \&c.$ that

is, as $-F^q$ to $-n \times T^{n-1} + n \times \frac{n-1}{2} \times T^{n-2} X, \&c.$

And because the Ellipsis in question is nearly circular, in which half the *Latus transversum*, and half the *Latus rectum*, or T and R , are equal, and all the Altitudes or Distances from the Center are nearly the same, therefore X , in respect of them, is almost equal to nothing; that is, the Terms affected by that vanishing; whence, by the mutual Destruction of the Terms, and the Neglect of those that vanish, the first Analogy will be chang'd into this, $R \times G^q$ is to T^n as $-F^q$ is to $n \times T^{n-1}$; and

Book IV. of ASTRONOMY. 487

and still putting T in the Room of R in the first Term, and $T \times G^q$ will be to T^n as F^q to $n \times T^{n-1}$; therefore G^q will be to T^{n-1} as F^q to $n \times T^{n-1}$; and again, G^q to F^q as T^{n-1} to $n T^{n-1}$; that is, as 1 to n : Therefore G will be to F (that is, the Angle ATI to the Angle ATL) as 1 to $\sqrt{n}$. Wherefore, since the Angle, which the Body in the unmoveable Ellipsis describes, in its Descent from the upper to the lower Apfis, is equal to two Right Angles; the Angle, which the Body (that is urg'd by a centripetal Force proportional to a Power of the Altitude, whose Exponent is p or $n-3$) moving in an Orbit nearly circular, describes in its Descent from the upper to the lower Apfis, will be equal to two Right Angles; the Angle, which a Body, (which is acted upon by a centripetal Force proportional to a Power of the Altitude whose Exponent is p or $n-3$) descending from its upper to its lower Apfis, in an Orbit nearly circular, describes, will be to two Right Angles as 1 to $\sqrt{n}$; and so in an Arithmetical Phrase equal to $\frac{180 \text{ deg.}}{\sqrt{n}}$: And an equal An-

gle being repeated, a Body will return from its lower to its upper Apfis; and so on continually.

And, on the contrary, if the centripetal Force be as any Power of the Height, that Power may be found from the Motion of the Apfes; viz. if the angular Motion, by which a Body descends from the upper to the lower Apfis, or the whole angular Motion by which a Body receding from an Apfis returns to it, be to the angular Motion of a whole Revolution (or to 360 deg.) as Unity is to the Number $\sqrt{n}$; then the centripetal Force will be as a Power of the Altitude, whose Exponent is $n-3$. For, if the centripetal Force be as A^{n-3} , by

488 The ELEMENTS Book IV.

the first Part of this Proposition, there will be the said Ratio between the angular Motions, or between G and F ; and therefore this Law of the centripetal Force will depend upon that Proportion, between G and F . Whence the centripetal Force requir'd is found. Q. E. F.

COROLLARY

We will give some Examples of this Proposition in some of the most remarkable Cases. The Angle is requir'd, which a Body makes in its Descent from its upper to its lower Apfis, in an Orb nearly circular, whilst it is urg'd by an uniform centripetal Force, or a Force which is the same in all Altitudes. Because in this Case the centripetal Force is constant, either as A^0 , or as A^{3-3} , n is $= 3$, and $\sqrt{n} = \sqrt{3}$; and so the Angle describ'd by a Body moving from its upper to its

lower Apfis, is $\frac{180 \text{ deg.}}{\sqrt{3}}$ or $103 \text{ deg. } 55'$, and that

Angle being again describ'd, it returns to the upper Apfis, the Apfides moving swiftly in *antecedentia*. But if the centripetal Force be as the Altitude itself, or A^1 , that is, as A^{4-3} ; n will be $= 4$, and the Angle between the upper and the lower Apfis $\frac{180 \text{ deg.}}{\sqrt{4}}$ or 90 deg. therefore

the Apfides of such an Orbit are carried in *antecedentia*, with the same angular Celerity, as the Body describing the Orb, is carried in *consequentia*. Likewise, if the centripetal Force be

as $A^{\frac{11}{4}}$ inversely, or as $A^{\frac{11}{4}}$, or as $A^{\frac{1}{4}-3}$; n will be $= \frac{4}{3}$ and $\sqrt{n} = \frac{2}{\sqrt{3}}$; therefore the Angle between the upper and the lower Apfis, which is $\frac{180 \text{ deg.}}{\frac{2}{\sqrt{3}}}$, is 360 deg. that is, a Body which

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Book IV. of ASTRONOMY. 489

is urg'd by such a Force, as it recedes from its upper Apsis, and then continually descends, will come to its lower Apsis when it has made one whole Revolution ; then by a constant Ascent, as it goes thro' another whole Revolution, it will return to the upper Apsis, and so on constantly

Suppose, on the Contrary, that the Motion of the Apses is given, and the Law of the centripetal Force be requir'd : for Example, let the Law of that centripetal Force be requir'd by which that Body is acted upon, which in one Revolution and a half, by an alternate Descent and Ascent, from the upper Apsis, returns again to the upper Apsis. Because in this Case, the angular Motion, by which a Body descends from the upper Apsis to the lower, is 270 Deg. the fourth Proportional to 270 Deg. 180 Deg. and 1, or $\sqrt{n}$, will be equal to $\frac{2}{3}$, and therefore $n = \frac{4}{9}$: and so (by the second Part of this Proposition) the centripetal Force is as $A^{\frac{4}{9}-4}$; that is reciprocally as $A^{-\frac{4}{9}}$ or $\frac{1}{A^{\frac{4}{9}}}$. Again, let the Law of that centripetal Force be requir'd, by which that Body is acted upon ; which, from the upper Apsis, to the upper Apsis again, makes 363 Deg. that is, which is carried in an Orbit almost circular, whose upper Apsis, every Revolution of the Body goes thro' three Degrees *in consequentia*. Because the whole angular Motion, by which a Body receding from the upper Apsis, goes back to it again, is to the angular Motion of 360 Deg. as 363 to 360 ; $\sqrt{n}$, or the fourth Proportional to 363, 360 and 1 will be $\frac{60}{181}$; therefore n will be $\frac{177}{181}$: and the centripetal Force as $A^{\frac{177}{181}-4}$ — 3, or as $A^{-\frac{60}{181}}$; that is, inverfly as $A^{\frac{60}{181}}$, or inverfly as $A^{\frac{1}{3}}$. Therefore the centripetal Force, by which a Body is urg'd,

490 *The* ELEMENTS Book IV.

urg'd, that describes an Orbit, whose Apfes, in one Revolution of the Body, are mov'd 3 deg. *in consequentia*, as the Body recedes from the Center, decreases in a Ratio something greater than the Duplicate, but 60 Times nearer to a Duplicate than a Triplicate.

COROLLARY.

Hence it is plain, that if the centripetal Force, by which the Body is urg'd in its Approach to the Center, encreases in a Triplicate, or a less Ratio of the Altitude diminish'd, the Body, if it begins to descend, will never come to the lower Apfis, but descend quite to the Center; but if, from the Apfis, it ascends ever so little, it will never come to the upper Apfis, but continue to ascend *in infinitum*. For because the centripetal Force is reciprocally as A^3 , that is as A^{-3} , that is $p = -3$; $n (= p + 3)$ will be $= 0$, and $\sqrt{n} = 0$; and the Angle, by the Description of which, a Body will go from the upper to the lower, or from the lower to the upper Apfis, will be equal to $\frac{180 \text{ deg.}}{0}$, or be in-

finite: Therefore it will never come to the other Apfis, but, after innumerable Revolutions, it will either descend to the Center, or ascend *in infinitum*. But if the Force, in the Recess of the Body from the Center, decreases in a Ratio of the Distance greater than Triplicate, n will be a negative Quantity, and $\sqrt{n}$ will be a Thing impossible; whence the Truth of the Proposition is plain. And, on the contrary, it appears, that if a Body ascending and descending, alternately, from Apfis to Apfis, never comes to the Center; the Force, as you recede from the Center, will either be encreas'd or diminish'd in a less Ratio of

Book IV. of ASTRONOMY. 491

of the Height than a triplicate one: And, by how much less the Angle is, which is describ'd by the Body, whilst it goes from Apfis to Apfis, by so much more will that Ratio of the Forces differ from a triplicate reciprocal Ratio.

PROPOSITION VII. LEMMA.

IN elliptical Orbits, which are nearly circular, to determine the Motion of the Apses from the given Law of the centripetal Force, supposing the centripetal Force to be as the Sum or Difference of any two Powers of the Altitudes.

Things being as before, let m , b and C be any given Quantities, of which m is the Exponent of the Power, and also n , as in the foregoing Prop. and let the given Law of the centripetal Force be such, that it may be every where as $\frac{bA^m + CA^n}{A^3}$; or else putting $T - X$ instead of A ,

as $b \times \frac{T - X^m + C \times T - X^n}{A}$. Reducing this Value

to an infinite Series, the centripetal Force will be as $bT^m - bT^{m-1}X + b m \times \frac{m-1}{2} T^{m-2} X^2 +$

$CT^n - CnT^{n-1}X + Cn \times \frac{n-1}{2} T^{n-2} X^2, \&c.$

$\frac{A^3}{A^3}$
We must lay down, as analogous, this Value of the centripetal Force, and the Value, before found, of that Force which makes a Body move in a moveable Ellipsis, $\&c.$
 $\frac{T \times F^q - X \times F^q + R \times G^q - R \times F^q}{A^3}$; that is,

when these Values have hitherto the same Denominator, the given Part of one Numerator is to have the same Ratio to its Part not given, as the given Part of the other Numerator, to its Part

492 *The ELEMENTS* Book IV.

Part not given; that is, $R \times G^q - R \times F^q + T \times F^q$ must be to $-F^q \times X$, as $bT^m + CT^n$ is to $-$

$$b \times m T^{m-1} X + b \times m \times \frac{m-1}{2} T^{m-2} X^2 - C \times n$$

$$T^{n-1} X + C \times n \times \frac{n-1}{2} \times T^{n-2} X^2, \text{ \&c. And as}$$

the Orbits are suppos'd almost circular, T and R will be equal. Besides, by applying the Consequents to $-X$, and neglecting the Terms affected with X , as being very small, (because X is infinitely little) $T \times G^q$ will be to F^q as $bT^m + CT^n$ to $b \times m \times T^{m-1} + C \times T^{n-1}$. Or, to make it more simple, that all the Powers of T may become equal, let T be put for Unity; then G^q will be to F^q as $b + C$ to $b m + C n$. And for that Reason G to F , or (as in the Fig. of *Prop. 4.*) the Angle ATl to the Angle ATL as

$$\sqrt{b + C} \text{ to } \sqrt{b m + C n}, \text{ or as } 1 \text{ to } \sqrt{\frac{b m + C n}{b + C}}.$$

And therefore when the Angle, between the upper and lower Apfis, in an immovable Ellipsis becomes equal to two Right ones; the Angle which the Body (by a centripetal Force, proportional to $\frac{b A^m + C A^n}{A^3}$) describes in its Descent from

its upper to its lower Apfis, or contrary Ascent, is to two Right Angles; as 1 to $\sqrt{\frac{b m + C n}{b + c}}$;

and, in an Arithmetical Phrase, that Angle is $\sqrt{\frac{b + C}{b m + C n}} \times 180 \text{ deg.}$ If the centripetal Force

be as the Difference of Two Powers of the Altitude, (namely, as $\frac{b A^m - C A^n}{A^3}$) putting

$-C$, in the Room of $+C$, in the Value already found, the Angle between the upper and lower Apfis,

Book IV. of ASTRONOMY. 493

Apsis, of a moveable Orbit nearly circular, is equal to $\sqrt{\frac{b-C}{bm-Cn}} \times 180^\circ$. The same Way will the Problem be solv'd in more difficult Cases.

SCHOLIUM.

If a Body acted on, by a centripetal Force, which is, reciprocally, as the Square of the Altitude, be revolv'd in an Ellipsis, that has its Focus in the Center of immoveable Forces, and to this centripetal Force be added, or from it be taken any other Force, whose Law and Quantity is given; the Motion of the Apsis, arising from that given Force, will be determin'd by

this Proposition, viz. if from the Force $\frac{1}{A^2}$ be taken an extrinfecal Force, which is as the Altitude or Distance from the Center, namely, as CA ; the remaining Force, by which the Body is urg'd, will be $\frac{A-CA^2}{A^3}$, where $b=1$, $m=1$, $n=4$: Therefore the Angle of the Revolution, between the Apfes, is $\sqrt{\frac{1-C}{1-4C}} \times 180^\circ$. For Ex.

Let the extrinfecal Force, which is taken away, be 357,45 Times less than the primary Force, by which the Body is revolv'd in an Ellipsis, which, to make it easy, we have call'd 1; and

C will be $= \frac{1}{357,45}$ or $\frac{100}{357,45}$, and $1-C = \frac{35645}{35745}$ and $1-4C = \frac{35345}{35745}$; and so $\frac{1-C}{1-4C} = \frac{35645}{35345} = 1,008487754$, and $\sqrt{\frac{1-C}{1-4C}} = 1,004234$. And

therefore $\sqrt{\frac{1-C}{1-4C}} \times 180$ deg. or the Angle between the Apfis, in a moveable Orbit, will be $1,004234 \times 180$ deg. or 180 deg. 76228, that is, 180 deg.

494 *The* ELEMENTS Book IV.

180° 45' 44". Therefore the Motion of the Apſes is found in one half Revolution, or whilſt the Body deſcends from the upper to the lower Apſis, viz. 45' 44" *in conſequentia*, and ſo much does the Body go forward, when it aſcends from the lower to the upper Apſis; that is, each Revolution of the Body the Apſis is carried *in conſequentia* 1° 31' 28".

And, on the contrary, if the Angle between the Apſes be given, you may find what Ratio the extrinſecal Force (whoſe Law is given) has to that Force, by which a Body wou'd be mov'd in an immoveable Ellipſis; as ſuppoſe, if a Body return'd from the upper Apſis to the upper Apſis again, makes, by its angular Motion, an Angle of 315°, the Apſis at the ſame Time moving 45° *in antecedentia*, and the Ratio of that ſuperadded Force (which is as the Altitude) to the Primary Force, by which a Body wou'd move in an unmov'd Ellipſis, be requir'd. For, becauſe, by this Propoſition it has been demonſtrated, that the Angle between the upper and the lower Apſis, is

$$\sqrt{\frac{b+Cn}{bm+Cn}} \times 180^\circ, \text{ or that the Angle between}$$

the upper, and the upper Apſis, is $\sqrt{\frac{b+C}{bm+Cn}} \times 360^\circ$, that is (in the preſent Caſe, where the

ſuperadded Force is as the Altitude) $\sqrt{\frac{1+C}{1+4C}}$

$$\times 36^\circ; \text{ therefore } 315^\circ \text{ will be } = \sqrt{\frac{1+C}{1+4C}} \times 360^\circ$$

And, by reſolving this Æquation, its Value C will be found $\frac{1}{4}$. But as this primary Force, by which a Body wou'd move in an immoveable Ellipſis, be ſuppos'd 1; this Force will be to the added Force (which is as the Altitude, and makes

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Book IV. of ASTRONOMY. 495

makes the Apfes go back fo fwiftly) as 44 to 5.

COROLLARY.

From the foregoing Scholium, and the 2d Corollary of *Prop. 5.* it follows, that if from a Force, which is in a duplicate Ratio of the Altitude inverfly, be fubtracted a Force which is directly as the Altitude, there will be left a Force, which, as you recede from the Center, decreases more (or is lefs) than in a duplicate Ratio of the Altitude encreas'd, or a Force which, as you approach to the Center, encreases more (or is greater) than in a duplicate Ratio of the Altitude diminish'd; for as well in that Cafe as in this, the Apfes of the Orb go forward, and for that Reason the two Cafes become the fame. And it appears by the fame Reasoning, that if to a Force, which is in a Duplicate reciprocal Ratio of the Distance from the Center, be added an extrinfecal Force, which is directly as the faid Distance, the Sum of 'em will be a Force, which, as you recede from the Center, decreases lefs (or is greater) than in a duplicate Ratio of the Distance encreas'd, or (what is the fame) which, as you come towards the Center, encreases lefs (or is lefs) than in a duplicate Ratio of the Distance diminish'd. But thefe alfo are an immediate Confequence of *Cor. 1. Prop. 4.*

PRO-

PROPOSITION VIII.

IF a Secondary Planet *L* describe an Elliptic Orbit, whose Focus is *T*, about the Primary Planet *T*; its greater Axis, or Line of the Apses, by its angular Motion, goes forward twice, and backward twice, every Revolution of the Satellite *L*; that is, it goes forward twice, when *L* is in the Syzygies *G* and *B*, but it goes back, when *L* is in *C* and *D* the Quadratures with the Sun. [Fig. 10.]

Let *S* denote the Sun, *T* a Primary Planet, moving in the Orbit *DC*, from *D* towards *C*, about the Sun, *L* a Secondary Planet, describing an Elliptic Orbit, about the Focus *T*, from *C* to *G* towards *D*. Now, because in these Three Bodies, the accelerating Attractions of any Two, on the Third, are, (by what has been shewn in *Book I.*) to one another, reciprocally, in a duplicate Ratio of the Distances from it; this brings us to the Case of the first Proposition of this Book (whose Construction we here suppose) in which it is shewn, that the Satellite *L* is urg'd by a double accelerating Force, the one from *L* to *T*, and the other from *L* to *S*. And as the first Force from *L* to *T*, arising from the mutual Attraction of the Bodies *L* and *T*, is reciprocally proportional to the Square of the Distance (by *Corol. 2. Prop. 51. Book I.*) the Body *L* will, by that Force alone, describe an Ellipsis, which has its Focus in *T*. But, because the last Force, by which *L* is acted upon, and which is directed from *L* to *S*, (and is, by what has been before shewn in *Prop. 1.* as the Right-line *AS*) is resolv'd into Two Forces, one of which is as *AM*, and the other as *MS*, which is always reduc'd to *MN*, as has been shewn, and in the direction of these Right-lines; of which that which is as *AM*,
tho'

Book IV. of ASTRONOMY. 497

tho' it has the same Direction with the first, by which $\mathcal{T}$ draws L , yet it does not observe the same Law; neither is AM reciprocally as the Square of the Right Line $L\mathcal{T}$, which is the Distance of the Body L from $\mathcal{T}$, but almost directly as $L\mathcal{T}$; for as $L\mathcal{T}$ encreases, things remaining as before, AM encreases almost in the same Ratio; for SL , by reason of the Distance of the Body S , is almost parallel to $S\mathcal{T}$;] the Force by which L when it is in the Quadrature C and D is urg'd toward $\mathcal{T}$, being compounded of two [namely, that first by which $\mathcal{T}$ draws L , and which is inversly as the Square of the Distance $L\mathcal{T}$, and that which being as AM , is almost directly as the very Distance $L\mathcal{T}$, for the third Force MN in that Case is nothing] is not the Square of the Distance inversly, but as you recede from $\mathcal{T}$, decreases in a less Ratio than a duplicate of the Distance encreased: When therefore to the Force, which is reciprocally proportional to the Square of the Distance from the Center or Altitude, namely, that by which L draws $\mathcal{T}$, an extrinssecal Force, which is as the Altitude itself, is added, there will (by *Cor.* of the foregoing *Prop.*) be compounded a Force by whose Action the Body describes an Ellipsis, whose Apses are mov'd *in antecedentia*. Therefore when the Satellite is in the Quadratures, the Apses of the Elliptic Orbit go backward.

But if the Satellite L (*Fig.* XI.) be in G the Conjunction, or B the Opposition with the Sun S , where the Force AM vanishes; besides the Force by which $\mathcal{T}$ draws L ; and which is reciprocally as the Square of the Altitude $\mathcal{T}L$; the Body L will be urg'd by the Force MN , tending from M to N , which is not as the Square of the Altitude $\mathcal{T}L$ inversly; but encreasing as

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the

498 *The* ELEMENTS Book IV.

the said Altitude encreases, and almost as TL itself directly. Therefore when from a Force reciprocally proportional to the Square of the Altitude, by which T draws L , is taken away an extrinsecal Force (*viz.* a Force arising from the Sun's Action) which is almost as the Altitude itself; there will (by the Corollary of the foregoing Proposition) be left a Force by whose Action a Body is carried in an Ellipsis, whose Apfes are mov'd *in consequentia*. Therefore when a Secondary Planet is in the Syzygies with the Sun, the Apfes of the Elliptic Orbit describ'd by it go forward.

But when the Place of the Satellite is between the Syzygies and the Quadratures with the Sun, the Motion of the Apfes depends upon the two foremention'd Causes jointly; namely, upon an extrinsecal additional Force, as AM , and an ablative one as MN ; in such Manner that the Apfes, according to the Excess of the one or the other, go forward or backward: Therefore in each Revolution of the Satellite L about T , the Apfes by Turns go forward twice, and backward twice.

PROPOSITION IX. LEMMA.

IN Fig. XI. the Force MN in the Syzygies is almost twice greater than the Force AM in the Quadratures.

In the Syzygies, SM is to SN , in a Duplicate Ratio of SN to SL : but the Distance of the Points N and T , when compar'd with ST the Distance of a Planet from the Sun, vanishes, therefore ST is equal to SN ; and so SM is to ST in a Duplicate Ratio of ST to SL : But Quantities Arithmetically proportional as SB , ST , SG , &c. when their Interval BT or TG is very little in respect of any one of 'em as SG ,
are

Book IV. of ASTRONOMY. 499

are also as it were Geometrically Proportional, and contrariwise ; wherefore if you suppose $B M$ (in the first *Fig. XI.*) and $G M$ (in the second *Fig. XI.*) equal to $T B$, you will find that (in the first *Fig. XI.*) $M S$, $B S$, $T S$, and $L S$, and (in the second *Fig. XI.*) $M S$, $G S$, $T S$, $L S$, will be nearly in a continued Geometrical Proportion: Therefore in each Figure $M S$, is to $T S$ in a Duplicate Ratio of $T S$ to $L S$. But $M T$ is (by Construction) twice greater than $T B$, which (by what foregoes) is equal to $A M$ in the Quadratures ; and $M N$ is almost equal to $M T$. Therefore $M N$ in the Syzygies, is nearly twice greater than $A M$ in the Quadratures. Q. E. D.

PROPOSITION X.

IN every Revolution of any Satellite the Apses for the most part go forward more than they go backward, and by the Excess of their going forward, are carried in consequentia.

For the Force $M N$ in the Syzygies, which being taken away from the Force with which T draws L , is the Cause that the Apses go forward, is (by the foregoing Lemma) nearly twice greater than the Force $A M$ in the Quadratures, by whose Addition to the said Force, by which T draws L , the Apses go backward : And therefore the Excess in one whole Revolution of the Satellite will be within the Power of $M N$, and will transfer the Apses in consequentia. Q. E. D.

These will happen so commonly, and *cæteris paribus* : But there are particular Cases, (as will be shewn in the next *Prop.*) in which, in one Revolution of the Satellite about its Primary Planet, the Apses go backward more than they go forward.

PROPOSITION XI.

IN one Revolution of the Satellite *L* about *T*, the Progress of the ApSES exceeds their Regress, when they are in the Syzygies with the Sun, more than if they were farther from it: But the Regress of the ApSES exceeds their Progress in one Revolution of the Satellite, when the ApSES are in the Quadratures with the Sun: Yet they go forward more and swifter in their Syzygies, but they recede less and slower in the Quadratures; and by the Excess of the Progress above the Regress, in one Revolution of the ApSES about the Sun, are carried in consequentia, [Fig. XI. and XII.]

Because (by *Cor. 2. Prop. 5.*) the Progress or Regress of the ApSES depends upon the Centripetal Force of the Satellite towards its Primary Planet, which decreases more or less than in a Duplicate Ratio of the encreas'd Distance as you recede from the Center, or in the Passage from the lower to the upper ApSES, as also upon a contrary Increment as you go towards the Center, or return to the lower ApSES; it is plain that the Progress or Regress is the greatest, when the Ratio between the Forces in the ApSES differs very much from an inverse Duplicate Ratio of the Distances. Suppose the Situation of the Orbit of a Satellite in respect to the Sun, to be such, that its ApSES may be in the Syzygies with the Sun: Therefore the extrinfecal Force *MN*, which urges the Satellite when it is in the Syzygies, being taken away from the Force by which the Primary Planet draws the Satellite, leaves a Force in the Satellite at *B*, whose Ratio to the Force likewise left in the Satellite at *G*, differs more from a Duplicate Ratio of the Distances inversly, than if the ApSES were not in the Syzygies; because the Ratio between the Distances *TB*, *TG*,
is

Book IV. of ASTRONOMY. 501

is more unequal, or differs more from the Ratio of Equality, than the Ratio between any other two in this Orbit: Therefore the Progress of the Apfes in this Case is the greatest. But when a Satellite is come to C , the extrinfecal Force AM being added to the Force by which T draws the Satellite, does not so much make it recede from a Duplicate inverse Ratio of the Distances in the Passage from C to D ; because the Distances TC , TD become equal: Therefore the Effect of this added Force is less; that is, the Apfes of the Orb go back less in this Case. Therefore, because when the Apfes are in the Syzygies, they go forward very swiftly when the Satellite goes thro' the Syzygies, and go back very slowly when the Satellite is in the Quadratures with the Sun; it is plain, that in one whole Revolution of the Satellite about the Primary Planet, the Apfes in this Case will be carried, *in consequentia*, very swiftly.

Now let the Apfes of the Satellite's Orb be in the Quadratures: in which Case, the foresaid extrinfecal Force AM , which acts upon the Satellite in the Quadrature C , being added to the Primary Force by which T draws the Satellite, makes up a Force compounded of both, by which the Satellite, when at C , tends towards T ; (*Fig. XII.*) whose Ratio to a Force compounded after the same Manner, by which a Satellite, when at D , is urg'd towards the said T , recedes more from a Duplicate reciprocal Ratio of the Distances, than if the Apfes were not in the Quadratures; because the Ratio between TC and TD is more unequal than the Ratio between any other two of this Orbit; therefore the Regress of the Apfes is the greatest in this Case. But when the Satellite is come to G or B , the

502 *The* ELEMENTS Book IV.

extrinfecal Force MN now urging the Satellite, being taken from the Force by which T draws the Satellite, does not so much make it recede from a Duplicate inverse Ratio of the Distances, whilst the Planet goes from G to B , and so *è contra*. Because the Distances TG , TB , become equal; whence the Effect of this ablative Force is less in making the Apfes go forward; that is, the Apfes of the Orb go forward slower. Therefore, from these two Causes, when the Orb is situated as here, in respect to the Sun, the Apfes of the Orb of a Satellite (whilst it makes one entire Revolution about its Primary Planet) are carried backward.

Besides, if the Motion of the Apfes of the Satellite about T , and the Motion of the Primary Planet T about the Sun S be such, that the Apfes of the Satellite's Orbit are nearly in the same Aspect with the Sun for a considerable Time, or whilst the Satellite itself changes all its Aspects with the Sun often; by reason of the Length of Time in which the Velocity of the Progress and the Slowness of the Regress of the Apfes are continu'd, namely, when the Apfes stick about the Sun's Syzygies, they will go forward remarkably enough: and likewise, by reason of the Length of Time that the Apfes of the Satellite stick near the Sun's Quadratures, and consequently by which the Velocity of the Regress and the Slowness of the Progress are continu'd, they will recede sensibly enough.

But because the extrinfecal ablative Force MN , that causes the Progress of the Apfes, has in *Prop. 9.* been shewn to be almost twice greater than the Force AM , which makes them go back; and besides, the Apfes stick longer in the Syzygies than in the Quadratures, because
being

Book IV. of ASTRONOMY. 503

being there carried *in consequentia*, they go forward with the Sun, but here being carried *in antecedentia*, they quickly pass thro' the Sun's Quadrature, which is carried *in antecedentia*; it is plain that the Apfes go forward faster, and for a longer Time in their Syzygies, and go back slower and not so long in their Quadratures; and, by the Excess of the Progress above the Regress, in one whole Revolution of the Apfes to the Sun (that is, whilst the Apfes seen from T go thro' the Aspects with the Sun) they are carried *in consequentia*.

All that has been said will almost happen the same Way, tho' the Species of the Secondary Orbit don't precisely continue the same, if so be that the Change of the Excentricity is not too great.

PROPOSITION XII.

IF a Satellite moves about its Primary Planet in an excentric Orbit, the Excentricity of it will be twice chang'd in every Revolution: And in one Revolution of the Satellite it will be the greatest, when the Satellite is in the Syzygies with the Sun; but the least, when in the Quadratures: And in the Passage of the Satellite from the Quadratures to the Syzygies, it is perpetually encreas'd, and, on the Contrary, in its Passage from the Syzygies to the Quadratures, it is perpetually diminish'd.

For if the Body which did describe a given Ellipsis, (namely, by being acted upon by a Centripetal Force, which, tending to one of the Foci or to the Center, is reciprocally proportional to the Square of the Distance from it) in its Descent from the upper to the lower Apfis, be urg'd by a Centripetal Force, which is encreas'd more than in a Duplicate Ratio of the

diminish'd Altitude or Distance from the Center ; it is manifest, that a Body, thus urg'd towards the Center, wou'd be more strongly push'd towards it, than if (as before) it was urg'd by a Force encreasing only in a Duplicate Ratio of the diminish'd Distance ; therefore it wou'd describe an Orb falling within the Elliptic Orbit before describ'd (except in the upper Apfis) and in the lower Apfis, it wou'd come nearer to the Center than before ; that is, it wou'd describe an Orb more excentric than the forementioned Orb, or such an one in which the Distance of a Body in the upper Apfis has a greater Ratio to its Distance in the lower Apfis, than in the former Orb : Now if the Force, in the Ascent of the Body from the lower Apfis, shou'd decrease with the same Degrees by which it increas'd in the Descent ; the Body wou'd return to the former Distance. But if the Centripetal Force shou'd decrease more than in the same Ratio, the Body then being less attracted, will ascend to a greater Distance, and describe an Orb, whose Excentricity is yet greater. Likewise, if a Body in its Descent from the upper to the lower Apfis be urg'd by a Force, which is increas'd less than in a Duplicate Ratio of the diminish'd Distance ; it is plain that the Body will describe an Orb falling within the Elliptic Orb before describ'd, (when the Force was reciprocally as the Square of the Distance) except in the upper Apfis, from whence it did set out, that is, less excentric ; and that that Excentricity will yet be diminish'd, if in the Ascent of the Body the Centripetal Force decreases less, or slower than before.

Let the Satellite *L* in the former Figures and in *Fig. XIV.* be suppos'd near the Quadrature in *C*, in which Case the Force *M N* vanishes, and
besides

Book IV. of ASTRONOMY. 505

besides the Force by which the Primary Planet T draws the Satellite L , which is reciprocally as the Square of the Distance, the Satellite will be urg'd by a Force as AM , which is as the Distance LT directly. And the Force made up of these (by *Cor. Prop. 7.*) as you recede from the Center, decreases less than in a Duplicate Ratio of the encreas'd Distance, and, as you go to the Center, encreases less than in a Duplicate Ratio of the diminish'd Distance : Therefore if the Body, during one whole Revolution, is impell'd by this Force towards the Center, it wou'd describe an Orb less excentric than if that extrinfecal Force AM was taken away, and it was urg'd only by the Force with which T draws L , which is inverfly as the Square of the Distance ; and therefore, in that Part of the Revolution, where it is impell'd by that Force, it describes a Part of an Orbit less excentric, than if the said Force AM was away.

If the Satellite be in the Syzygies, as in B or G , when the Force AM vanishes, the Force MN is taken away from the Force by which T draws L ; that is, a Force which is as the Distance is taken away from a Force reciprocally proportional to the Square of the Distance ; therefore the remaining Force by which the Satellite is now urg'd (by *Cor. Prop. 7.*) as you approach to the Center, encreases more than in a Ratio of the diminish'd Distance, and decreases in the same Manner as you recede from the Center ; and besides, by what has been shewn, if the Satellite, during one whole Revolution, was impell'd towards the Center by that remaining Force, it wou'd describe an Orb more excentric than that which it wou'd describe, if it was urg'd only by the attracting Force

Force of the Body T ; and therefore much more excentric than that it wou'd describe, if it was urg'd by that Centripetal Force which acted upon it when it was in the Quadrature at C : Whence in that Part of its Revolution when the extrinfecal Force MN is the greatest (that is, in the Syzygies) the Satellite describes a Part of an Orbit more excentric than when it is in the Quadratures; or, in other Words, the Excentricity of an Orb describ'd by a Satellite, is greater when the Satellite is in the Syzygies, and less when it is in the Quadratures.

Besides, because the extrinfecal additional Force AM is greatest of all, and the Force MN is least, or none at all, when the Satellite is in the Quadratures, the Force made up of that and of the Force by which T draws L , is (*ceteris paribus*) greatest, and very much differs from a Duplicate reciprocal Ratio of the Distances; therefore in that Case, the Part of the Orb which is describ'd, will belong to an Orb very little excentric. When the Satellite is gone from C towards B , the extrinfecal ablative Force MN encreases also, and therefore exerts its Effects more and more; that is, makes the Satellite describe Part of an Orbit belonging to a more excentric one: wherefore as a Satellite goes from the Quadratures to the Syzygies, its Orbit becomes more and more excentric, till the Satellite being come to the Opposition or Conjunction with the Sun, the Orb becomes most excentric of all, the ablative Force MN then acting alone, and being greatest and most direct. But afterward, as the Satellite goes towards the Quadrature, the Excentricity begins again to diminish, because the Force AM , which diminishes it is encreas'd. Therefore the Excentricity of the Orb, as the Satel-

lite goes from the Quadratures to the Syzygies, is continually encreas'd ; and as it goes from the Syzygies to the Quadratures, is continually diminish'd : And therefore is the greatest (*ceteris paribus*) in the Syzygies, and the least in the Quadratures.

PROPOSITION XIII.

THE same Things being laid down as in the foregoing Prop. I say, that if several Revolutions of the Satellite be compar'd together, the Excentricity of the Orbit of the Satellite is greatest, when its Apses are in the Syzygies ; and from thence it is continually diminish'd when the Apses pass from the Syzygies to the Quadratures with the Sun, where it becomes the least ; but when the Apses go from the Quadratures to the Syzygies, the Excentricity continually encreases.

This is prov'd the same Way as the last Proposition but one. For (by Cor. Prop. 7.) the Forces of the Satellite in the Quadratures, encrease or decrease less than in a duplicate Ratio of the diminish'd or encreas'd Distance : Wherefore if the Apses of an Orb are in the Quadratures, the said Ratio of the whole Encrease or Decrease is the least in the Passage between the Apses. Likewise, the Forces of a Satellite in the Syzygies increase or decrease more than in a duplicate Ratio of the diminish'd or encreas'd Distance ; and therefore if the Apses of the Orbit happen to be in the Syzygies, the said Ratio of the whole Encrease or Decrease of the Forces is greatest of all between the Apses ; that is, the Ratio of the whole Encrease or Decrease of Forces is least when the Apses are in the Quadratures, and greatest when they are in the Syzygies ;

zygies; and therefore it continually encreases in every Revolution of the Satellite about the Primary Planet, whilst the Apfes go from the Quadratures to the Syzygies, and continually diminishes whilst they go from the Syzygies to the Quadratures. But it has been shewn by the foregoing *Prop.* that if in each Revolution of a Satellite, the Ratio of the Encrease or Decrease of the Centripetal Force be augmented, the Excentricity of the Orb will likewise be augmented; and on the Contrary, the Excentricity will be diminish'd, if that Ratio decreases: And therefore whilst the Apfes pass from the Quadratures to the Syzygies, the Excentricity of the Orb continually encreases, and, in their Passage from the Syzygies to the Quadratures, perpetually diminishes; therefore the greatest Excentricity of the Orb is when the Apfes are in the Syzygies, and the least when they are in the Quadratures. And by reason of the Length of the Time that the Apfes stick near the Quadratures or the Syzygies, the Encrease or Decrease of the Excentricity becomes very sensible, and the Inequality between the Excentricity in one Case, and the Excentricity in the other, is the greatest of all. Q. E. D.

SECTION

SECTION III.

Of the Errors which the Sun produces in the Motion of a Satellite, when the Plane of the Orbit describ'd by it is inclin'd to the Plane of the Orbit of a Primary Planet describ'd about the Sun.

IN the first Book, we have generally treated of the Errors of a Satellite in Latitude, viz. in the 61st Prop. and more particularly afterwards in the 64th Prop. when we shew'd the Reason of the Precession of the Equinoctial Points arising from the same Cause. Yet we'll take the Liberty to transfer some of those Things hither, as to their proper Place, and explain other Things more particular concerning these Errors.

PROPOSITION XIV.

IF a Satellite revolves about a Primary Planet, in an Orb whose Plane is inclin'd to the Plane of the Primary Orbit of the Planet about the Sun, the Line of the Nodes will move by an angular Motion in antecedentia, with an unequal Velocity; the greatest being when the Nodes are in the Quadratures with the Sun, and afterwards it will move slower, till the Nodes being in the Syzygies, it is altogether at rest. In the Places that are between the Quadratures and the Syzygies, the Nodes participating of both Conditions recede slower; therefore being always either Retrograde or Stationary in every Revolution of a Satellite about the Primary Planet, they are carried in antecedentia; and in the same Revolution of a Satellite they go back faster, cæteris paribus, when the Satellite is in the Syzygies.

Let

Let S (as often before) denote the Sun; (*Fig. XIV.*) ETE the Orbit of a Primary Planet T ; and $CB DG$ the Orbit of a Secondary Planet L about the Primary One: Let the right Lines SL , ST , be drawn, and if there be Occasion, let 'em be produc'd, and in them be taken the Points K , A , M , N , and let AM be drawn. It is plain from *Prop. 60. Book I.* that all the Errors of the Satellite L arise from the Forces of it that are as AM , MN , being in the Direction of those right Lines. Now the Force acting on L , according to the Direction of the right Line AM (that is, according to the right Line LT) does not disturb the Motion of L in Latitude, when LT is in the Plane of the Orb $CB DG$. Again, if the Direction of another Force, as MN , lies also in the Plane of the Orb $CB DG$ (that is, if a right Line, drawn thro' L and parallel to MN , lies in a Plane parallel to $CB DG$) neither will it then disturb the Motion of the Satellite in Latitude. But MN or ST , when it is in the Plane of the Orb SET , will also be in the Plane $CB DG$, if it be the common Intersection of those two Planes, namely, of the Orbits of the Primary and Secondary Planets; that is, if the Nodes be in the Syzygies. Therefore the Nodes being in the Syzygies, there will arise no Error of the Motion in Latitude, neither from the Force AM , nor from the Force MN , which are the only Forces that disturb the Motion of a Satellite; therefore there will be no Error in Latitude. Whence the Line of Nodes, and the Nodes also in this Case are at rest, when their Motion is an Error arising from some disturbing Force.

But when the Nodes of the Orbit $CB DG$ [*Fig. XV.*] are in the Quadratures C and D , the Force MN most of all disturbs the Satellite's Motion

Motion in Latitude ; because in this Case, the Line MN (and LF drawn thro' L parallel to it, along which the Force MN of the Satellite L exerts itself) is most of all inclin'd to the Plane of the Orbit $CB DG$, viz. as much as the Planes of the Orbs $ST E$ and $CB DG$ are inclin'd to one another. In this Case, let us suppose one Part of the Plane $CB DG$, viz. $CG D$, to be rais'd above the Plane of this Scheme (which agrees with the Plane of the Primary Orbit $ST E$) and the other $CB D$ to be depress'd below it : And let the Orbit $Cb Dg$, be suppos'd equal, similar, and alike posited, in respect of the Center T , to the Orb $CB DG$: Therefore in this Case, as soon as the Satellite L passes the Node C , that it may come to the Conjunction with the Sun in G , the Force MN , in the Direction of the right Line MN or its Parallel LF , which acts from L towards F , draws it from the Orbit $CG D$, which otherwise it wou'd describe, and receding continually towards S from the Plane $CG D$ to the next Node, it does not now pass thro' the Plane $ST E$ or $Cg D$ in the Point D , (thro' which it wou'd pass, if that Force was away) but a little on this Side of it in respect to the Sun S ; that is, the Node of an Orbit describ'd by a Satellite (when urg'd by the Force MN) is nearer to the Sun than the Point D , in which it was found in the foregoing Revolution of the Satellite.

Likewise, whilst the Satellite goes from the Node D , to its Opposition with the Sun in B , the Force MN acting in the Direction MN from M towards N , or from L towards F makes the Satellite L continually recede from the Orbit $DB C$, and describe an Orbit less inclin'd to the Plane SET , or DtC . And since this Force on
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the Satellite *L* exerts itself till it vanishes, namely, when the Satellite is come to the Quadrature in *C*, it also makes the Orbit truly describ'd by the Satellite sooner to intersect the Plane *b C g D* than the Orbit *D B C G*, or a little nearer to *b* than the Point *C*, and there makes the Node of the Orbit of a Satellite with that of its Primary Planet. And the same will again happen when the Satellite goes the second Time thro' the same Circle *C G D*; namely, the Node or Point in which the Satellite goes thro' *b C g D*, is in *antecedentia* in respect of the Point in which it had gone thro' it the Time before, and so on; that is, this Force *M N* forces the Satellite to describe an Orbit, whose Nodes go in *antecedentia* at every Revolution of *L* about *T*; but the Nodes go backward the fastest of all when they are in the Quadratures *C* and *D*, because (as it has been before shewn) the right Line *M N* or *L F* according to which the Force *M N* acts, which causes the Regress of the Nodes, is most of all at right Angles or perpendicular in that Case to the Orbit *C G D B*.

Besides, what is asserted of the Situation of the Nodes between the Quadratures and Syzygies, is self evident, because the right Line *M N* is in that Case more oblique to the Plane *CBDG*, and therefore has not so much Power to disturb the Satellite from it. And since the Regress of the Nodes depends upon the Force *M N*, that Regress is (*cæteris paribus*) greatest, when that Force is greatest, that is, when that Force is in the Syzygies, Q. E. D.

If the Inclination of the Orb of a Satellite to the Plane of the Orb of the Primary Planet about the Sun be chang'd, the Motion of the Nodes is not sensibly chang'd, unless the Change of the Inclination be too great; for the Force that gives

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Book IV. of ASTRONOMY. 513

Motion to the Nodes is not much chang'd in that Case.

PROPOSITION XV.

THE same Things being laid down, I say, that the Inclination of the Plane of the Orb of a Satellite to the Plane of the Orb in which the Primary Planet revolves about the Sun, is continually chang'd, and is the greatest when the Nodes are in the Syzygies with the Sun; and the least (*cæteris paribus*) when they are in the Quadratures: And besides, that in this Case, the said Inclination is diminish'd whilst the Satellite passes from the Quadratures to the Syzygies, and that it is again augmented, whilst the Satellite passes from the Syzygies to the Quadratures. Whence it happens, that when the Satellite is in the Syzygies, the Inclination of the Planes is the least, and returns nearly to the former Quantity when the Satellite is come to the next Node. I say moreover, that when the Nodes pass from the Syzygies to the Quadratures, that Inclination of the Planes diminishes, and becomes least of all (*cæteris paribus*) when the Nodes are in the Quadratures; and then that it encreases by the same Degrees that it decreas'd before; and the Nodes being again return'd to the Syzygies, it returns to the first Magnitude.

'Tho' all the Parts of this Proposition follow easily enough from the 61st Prop. B. 1. yet we shall in this Place explain them more fully, and demonstrate each of them by a particular Scheme.

First, let the Nodes of the Satellite *L* [Fig. XVI.] be in the Syzygies with the Sun *S*; that is, let the Line of the Nodes produc'd pass thro' the Sun; in which Case the right Line *MN* (which has been demonstrated in Prop. 61. B. 1.

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to shew by its Position the Direction, and by its Magnitude the Quantity of that Force, which only affects the Motion of a Satellite in Latitude) is in the Plane of the Orbit of the Satellite L : And therefore howsoever it disturbs the Motion of the Satellite in the said Plane, it does not impel it to any Motion out of the said Plane $BCGD$; that is, it does not at all affect the Motion of the Satellite in Latitude, but leaves the Satellite undisturb'd as to its Motion in Latitude, to describe his Orbit about its proper Primary Planet. And as that disturbing Force MN by its Nature always diminishes the Inclination of the Orb of the Satellite to the Orb of the Primary Planet, and never encreases it but by Accident, upon Account of its making the Nodes to go *in antecedentia*; it is plain, that the Inclination of the said Orbs is greatest of all, when the Nodes are in the Syzygies.

Now supposing the Nodes to be in the Quadrature with the Sun, as soon as the Satellite has pass'd the Node C or D , being impell'd by the Force MN , and according to the Direction LF , from L to F , it is continually drawn from the Plane of the Orbit $CGDB$ (which, without this, it wou'd describe) and describes an Orbit, whose Plane is less inclin'd to the Plane of the Orbit of the Primary Planet $CgDb$. That this may appear the plainer, let ST represent (in *Fig. XVII.* the Eye being suppos'd in the Line of the Nodes) the Plane of the Primary Planet's Orbit about the Sun, TL the Plane of the Secondary one, inclin'd to the Plane ST by the Angle LTS , or the Complement of it to two right Ones. The Satellite at L drawn by the Force MN from L towards F , that is, from the Place L to the Place I , describes an Orbit, whose
Plane

Plane TL is inclin'd by a less Angle lTS , to ST the Plane of the Orbit of the Primary Planet about the Sun. And this Change of the Inclination of the Orb of a Satellite, to the Orb of the Primary Planet, is (*cæteris paribus*) by so much greater as the Force MN which produces it is greater: But the Force MN is greater the nearer the Satellite is to the Syzygies G and B . Therefore the Nodes sticking at the Sun's Quadratures, the Inclination of the Orb of the Satellite is the least, when the Satellite is in the Syzygies. Again, because the right Line MN and its Parallel LF (*viz.* the Direction of the Force which disturbs and diminishes the Inclination) is nearer to a Perpendicular to the Plane of the Orbit of the Satellite, when the Nodes are in the Quadratures, than in any other Situation of the Nodes; in that Case, the Change of the Inclination will be the greatest, therefore the Inclination of the Planes will be the least: For the nearer the Angle FLT comes to a right One, the greater Angle does the little right Line Ll (of a given Length) subtend. Therefore the Inclination of the Orbit of a Satellite, to the Orbit of its Primary Planet, will be least of all, when the Nodes are in the Quadratures, and the Satellite, at the same Time, in the Syzygies.

Moreover, let STb (*Fig. XVIII.* the Eye being suppos'd in the Line of the Nodes produc'd, as before) represent the Plane of the Orbit of the Primary Planet T revolving about the Sun; G TB the Plane of the Orbit of a Satellite, whilst it is inclin'd to the former Orb by this small Angle, (*viz.* GTg , or BTb) which happens (as has been shewn) when the Satellite in the Syzygies with the Sun, is seen from T . And since it has

been shewn, in the foregoing *Prop.* that the Nodes of the Orbit of a Satellite go *in antecedentia*, the Orbit which the Satellite, going from G , (when seen from T , it was in Conjunction with the Sun) describes, will meet with STB on this Side T , in respect of the Sun, and will be the Line Gd ; and the Orbit which it describes going from B , where it was in Opposition with the Sun, *viz.* the Line Bs , will meet with the Plane STb at s , *viz.* beyond T , in respect of the Sun. Now it is plain, that the Line Gd or Bs contains a greater Angle with STb at d and s , than that which GT or BT contains with the same Plane STb ; *viz.* one about equal to that which the Orbit of the Satellite contain'd with the Plane STb , when the Satellite was near a Node. Therefore when the Nodes are in the Quadratures, the Inclination of the Plane of the Orbit of the Satellite diminishes, as the Satellite passes from the Quadratures to the Syzygies; and, on the contrary, encreases as the Satellite passes from the Syzygies to the Quadratures; and returns to the first Quantity when the Satellite comes to the next Node.

Now, since it has been shewn that the Force MN (disturbing the Satellite's Motion in Latitude, and especially diminishing the Inclination of the Plane of the Orbit of the Satellite to the Plane of the Orbit of the Primary Planet) is the greatest when the Satellite is in the Syzygies, and least (because none at all) when it is in the Quadratures, and that it does most powerfully produce this Effect, (*cæteris paribus*) when the Satellite is in the *Limit*; it is plain, that by how much the *Limits* are nearer to the Quadratures when the Satellite is in the Syzygies, by so much less does this Force MN diminish the Inclination
of

Book IV. of ASTRONOMY. 517

of the said Planes, because its Quantity is none when the Satellite is in a Situation which is the most convenient for producing of this Effect, and its Quantity is greatest when, by reason of the Satellite's Situation, this Effect ceases. And on the other Hand, since it has been lately shewn that the Force MN , by bringing round the Nodes *in antecedentia*, does again encrease the said Inclination, and very powerfully encreases it, when the Satellite is near either of the Nodes, and that in the mean Time, the Force itself is greatest when the Satellite is in the Syzygies; it is plain, that by how much they are nearer to the Syzygies with the Sun, by so much more does the Force MN encrease the Inclination of the Planes. And therefore, since the *Limits* are equally near to or remov'd from the Quadratures, as the Nodes are to or from the Syzygies, it is plain, that by how much the Nodes are nearer to the Syzygies, by so much is the Inclination of the Plane of the Orbit of the Satellite, to the Plane of the Orbit of the Primary Planet greater, and so much less as the Nodes are nearer to the Quadratures. And because (by the foregoing *Prop.*) the Nodes are carried *in antecedentia*, this Inclination of the Planes is diminish'd, whilst the Nodes pass from the Syzygies to the Quadratures, and becomes least of all (*cæteris paribus*) when the Nodes are in the Quadratures; then it encreases by the same Degrees by which it had before decreas'd, and the Nodes being again got to the Syzygies, it returns to its first Quantity. Q. E. D.

PROPOSITION XVI.

ALL the Inequalities before describ'd, which happen in the Motion of a Satellite, as well those that affect its Motion in Length, as those that affect it in Breadth or Latitude; and as well those which happen to a concentric, as those that happen to an excentric Orbit, are something greater in the Conjunction of a Satellite with the Sun, than in the Opposition.

Because (by Prop. 25. El. 5.) the Sum of the greatest and least of four Quantities Geometrically Proportional, always exceeds the Sum of the two others; by taking away on either Side the Sum of the two least, the Interval of the greatest and next will exceed the Interval of the next greatest and the least. And because, (by the Construction of Prop. 14.) the Satellite [Fig. XIX.] being in Conjunction with the Sun at G , SM is to SN (by reason of the Smalness of NT in respect of SN) that is, to ST in a Duplicate Ratio of ST to SG ; the right Line MT exceeds the Double of TG : For that which is next to the greatest exceeds SB . Again, since when the Satellite is in Opposition, as at B , Sm is to ST , in a Duplicate Ratio of ST to SB ; the right Line Tm (for the same Reason) wants of being equal to twice TB : therefore TM much more will want of being equal to Tm , that is, NM exceeds Nm . But NM , Nm represent the Forces respectively disturbing the Satellite in the Conjunction and Opposition; and therefore the Effects of the former are greater than the Effects of the latter. And besides, since the Forces represented by NM , Nm (which are the only Forces that disturb the Motion of the Satellite when it is in the very

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Book IV. of ASTRONOMY. 519

Conjunction and Opposition) and that another Force, as AM (which disturbs the Satellite near the Syzygies) is greater in the Conjunction than in the Opposition, *viz.* in that greater than the Radius LT , and in this less; it is evident that all the Inequalities explain'd in the foregoing *Prop.* are (*cæteris paribus*) something greater near the Conjunction of a Satellite with the Sun, than near the Opposition with it. *Q. E. D.*

SCHOLIUM.

The foregoing Proposition wou'd always hold true, if the Orbit of the Satellite was concentric with the Primary Planet; otherwise it would only hold in an entire Revolution of the Apfes about the Sun. But the Motions of the Satellites, when nicely observ'd, seem to shew that the Center of the Circular Orbit of a Satellite is something farther from the Sun, than the Center of the Primary Planet; that so the Satellite may all along accompany the Primary Planet. For if the Center of the Satellite's Orbit was the same as the Center of the Primary Planet, the Satellite leaving the Primary Planet wou'd fall into the Sun. For the Satellite describing its Orbit, makes that Orbit to be look'd upon as an heavy Ring, and the Gravity of each Part of that Ring is to be look'd upon as the same with the Gravity of the Satellite in it, and that Place to be reckon'd heavy, which is the Center of Gravity of it. And, since by *Prop.* 16. the accelerating Gravity of a Satellite towards the Sun, over and above the accelerating Gravity of the Primary Planet towards it, is more encreas'd in the inferior Part of its Orbit near the Conjunction, than it is diminish'd in the superior Part of it near the Opposition, and the Satellite dwells as long in the one as in the other; the

Center of Gravity of the said heavy Ring or Orbit, and therefore the Orbit itself is urg'd by a greater accelerating Gravity to the Sun than the Primary Planet ; therefore leaving the Primary Planet, it will come nearer to the Sun; which also will much more be true, if the Center of the Circular Orbit be nearer to the Sun than the Center of the Primary Planet. But the Center of the Orbit is so plac'd by the All-wise CREATOR, according to the Distance of each Primary from the Sun, that the Center of Gravity of the said Ring (look'd upon as heavy) is as far distant from the Sun as the Center of the Primary Planet, lest the one shou'd be left by the other: This also holds good in Eccentric Orbs, the Gravity of each small Part of the Orbit being estimated by the Time that the Satellite stays in it.



SECTION

SECTION IV.

Of the Errors which the Sun produces in the Motion of a Satellite, when a Primary Planet is mov'd in an Excentric Orbit about the Sun.

PROPOSITION XVII.

IF, by encreasing and diminishing by turns the Distance between the Sun S and a Primary Planet T, the Action of S be encreas'd and diminish'd by turns; L T the Radius of the Orbit of the Satellite will at the same Time be encreas'd and diminish'd, and the Periodical Time of the Satellite, about the Primary Planet, will be encreas'd and diminish'd, in a Ratio compounded of a Sesquiplicate Ratio of the Radius, and the Subduplicate Ratio, by which the attracting Force T of the Primary Planet is diminish'd, or augmented by the Encrease or Decrease of the Sun's Action. (Fig. XX.)

The Force with which the Primary Planet *T* draws the Satellite *L* (namely that by which *L* is kept in its Orb *BCGD*) is encreas'd (as it has been often shewn above) when *L* is in the Quadratures *C* or *D*, by the Addition of an extrinfecal Force arising from the Sun *S*, which is as *AM*; and whilst *L* is in the Syzygies, that attracting Force of the Body *T* is diminish'd, by taking away a Force which is as *MN*. And because *MN* perpendicular in the Syzygies (by Prop. 9.) has been shewn, as it were, to be twice greater than *AM* in the Quadratures, by reason of these Extrinfecal Forces arising from the Sun, the attracting Force of the Primary Planet is less diminish'd

minish'd than it is encreas'd ; and therefore it must simply be look'd upon as diminish'd. And because the attracting Force of the Body $\mathcal{T}$ is that by which L is kept in its Orbit ; if it shou'd decrease by degrees, the Satellite L (being less and less attracted) would continually recede from $\mathcal{T}$; and, on the contrary, if that Force was encreas'd in the Primary Planet, then it wou'd approach nearer to $\mathcal{T}$. Therefore the Action of $\mathcal{S}$, the Sun's Body, being encreas'd as $\mathcal{T}$ comes nearer to it, the *Ablatitious* Force (or Force which is to be taken away) will be encreas'd, and consequently the attracting Force of the Primary Planet $\mathcal{T}$ will be diminish'd by degrees, and for that Reason $\mathcal{T}L$ the Distance of the Satellite will be encreas'd ; and on the contrary, the Action of $\mathcal{S}$ itself being diminish'd by the Recess of the Primary Planet $\mathcal{T}$ from it, MN will be diminish'd at the same Time ; and again, $\mathcal{T}$ its attracting Force will be encreas'd, and the Satellite will again approach the Primary Planet. And these Things will happen by Turns ; namely, the Orbit of the Satellite will be dilated whilst the Primary Planet is in the *Perihelium*, and will be contracted again when it returns to the *Aphelium*. Besides, the Force in the Primary Planet which draws L posited at different Distances, or the Centripetal Force of the Satellite L towards $\mathcal{T}$, is (by *Prop. 26. B. 1.*) in a Ratio compounded of the simple Ratio of the Radii directly, and the Duplicate Ratio of the Periodical Time of L about $\mathcal{T}$ inversly : therefore if the Radius remains the same, the inverse Duplicate Ratio of the Periodical Time is the same with the Ratio of the Centripetal Force of the Satellite L ; that is, the Square of the Periodic Time is encreas'd in the same Ratio in which that Centripetal Force

is

Book IV. of ASTRONOMY. 523

is diminish'd, or the Periodical Time itself is encreas'd in the Subduplicate Ratio in which the said Centripetal Force is diminish'd ; and on the contrary, the Periodical Time is diminish'd in the Subduplicate Ratio in which the Centripetal Force is encreas'd. And besides, as the Law of Attraction in the Primary Planet $\mathcal{T}$, or of the Centripetal Force in the Satellite L , is such, as to be reciprocally as the Square of the Radius $\mathcal{T}L$, and the extrinfecal *Ablatitious* Force be in respect of it very small ; the remaining Force by which L does yet tend toward $\mathcal{T}$, is almost in a Duplicate Ratio of the Radii inversly ; therefore (by *Prop. 28. B. 1.*) the same absolute attractive Force remaining in $\mathcal{T}$, (tho' the Radius be any Way chang'd) the Squares of the Periodical Times are as the Cubes of the Radii, and therefore the Periodical Times are in a Sefquiplicate Ratio of the Radii. Therefore if neither the attractive Force remains the same in the Primary $\mathcal{T}$, but is by Turns encreas'd and diminish'd by the Action of the Sun, nor the Radius $\mathcal{T}L$ continues the same (which we have before shewn to be liable to Change) the Periodical Time will also be encreas'd and diminish'd in a Ratio compounded of the two former Ratio's : that is, if the Action of the Sun S , by which the attracting Force of the Primary Planet $\mathcal{T}$ is diminish'd, be diminish'd and encreas'd by Turns ; likewise the Periodical Time will be encreas'd and diminish'd in a Ratio compounded of the Sefquiplicate Ratio of the Radii, and the Subduplicate Ratio in which the attracting Force of the Body $\mathcal{T}$ is diminish'd or encreas'd. $\mathcal{Q}. E. D.$

SCHOLIUM.

If the Force attracting the Satellite $\mathcal{T}$ be augmented any other Way than by an extrinfecal Force

524 *The* ELEMENTS Book IV.

Force arising from the Sun, as for *Ex.* if by a new Accession of Matter, the Primary Planet becomes bigger, and for that Cause (by *Prop. 49. B. 1.*) its Attraction becomes greater in that Proportion; the Satellite *L* will both revolve in a less Orbit with the diminish'd Radius $\mathcal{T}L$, and its Periodical Time will be diminish'd in a Ratio compounded of the Sesquiplicate Ratio of the Radii directly, and a subduplicate Ratio of the attracting Forces of the Body $\mathcal{T}$ inversely; as before: likewise, if the Primary Planet be diminish'd by the taking away some of its Matter, the Periodical Time of the Satellite will be encreas'd. For the compound, or the remaining Force, by which the Primary Planet, being encreas'd or diminish'd draws the Satellite, is precisely in a Duplicate Ratio of the Distance inversely, which only holds good in the Case of the foregoing *Prop.* The same will happen in respect of a Primary Planet, which has been here shewn of a Secondary One, if the Sun shou'd in any wise be encreas'd or diminish'd.

PROPOSITION XVIII.

THE same Things being laid down, in a dilated, or contracted System of a Satellite, the Periodical Effects of the disturbing Forces, or the Linear Errors of a Satellite in any Revolution, are as the Radii of the Orbits, and the Squares of the Periodical Times jointly: But the angular Errors of it seen from the Center of a Primary Planet are as the said Squares of the periodical Times. [Fig. 21.]

For the disturbing Forces, or the Forces which cause Errors, viz. *AM*, *MN* (other Things being as before) are as the Radius $\mathcal{T}L$; and therefore the Periodical (Errors or) Effects of'em, are (by *Cor. Prop. 23. B. 1.*) as the Forces and the Squares of the Periodical Times jointly.

But

Book IV. of ASTRONOMY. 525

But these Errors are Linear Errors, *viz.* such as are measur'd by the Distances of Bodies from similar Figures in those Places, to which the Bodies in the same proportional Times wou'd come without those Forces. And for that Reason, the angular Errors seen from the Center of the Primary Planet *T*; that is, the Angles, under which the said Linear Errors appear from the said Center, are in every Revolution of the Satellite *L*, as the Squares of the Times of the Revolution nearly; the other of the compounding Ratio's (*viz.* the Ratio of the Radii) vanishing, because the Linear Errors are seen at the Distance of the Radii.

SCHOLIUM.

Tho' all the angular Errors in the Motion of a Satellite, seen from the Center of the Primary Planet, vary as the Satellite's Period varies; namely, the Motion of the Apses or Apfides, the Regress of the Nodes, and the other before-mention'd Deviations, as well in Longitude or Length, as in Latitude, arising from the extrinsecal Forces of the Sun; yet, as in one Period of a Satellite, some of those Errors are scarce sensible, and not remarkable till after several Revolutions, and some Errors compensate for others: therefore they have been the less known to Astronomers; at least one Error describ'd in *Prop. 1.* by which the Motion of a Satellite in the Quadrants of its Orbit before the Syzygies is accelerated, and retarded in the Quadrants after them, which happens every Revolution, or rather every half Revolution: for in the Quadratures of the Sun this Error ceases, the Cause ceasing. Therefore this Error (as it is deduc'd from the said *Prop. 1.* or rather as it is corrected by *Cor. Prop. 2.*) by this 18th *Prop.* its Periodical Time

526 The ELEMENTS Book IV.

Time being encreas'd or diminish'd, in one of its whole Periods is encreas'd or diminish'd in a Duplicate Ratio of the Time between the Quadratures. But the Time between the Quadratures does not depend upon the Period alone, but is greater or less according as the Satellite's Place is near to the upper or lower Apsis in its proper Orbit.

PROPOSITION XIX.

If a Primary Planet be mov'd about the Sun at a great Distance from it in an excentric Orbit, the Forces of the Sun disturbing the Motion of a Satellite and their Effects, viz. all the above describ'd Errors in Longitude, Latitude, and Altitude, are reciprocally as the Cubes of the Distances of the Primary Planet from the Sun; that is, directly as the Cubes of the apparent Diameters of the Sun seen from the Primary Planet. [Fig. XXII.]

Because the Sun is at a great Distance, the Distance between the Points T and N , in Comparison of ST , vanishes, or SN , ST are equal. And since the Excentricity of the Orbit $CB D G$ (if it has any) being compar'd with TS will also vanish, SK will likewise become equal to ST . Besides, TS being encreas'd or diminish'd, the Orbit $CB D G$ together with the Lines TL , MN , AM , &c. that depend upon it, are encreas'd and diminish'd by the foregoing Prop. And indeed, by reason of the great Distance of the Sun S , they are all encreas'd or diminish'd in the same Proportion; that is, the right Line AM (which in its middle Quantity is equal to TL , because when the Satellite is in Conjunction with the Sun it becomes greater, and about the Opposition less) is to LT as SK to ST ; therefore a Force as AM is to a Force as SK in the same Ratio as LT is to ST . Likewise

Book IV. of ASTRONOMY. 527

wife, a Force as MN is to a Force as SK almost as LT to ST ; that is, the Forces as MN AM , are nearly as the Force SK and the Ratio of LT to ST jointly; that is, as the Force $SK \times \frac{LT}{ST}$. And since the Force constantly

accelerating towards the Sun's Body is (by *Prop. 42. B. 1.*) inversly, as the Square of the Distance from it; that is, at the Distance SK as $\frac{1}{SK^2}$;

putting $\frac{1}{SK^2}$ the Force SK for the Forces

AM , MN , will be as $\frac{1}{SK^2} \times \frac{LT}{ST}$; that is, as

$\frac{LT}{SK^2 \times ST}$. But it has been before shewn, that

SK is in that Case equal to ST : Whence the the Forces AM , MN are as $\frac{LT}{ST^3}$. Now if

LT be also look'd upon as constant and unchangeable, the disturbing Forces will be as AM , MN , therefore the Effects of those Forces, namely, all the Errors (of which we have before made mention) will be as $\frac{LT}{ST^3}$, that is,

reciprocally as the Cube of the Distance of the Primary Planet from the Sun. And it is commonly known, that the apparent Diameters of a Sphere of a given Magnitude are inversly as the Distances; and therefore the said disturbing Forces, and the Errors of a Satellite arising from it, are as the Cubes of the apparent Diameters of the Sun seen from the Primary Planet. $\square$ E. D.

PROPOSITION XX.

TO find the Ratio between the Sun's Forces, which disturbs the Motion of a Satellite, and the Forces by which a Satellite is retain'd in its Orbit about a Primary Planet. [Fig. XXIII.]

It has been shewn in the foregoing Propositions, what Ratio the Forces of the Sun disturbing the Motion of a Satellite have to the Force by which the Sun draws the Primary Planet, or by which the Primary Planet is retain'd in its Orbit about the Sun; namely, that which the right Lines AM , MN , have respectively to SN , or AM , MT (the Points N and T coinciding) to ST . What remains is to determine the Ratio between the said disturbing Forces, and the Force by which the Satellite L is retain'd in its Orbit $CBDG$ about its Primary Planet T . The right Line AM in its middle Quantity is equal to TL ; wherefore the Mean Force, as AM , is to the Force SN or ST , by which T is retain'd in its Orb, as TL to TS : And (by Prop. 26. B. 1.) the Force by which T is retain'd in its Orbit ETE , is to the Force, by which L is retain'd in its Orbit $CBDG$, in a Ratio compounded of the Ratio of ST to TL , and a Duplicate Ratio of the Periodical Time of the Satellite about the Primary Planet, to the Periodical Time of the Primary Planet about the Sun. And therefore, *ex æquo*, the foresaid Mean Force AM is to the Force, by which L is detain'd in its Orbit, in a Ratio compounded of the Ratio of TL to TS , ST to TL , and a Duplicate Ratio of the Periodical Time of the Satellite about the Primary Planet to the Periodical Time of the Primary Planet about the Sun; that is, (the two foregoing Ratio's destroying each other) the Mean Force AM , is to the
Force

Book IV. of ASTRONOMY. 529

Force, by which L is retain'd in its Orbit about T , is in the foresaid Duplicate Ratio of the Periodical Times. If the Ratio between MN and AM be found out, the Ratio of each Force disturbing AM and MN to the Force by which L is retain'd in its Orbit will be also found. Nay, you may find the Force AM not only in its Mean Quantity, but very exactly according to the various Position of the Satellite L , and from it nicely determine the Ratio between the disturbing Forces and the Force, by which L is retain'd in its Orbit.

C O R O L L A R Y.

If the Sun be suppos'd at a great Distance, as it happens in the Circumsolar Planets, at least in those that have Satellites; AM will always be equal to TL , the right Lines MT , AL , being parallel; and MN or MT equal to AL . But because SA is to SK in a Duplicate Ratio of SK to SL , and SL infinite in Respect of KL ; AK (the Difference of AS and SK) will be the Double of KL (the Difference of SK and SL) and AL the Triple of it: But KL is the Right Sine of the Arc CL , namely, of the Distance of the Satellite from the next Quadrature C , supposing TL the Radius. Therefore in that Case you have always the Ratio of the Force AM to MN given, viz. that of the Radius to the Triple of the Right Sine of the Satellite's Distance from the Quadrature.

S C H O L I U M.

Much after the same Manner that the Sun, plac'd without the Orbit of a Satellite, disturbs its Motion, and that several Ways, as it appears from several Propositions of this Book; the Su-

perior Planets will disturb the Motion of the Inferior, and the Comets will disturb the Motion of all the Planets. For the afore describ'd Disturbance of the Satellites arising from the Sun, is owing to the Gravitation of each Satellite towards the Sun, by which (amongst other Things) it happens, that the Gravitation of a Satellite towards its Primary Planet is no longer in a Duplicate reciprocal Ratio of the Distance from it. And the Actions of the Planets on one another, will produce the like Effects, tho' far less sensible, by reason of the Smallness of their Bodies when compar'd with the Sun, and their immense Distances. Yet these Effects (how small soever) will be produc'd, and if they continue, and are always directed the same Way, they will at length become sensible: for Example, it is found that the Apses of the Earth's Orbit do, after several Years, sensibly appear to have been carried *in consequentia*, tho' this Motion be very small, if it be compar'd with that of the ApSES of the Moon *in consequentia*; because in twenty Years they don't move quite four Minutes and a half, whilst the ApSES of the Moon in the same Time have made more than two whole Revolutions and a Quarter.

Besides, the Disturbance of the Motion of the Planets, arising from other Planets and Comets will be very unequal, because the Distances of the disturbing Bodies are very unequal. But as the Effects of all this Disturbance are small, their Inequalities will be almost insensible; and the Mean or middle Quantity of each will suffice for Astronomical Calculations.

SECTION.

SECTION V.

Concerning the Motion of the Moon, seen from the Earth.

IT has been shewn at large, in the foregoing Propositions, what the Errors produc'd by the Sun in the Motion of the Secondary Planets are; of all which Errors we will give an Example in the Moon, the Earth's Satellite, and summarily shew by what Inequalities it is affected; how they are observ'd and distinguish'd from each other, and also how they may by Computation be trac'd from their Springs; namely, the Sun's Forces which cause them; and last of all we shall give a Method for calculating and making use of Tables of the Moon's Motions, where it will appear, by the by, what Errors the Constructors of Tables have hitherto taken notice of, and what Errors they have pass'd by.

PROPOSITION XXI.

TO describe and explain the Motion of the Moon and its Inequalities hitherto observ'd, whether distinguish'd or mix'd with others, from what has been said and prov'd.

That the Earth, which is a great Body and a Primary Planet, as it is carried about the Sun, can (whilst it self is carried about the Sun) carry the lesser Body of the Moon, revolving about it under the Form of a Satellite, in an Orb any way inclin'd to the Plane of the Ecliptic in which the Earth is carried about the Sun; and that the Moon can revolve in an Ellipsis having

a Focus nearly in the Earth's Center; is plain from the 59th *Prop.* of *B. 1.* and its Corollaries: And these are *Phænomena* which every Body plainly sees; for from them the Phases of the Moon, and the Eclipses of the Luminaries, are explain'd in the 16th, 18th, and 19th *Prop.* of *B. 1.* But the Motion will be very much disturb'd by the Action of the Sun and the Moon, and affected with these Inequalities which a constant and more careful Observation has discover'd, whose Effects will nevertheless almost vanish, if they be compar'd with the first Inequalities; whence it happens, that in the forementioned Propositions the Motion of a Satellite about its Primary Planet is suppos'd very like the Motion of a Primary Planet about the Sun, tho' upon a nice Observation it differs a little in some Things.

For first of all, if it was not for the Sun's Action, the Moon wou'd be so mov'd about the Earth, as to describe Areas proportionable to the the Times, by a Radius drawn to the Earth: But by the said Action, it happens (by *Prop. 1.*) that it describes an Area greater for the Time (that is, it is carried swifter) in the Syzygies with the Sun, than in its Quadratures. And therefore this Error is also chang'd, and in every Period of it in a Duplicate Ratio of the Time between the Quadratures, by *Schol. of Prop. 18.*

Secondly, if the Sun's Action did not disturb the Moon's Motion, the Moon would describe an Ellipsis having a Focus in the Earth's Center: But now it is plain by *Prop. 2.* that the Orbit of the Moon will (*cæteris paribus*) become less curve in the Syzygies, and more in the Quadratures; and therefore it is nearer the Earth in the Syzygies than in the Quadratures.

Thirdly,

Book IV. of ASTRONOMY. 533

Thirdly, the Moon wou'd constantly describe an immoveable Ellipsis about the Earth, if the Sun did not disturb it; but now from its Disturbance it happens, that this Orbit is very far from being an Ellipsis; neither can it be reduc'd to one but by the artful Industry of the Astronomers. For its Orb is a Curve, which is generated of an Ellipsis mov'd about a Fire Focus, or the Center of the Earth, its greater Axis in the mean Time vibrating with an angular Motion sometimes forward, and sometimes backward; as it is plain from *Prop. 8.* But the immoveable Curve *AEGH* (in *Fig. III.*) is truly the Moon's Orbit, which is no more to be reckon'd an Ellipsis than an Helix is a Right Line, because it is generated by a Point moving in a Right Line, whilst the Right Line is carried about by an angular Motion. But if out of Respect to the old Astronomers (who are to be valu'd) we call the Orbit of a Planet an Ellipsis, the Line of its Apfides (by *Prop. 8.*) does in each Lunation go forwards twice, namely, when the Moon is in the Syzygies, and goes back twice when the Moon is in the Quadratures; and this Progres (by *Prop. II.*) is greater than the Regress, when the Moon's *Apo-gæum* and *Perigæum* is in the Syzygies, and on the other hand the Regress does exceed the Progres, when it is in the Quadratures. Therefore in the former Case, if you observe one whole Revolution of the Moon, it will simply appear that the *Apogæum* goes forward; but in the latter it goes back: But as the first Case is most lasting in one whole Revolution of the Apfes about the Sun, they go forward simply.

Fourthly, If the Sun's Action had no Power to disturb the Moon's Motion, the Moon wou'd always describe the same Ellipsis about the

Earth; whereas now it continually changes its Shape; for the Excentricity of the Orbit is every Day chang'd, and (by *Prop. 12.*) it is greatest in every Revolution when the Moon is in the Syzygies, and least when the Moon is in the Quadratures. But if the Moon's *Apogæum* and *Perigæum* be in the Syzygies, and the Moon be at the same time in the *Apogæum*, that is, if the Excentricity be greatest of all; the Moon in the Syzygies will be both most remote and most slow, contrary to what was said in the first and second Paragr. of this *Prop.* But as there we did not consider that Excentricity, and we only say that, *cæteris paribus*, what was there said holds good, what we say now does not contradict it. But even in this Case the Moon does not move so slowly, neither does the Excentricity of the Orbit become so great, as it would be, if the Errors explain'd in the said Paragraphs were away.

Fifthly, if the Sun did not disturb the Moon's Motion, the Moon wou'd describe an Orbit about the Earth in an immoveable Plane, as a Primary Planet does about the Sun; which appears from *Prop. 11. B. 1.* to be the Property of the Way of a Projectile: But the Sun causes the Position of this Plane to vary differently, either by the Change of its common Section or Line of the Nodes with the Plane of the Ecliptic, or of the Inclination it self with the fix'd Plane of the Ecliptic. And by *Prop. 14.* in one of the Moon's Revolutions about the Earth, the Nodes go back very fast when the Moon is in the Syzygies, and very slowly when it is in the Quadratures; and if several Revolutions be compar'd, the Regress of the Nodes is the swiftest when they are in a Quadrature with

Book IV. of ASTRONOMY. 535

with the Sun, and slowest (because none) when in the Syzygies.

Sixthly, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is also changed. For (by *Prop. 15.*) in the same Revolution of the Moon this Inclination is the least when the Moon is in the Syzygies, and greatest when it is in the Quadratures; but if several Revolutions be compared together, it is least when the Nodes are in the Quadratures, and greatest when they are in the Syzygies.

Seventhly, all the afore describ'd Errors arising from the Sun's Action produc'd in any small Time; that is, the Velocities or Horary Motions of the *Apogæum*, Nodes, &c. and of the Moon are chang'd according to the Earth's Distance from the Sun, which causes those Errors, and that (by *Prop. 19.*) in a Triplicate Ratio of the Distance of the Earth inversly, or in a Triplicate Ratio of the apparent Diameter of the Sun seen from the Earth.

Eighthly, the Periodical Time of the Moon varies also according to the Earth's different Distance from the Sun: For it is least (by *Prop. 17.*) when the Earth is in the *Aphelium*; and greatest when the Earth is in its *Perihelium*. And therefore, besides all the Inequalities above mention'd in the Moon's Motion, which happen at those Times, the Moon's Mean Motion it self is swifter when the Earth is in the *Aphelium*, than when it is in the *Perihelium*, and proportionably slower or swifter as the Earth is nearer or farther from it.

Perhaps there are some other Inequalities not yet observ'd (or at least not yet distinguish'd from the others) in the Moon's Motion by which it is so disturb'd, that it can no way be reduc'd

to a certain Rule; *viz.* those of which Mention is made in the *Schol.* of *Prop.* 18. &c. After these which are enough known, the rest will more easily be observ'd, and their Causes deduc'd from the Principles before laid down.

PROPOSITION XXII.

TO find the Distance of the Moon from the Center of the Earth, by one Eclipse of the Moon, without any previous Observation of the Moon.

The common Investigation of the Moon from the Earth (built upon the Methods laid down in the 2d and 3d Books) presupposes the Theory of the Moon to be in some Measure establish'd, because this Theory depends upon it, and cannot be obtain'd without it; as *Ptolemy* observes, in the 11th Chapter of the 5th Book of his *Almagest*. He, by an Observation of the Moon in the Zenith, when at the same Time it has its greatest Latitude, performs the Matter. Few Places are fit for such an Observation, certainly not our Latitude. Therefore we must have Recourse to Geometry, and see if it will afford us any Help, and find the Parallax of the Moon without any previous Theory or Observation of the Moon.

At the Moment that the Horns of the Moon (in its Immersion into the Earth's Shadow, or its Emergence from it) are parallel to the *Æquinoctial*; let the apparent Altitude of the Moon, and (by Help of the neighbouring Fix'd Stars) its apparent Right Ascension and Declension be observ'd. Now the Phasis of the Eclips'd Moon is very well distinguish'd, if the Horns appear to be in a Right Line perpendicular to the Gnomon of a Sun-Dial, which is parallel to the Axe of diurnal Rotation, which Line represents the *Æquator*;

Book IV. of ASTRONOMY. 537

Æquator ; in which Case it is plain that the Moon and the Shadow of the Earth have the same Right Ascension. In *Fig. XXIV.* let the Circle $ZPHO$ represent the Meridian, in which the Pole is P , the Zenith Z . Let EQ be the Æquator, in which the Place from which you begin to count the Right Ascension is γ ; let HO be the Observer's Horizon, and A the Moon's apparent Place, and V its true one at the Time of the Observation ; which Places will be in the same Vertical Circle, because the Parallax does something depress the Phenomenon. Let the Circles of Declination PAB , PVD meeting with the Æquator in B and D , be drawn thro' A and V . Then, from the given Place of the Sun will be given its Right Ascension, and consequently the Right Ascension of the Point opposite to it, *viz.* the Center of the Shadow ; that is, γD : therefore BD the Difference of γD the true and γB the apparent Right Ascension will be given, or the Angle $BP D$, which is measur'd by this Arc. In the Spherical Triangle APZ three Sides are given ; namely, PZ the Complement of the Pole's Height, ZA the Complement of the observ'd apparent Altitude of the Moon ; and AP the Complement of the observ'd apparent Declination of the Moon ; therefore the Angle ZAP will be given. Again, in the Triangle VAP are given the two Angles, VAP and $VP A$, lately found with the intercepted Side AP ; therefore the Side AV , the Moon's Parallax will be found, which being given, you may, by *Prop. 46. of B. 2.* find the requir'd Distance of the Moon from the Center of the Earth.

PROPOSITION XXIII.

TO determine the Distance of the Moon from the Center of the Earth, at a given Time, by Observation. [Fig. XXV.]

Let OR denote the Earth, whose Center is T ; O the Place of the Observer; and let L be the Moon in the former Observation, when you have determin'd its Distance as well from the Earth's Center as from the Observer's Eye, (namely, the Right lines TL , OL) by the foregoing; and immediately before or after the Eclipse of the Moon, on which the said Observation has been made, let the Moon's apparent Diameter in L be observ'd. Now let the Moon be suppos'd in any other Place of its Orbit, as at l : its Distance Tl from the Earth's Center is requir'd. Let the Moon's apparent Diameter in l be observ'd; now (as it is commonly known) Ol is to OL as the apparent Diameter of the Moon at L is to its apparent Diameter at l ; that is, in a given Ratio; therefore also Ol is given: But by the apparent Altitude of the Moon in l , the Angle lOT is given; if therefore in the Triangle lOT the sides Ol and OT , as well as the Angle lOT be given, you will find Tl the Distance of the Moon from the Earth's Center. *Q. E. F.*

There is no Need of expressing the apparent Diameters exactly in Parts of a Circle; it is sufficient if their Ratio to one another be known: For these Arcs or Angles are not compar'd with other Angles or Arcs express'd by Parts of a Circle; but only one with another; for their inverse Ratio is the Ratio of the Distances of the Moon from the Eye at the Times of the Observations. Therefore it is enough if they be observ'd

Book IV. of ASTRONOMY. 539

serv'd with a Micrometer fitted to a Telescope, and the Distances of the Threads being divided into equal Parts, they will be denoted by the Number of those Parts.

PROPOSITION XXIV.

TO determine the Inequalities and Errors of the Moon by Observation and distinguish them one from another.

The Moon's apparent Places are determin'd by Help of the neighbouring Fix'd Stars, whose apparent Places are before exactly known, by *Prop. 29. B. 2.* and true Places from the apparent by the 68th *Prop.* of the same *Book*: But the Distance from the Earth's Center is known at any Time, by the foregoing *Prop.* Therefore all that regards the Moon's Motion is known.

Since the Inequalities in the Moon's Motion, which are taken notice of in the 21st *Prop.* are known, and we know how to look for 'em, by comparing several of these Kind of Observations made constantly and for several Years, their Quantities will be determin'd: as for Examp. we may this way determine whether (*cæteris paribus*) the Moon moves swifter in the Syzygies with the Sun than in the Quadratures; whether it be farther distant from the Earth in these than in those; whether the Moon's mean Motion be swifter in the Earth's *Aphelium* than in its *Perihelium*, &c. some of 'em being determin'd (*viz.* those which depend upon the Syzygies with the Sun, and perhaps those which end and begin again twice in a Synodical Month) the rest will more easily be distinguish'd from one another, which either depend upon the Earth's Annual Motion, or upon the Revolution of the Apes or Nodes of the Moon in respect of the Sun.

These

These being separated from one another, there is left that Inequality of the Moon, of which we have spoken in general in the *Schol.* of *Prop.* 16. whose Quantity being known by its Period will be gather'd from Observations. And at length by comparing the Period of the Moon which is now observ'd, with that that was known several Ages ago by Observations, it will appear, whether the Globe of the Earth is in this Process of Time encreas'd or diminish'd, (as I suspect) and we may Geometrically determine, by *Schol. Prop.* 17. what Increment or Decrement has happen'd in the whole Time. But if other Inequalities, hitherto not observ'd, affect the Moon's Motion, taking away the foremention'd ones, they will appear, and being made plain will encrease our Astronomy, by making up the Moon's Theory.

PROPOSITION XXV.

TO find the Forces of the Sun which disturb the Moon's Motions, or to determine the Ratio between those disturbing Forces, and the Force of our Gravity. [Fig. XXVI.]

The same Things being laid down as before, the Forces which disturb the Moon's Motion are represented by the Right Lines AM , MN , and by reason of the Sun's great Distance, the Line MN does not differ from MT , and is the Triple of the Right Sine of the Arc CL of the Moon's Distance from the Quadrature, supposing TL the Radius; and AM is equal to TL , as it has been shewn in *Cor. Prop.* 20. Also similar, but less Forces disturb the Motion of the Earth, because it is not the Center of the Earth it self, but the common Center of Gravity of the Earth and Moon, that describes the *Magnus Orbis* (or great Orbit) about the Sun, as it has been

Book IV. of ASTRONOMY. 541

been shewn in the 63d *Prop. B. I.* so that the Earth and Moon are as it were Satellites revolving about the said Center of Gravity. But we refer both the Forces to the Moon, for the Reasons which have been demonstrated in the 50th and 51st *Prop. of B. I.* Moreover, by *Prop. 20.* the Force of the Sun which disturbs the Moon, being as AM , is to the Force, by which the Moon is retain'd in its Orbit, in a Duplicate Ratio of the Moon's Periodical Time about the Earth, to the Earth's Periodical Time about the Sun: And the Force by which the Moon wou'd revolve about the Earth, if it was immoveable, at its present Distance from it, (which, by *Prop. 51. B. I.* is equal to the Force, by which it is now retain'd in its Orbit about the foresaid Center of Gravity,) is, by *Cor. 1. Prop. 26. B. I.* to the Force by which it might in the same Time revolve about the Earth, at its Distance from the said Center of Gravity, as the Distance of the Moon from the Earth, to the Distance of the Moon from that Center of Gravity; for the Periodical Times of the Earth and the Moon about their common Center of Gravity are the same: And the foresaid Force by which the Moon wou'd be retain'd in its Orbit about the Earth, at rest at its Distance from that common Center of Gravity, is to the Force which we call Gravity (which is indeed the same Force, but propagated to a less Distance from the Center, namely, to the Earth's Surface only) in a Duplicate Ratio of the Semi-Diameter of the Earth to the foresaid Distance of the Moon from the common Center of Gravity of the Moon and Earth: And therefore the Force of the Sun as AM is to the Force of Gravity in the Earth's Surface, in a Ratio compounded of

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these

§42 *The* ELEMENTS Book IV.

these three Ratio's ; namely, of the Duplicate Ratio of the Moon's Periodical Time about the Earth, to the Periodical Time of the Earth about the Sun ; of the Ratio of the Moon's Distance from the Earth, to the Distance of the Moon from the common Center of Gravity of the Moon and Earth ; and of the Duplicate Ratio of the Semi-Diameter of the Earth, to the Distance of the Moon from the common Center of Gravity of the Earth and Moon. Now the three compounding Ratio's are given ; therefore we have the Ratio compounded of them, *viz.* the Ratio between the Sun's Force, as AM , and the Force of Gravity. But the Ratio of the other disturbing Force, as MN or MT to AM or LT is known, namely, that of the Triple of the Sine of the Arc CL of the Moon's Distance from the Quadrature to the Radius : Therefore the Ratio of each Force of the Sun which disturbs the Moon's Motion to the Force of our Gravity here is found.

The Mean compounding Force may without any sensible Error be neglected, as coming near to the Ratio of Equality : which makes the Solution so much the easier.



SECTION

SECTION VI.

Of the Moon's Tables, and their Use.

PROPOSITION XXVI.

TO explain the Order of the Lunar Tables, design'd for readily finding of the Place of the Moon seen from the Earth.

First of all, the *Epocha* or Radix of the Moon's Motion is settled: that is, (as has been above explain'd in B. 3.) by taking a remarkable Point of the Mean Time, number'd in the Place for which the Tables are design'd; then you must set down the Mean Longitudes of the Moon, and its *Apogæums*, and ascending Nodes agreeing with the said Time. Likewise, for readily finding the Moon's Aspects with the Sun, the Radix of the Moon's Mean Motion from the Sun is settled; that is, the Mean Distance of the Moon from the Sun is put down agreeable to the said *Epocha*.

Secondly, the Mean Motions of the Moon it self, and its *Apogæums* and Nodes are to be set down in the Tables for Years, Tens of Years, and Hundreds of Years, &c. and also for Months, Days, Hours, and Parts of Hours. Likewise the Mean Motions of the Moon from the Sun, for the same Numbers of Years, Months, Days, &c. must be set down; which are easily found, by taking away the Sun's Mean Motion from the Moon's Mean Motion, agreeable to the same Time.

Thirdly, you must set down the Correction of the Moon's Mean Motion, according to the
Earth's

544 *The* ELEMENTS Book IV.

Earth's Distance from the Sun. For the Moon's Mean Motion being taken from the foregoing Table, is the Mean Motion in a larger Sense; that is, in respect of a Course of several Years. Now (by *Prop. 17.*) the Moon's Periodical Time is shorter when the Earth is in its *Aphelium*; but as it recedes from it, the Moon's Periodical Time is encreas'd, till it be the greatest of all, when the Earth is come to the *Perihelium*. Therefore its Mean Place before found is to be corrected by the Addition or Substraction of an Arc or Angle depending upon the Mean Anomaly of the Earth: Yet the Place of the Moon so corrected is still the Mean Place. This Table of the Correction of the Moon's Mean Motion depending upon the Earth's Anomaly, is settled by means of the 17th, 20th, and 25th *Prop.*

Fourthly, the Corrections (called *Æquations* in the Astronomic Phrase) of the Mean Place of the Moon's *Apogæum*. And these *Æquations* are Three; the First depends upon the Aspect of the *Apogæum* and the Sun; for it has been shewn in *Prop. 11.* that the *Apogæum* goes forward when it is in the Syzygies, and goes backward when it is in the Quadratures with the Sun; therefore to the Mean *Apogæum* is to be added, or from it is to be subtracted the *Æquation* depending upon the Situation of the *Apogæum*, in respect of the Sun: The Quantity of this *Æquation* is proportional to the Quantity of the Forces that produce those Effects, which are to be determin'd by the 11th, 20th, and 25th *Propositions*. The second *Æquation* of the Moon's *Apogæum* is Monthly, and depends upon the Sun and Moon's Aspect. For tho' near the Conjunction or Opposition of the Moon's *Apogæum* with the Sun, the *Apogæum* goes forward
in

Book IV. of ASTRONOMY: 545

in one whole Luration, yet that Progreſs of the Apogæum is not equable, but when the Moon is in the Quadratures, it goes forward ſlower, or by chance it may alſo go back; when the Moon is in the Syzygies, its Apogæum goes forward much faſter. Likewise, tho' near the Quadrature of the Apogæum of the Moon with the Sun, the Apogæum of the Moon goes back in reſpect of one whole Luration, yet it does not equably go back, but the Moon in the Syzygies either makes it go forward, or at leaſt go back more ſlowly. Theſe Things are plain from *Prop. 10.* whence the Place of the Moon's Apogæum being firſt (by the foregoing Correction) brought to an Equality, muſt alſo be corrected by this monthly *Æquation*. The Quantity of this *Æquation* is to be gather'd from the different Aspects of the Moon to the Sun (by *Prop. 8th, 20th, and 25th.*) But there is alſo Need of a third *Æquation* fully to rectify the Place of the Apogæum: For the Motion of the Apogæum, whether progreſſive or regreſſive, is greater or leſs according to the leſs or greater Diſtance of the Earth from the Sun; and its Velocity (by *Prop. 19.*) is in a Triplicate Ratio of the Sun's apparent Diameter: Therefore this depends upon the mean Anomaly of the Earth, as alſo the Correction of the mean Motion, of which we have ſpoken above. Its Quantity may be drawn from the Knowledge of the Degree of the Anomaly in *Prop. 19th, 20th, and 25th.*

Fifthly, the Excentricity of the Lunar Orbit is liable to a continual Change, and the Times of that Change are the ſame with thoſe of the progreſſive and regreſſive Motion of the Apogæum. Becauſe (by *Prop. 13*) if you compare one whole Luration with another whole one,

the greatest Excentricity is when the Apfes are in the Syzygies, and the least when they are in the Quadratures. And in one Luration (by *Prop. 12.*) it is greatest when the Moon is in the Syzygies, and least when the Moon is in the Quadratures: For Example, in that Month that the Apfes are in the Syzygies, the Excentricity is diminish'd, when the Moon itself is come to the Quadrature, at which Time it is least of all in that Luration. And likewise, when the Apogæum is in the Quadrature with the Sun, the Excentricity, which already was little, is diminish'd, whilst the Moon goes from the Syzygies to the Quadratures. Therefore the Excentricities of the Orbit, which agree with the Aspect of the Apogæum of the Moon are put down in the Tables, and that very conveniently over-against the *Æquations* of the Apogæum, which depend upon the said Aspect, and as it were make the same Table with them. Moreover, these Excentricities are again to be corrected by the Monthly Equation depending upon the Moon's Distance from the Syzygies: For when the Moon is in the Syzygies, the Excentricity (*cæteris paribus*) is greatest. But the Excentricity thus equated must again be (by *Prop. 18* and *19.*) corrected according as the Sun's Distance from the Earth is increas'd or diminish'd; and this *Æquation* depends upon the Earth's Anomaly: But these three Tables are calculated by Help of the Respective aforesaid Propositions, together with the 20th and 25th. And as the Mutations of the said Excentricity have the same Terms and Times with the aforesaid Motions of the Apogæum, the two last *Æquations* of it, namely, the Monthly and Annual, will also very conveniently in the same Table be

join'd

Book IV. of ASTRONOMY. 547

join'd to the two last *Æquations* of the *Excen-*
tricity. These *Corrections* are made very well
 and easily by the *Addition* or *Subtraction* of the
Particulars set down in the *Table*, according
 to the various *Distance* of the *Moon* from the
Syzygies with the *Sun*, or the various *Distance*
 of the *Earth* from the *Aphelium*; that is, un-
 der the same *Titles* as you had before set down
 the *Annual* and *Monthly Corrections* of the
Apogæum.

Sixthly, You must add the said *Table of the*
Æquation of the Orbit, by which, from the gi-
 ven *Time* of the *Moon's* being in the *Apogæum*,
 you may find the *Angle* by which the *Moon*,
 seen from the *Earth*, is distant from the *Apo-*
gæum. For as the *Moon* in its temporary *El-*
liptic Orbit moves after the same *Manner* about
 the *Earth* in the *Focus* of an *Ellipsis*, as any
Primary Planet moves about the *Sun*; that is,
 describes (by a *Radius* drawn to the *Earth*) an
Area proportionable to the *Time*; (at least we
 now abstract from all other *Inequalities*, having
 their *Proportion* in the other *Tables of Æqua-*
tion of the Moon's Motion;) if the *Orbit* be
 given, its true *Place* (when it is seen from the
Earth) will after the same *Manner* be found for
 a given *Time*, as the *Place* of the *Primary Planet*,
 seen from the *Sun*, is found; that is, the *Excen-*
tricity or *Ratio* of the *Distance* of the *Foci* to
 the greater *Axis* being given, from the given
 mean *Anomaly* of the *Moon* (which consists in
 an *Area* proportionable to the *Time*) the *coæ-*
quated Anomaly will be found in this *Place* after
 the same *Manner* as in that; namely, by *Help*
 of *Prop. 3. B. III.* or, if we may be allow'd to use
Approximation, we may make use of some of the
Propositions that follow in the 3^d *Book*. *Ward's*

Approximation is sufficient, in which Case the *Æquation* of the Orbit, or the *Æquation* of the Excentric (which is also call'd the *Æquation of the Moon's Center*) is (by *Prop. 6. B. III.* found according to the Degree of the Moon's mean Anomaly, and the *Æquations* being found for every Degree, are set down in the Tables, to be added or subtracted according to the Semi-Orbit, from the Apogæum to the Perigæum, or from the Perigæum to the Apogæum. But because the Excentricity of the Moon's Orbit is continually chang'd, and it would be impossible to construct a Table for every Excentricity; therefore Astronomers commonly construct *Æquation* Tables of this Kind for every Degree of the Mean Anomaly for the greatest and least Excentricity of the Orbit, and sometimes also for the Mean; and they also take a proportionable *Æquation* between the greatest and the least, and the Excentricity belonging to the Lunar Orbit is between the greatest and the least.

Seventhly, a Table must be added shewing that Inequality of the Moon, which we have (in the 1st *Prop.*) shew'd to belong to every Satellite, according to the Moon's various Distance from the Syzygies. For the Moon's Motion in the first Quadrant of the Month (or as the Moon goes from the Conjunction to the next Quadrature) is retarded; accelerated in the second, retarded again in the third, and then accelerated again in the fourth.

Tycho was the first that discover'd this Inequality in the Moon's Motion, and gave it properly enough the Name of *Variation*. But a Table of Variation is to be made from what has been said in *Cor. Prop. 2.* according as the Moon's Orbit is
2 broader

Book IV. of ASTRONOMY. 549

broader between the Quadratures, and narrower between the Syzygies, which has been shewn to be the true Track of the Moon (by *Prop. 2.* and its *Corol.*) The greatest Quantity of that Variation is drawn from the 20th and 25th *Prop.* and the Quantity belonging to every Distance of the Moon from the Syzygies (from *Prop. 1.* and 2. and the *Cor.* of the 2d.) Besides, the Variation is again to be corrected, by Help of the Table made by *Prop. 18.* and 19. For when the Earth is in the Perihelium, the Moon's Periodical Time is encreas'd; and among all the other encreas'd Errors, this Variation (which we now speak of) is encreas'd every Revolution in a Duplicate Ratio of the Time between the Moon's Quadratures (by *Schol. Prop. 18.*) This Correction depends upon the Earth's Mean Anomaly, and also the other by which the Variation is encreas'd or diminish'd in a Triplicate Ratio of the Sun's apparent Diameter, seen from the Earth.

Eighthly, you must set down three Æquations of the Moon's Nodes in Tables. The first of 'em depends upon the Aspect of the Nodes with the Sun: For (by *Prop. 14.*) the Nodes are altogether at rest in the Syzygies; but when they stick about the Sun's Quadratures, they are carried very swiftly *in antecedentia*; and especially so, if you compare one whole Lunation with another whole one. For if you regard one by itself, the Nodes go back swiftest (by *Prop. 14.*) when the Moon is in the Syzygies: And from hence the Monthly Æquation of the Nodes is necessary, which depends upon the Aspects of the Sun and Moon. The third Æquation is Annual; for the Nodes (*cæteris paribus*) go back so much the faster, as the Earth is nearer to the Perihelium,

lium, by *Prop.* 19. The Quantities of those *Æ*-quations (whether they be the greatest, or those that agree with any Aspect of the Nodes, or of the Moon with the Sun, and the Anomaly of the Earth) are determin'd by the 20th and 25th *Prop.* together with the 14th and 19th.

Ninthly, there is Need of the same Corrections to *equate* the Inclination of the Moon's Orbit to the Plane of the Ecliptic: For (by *Prop.* 15.) it is least when the Nodes are in the Quadratures with the Sun, and greatest when they are in the Syzygies; and in the same Lunation, least when the Moon is in the Syzygies, and greatest when it is in the Quadratures. Since therefore these have the same Times as the *Æ*-quations of the Nodes (as we have said before) in respect of the Aspect of the Nodes, as well with the Moon as with the Sun, they easily fall in together in the same Table. And the Correction of the Inclination of the Planes is made easily, by the *Particles* to be added in the least Inclination (as we have said before, concerning the correcting of the Excentricity) which vary according to the said Aspects of the Nodes and of the Moon to the Sun (or, in the Astronomers Phrase, according to the Distances of the Nodes and of the Moon from the Sun) or perhaps more fitly by the *Particles* to be deducted from the greatest Inclination: For this seems natural to the Moon's Orbit, because it would belong to it if the Sun did not disturb its Motion. But this Inclination also is to be *equated* by the Annual Correction, which depends upon the Earth's Distance from the Sun, or the Earth's Anomaly, and which will fall into one Table with the Annual *Æ*quation of the Nodes. The greatest Quantities of those

three

Book IV. of ASTRONOMY. 551

three *Æquations*, and those which belong to every Aspect or Distance, are drawn from the 15th, 18th, 19th, 20th, and 25th Propositions.

Tenthly, a Table is to be made to shew the Proportion of all the foregoing Monthly Corrections, or Increments or Decrements made in the Syzygies, in the Conjunction and Opposition with the Sun. For (by *Prop. 16.*) all the Effects that are produc'd in the Syzygies of the Luminaries, are greater in the Conjunction, and less in the Opposition. The Proportion may be drawn from the Ratio of the generating Forces, by the foregoing Propositions.

Eleventhly, there is added a Table of the Moon's Latitude, according to every Degree of the Distance from the said Node. Its Construction depends upon the Doctrine of Spherics. For in a Spherical Rectangular Triangle, besides the Right Line, the Hypothenuse is given, namely, the Moon's Distance from the Node, and the Angle at the Node, namely, the Inclination of the Plane of the Moon's Orbit to the Plane of the Ecliptic, (which is also call'd the *Inclination of the Limits*;) therefore the Side opposite to that Angle will be found; that is, the Moon's Latitude requir'd, according to the said Distance from the Node. But because the Inclination of the Plane of the Moon's Orbit to the Plane of the Ecliptic is different at different Times, therefore commonly one Table only of the Moon's Latitude is made for the least Inclination of the said Planes, those Particles being join'd, which are to be added to the Latitude, when the said Inclination is the greatest. In the Mean Inclinations, such a Part of the annex'd Particle is to be added, as is to the whole, as the Excess of the given Inclination

tion above the least, to the Excess of the greatest above the least; as it is usual in the like Cases.

Twelfthly, you must also subjoin a Table, shewing the Moon's Place reduc'd to the Ecliptic for every Degree of the Moon's Distance from the Node. Its Construction also depends upon the Doctrine of Spherics. For in the same Triangle in which the Latitude is first found out, the third Side, or the Distance between the Node and the Point where the Circle of Latitude, passing thro' the Moon's Place, cuts the Ecliptic, is the Moon's Distance from the Node reduc'd to the Ecliptic: And the Interval between that and the Moon's Distance from the Node, computed in its Orbit, is call'd the *Reduction*, and is dispos'd into a Table according to Art. And as it varies according to the various Inclination of the Lunar Orbit to the Ecliptic, (after the same Manner as before) the Excess of the foresaid Reduction is mark'd when the Inclination of the Limits is the greatest, that its proportional Part may be taken when the Inclination of the Limits is a Mean between the greatest and the least: For the Reduction itself is fitted to the least Inclination, as the Latitude was before.

But also when other Errors are discover'd, other Tables must be constructed, fully and perfectly to rectify the Moon's Motion, which is far the hardest Work in Astronomy. Let what we have said, of the Order of these Tables, suffice in the mean Time.

PROPOSITION XXVII.

TO explain the Order of Lunar Tables, for finding readily the Place of the Moon seen from a given Point in the Earth's Surface.

Because the Moon's Place seen from a Point in the Earth's Surface differs sensibly from its Place when seen from the Center, which last Place alone is to be found by the foregoing Tables; therefore Astronomers make Tables also to discover the first from the Knowledge of the last. And as the Parallax is all the Difference between the said Places, the Knowledge of it is sufficient. And because when you know the Horizontal Parallax, you may find the Parallax of any other Elevation above the Horizon, by *Prop. 47. B. II.* therefore we have Need only of the Moon's Horizontal Parallax. And as the Moon's Horizontal Parallax varies according to the Moon's greater or less Distance from the Earth, there must be made a Table of those Parallaxes. Now the Parallax for one Distance is known by *Prop. 22.* and the Moon's Distances from the Earth in the other Points of its Orbit, in respect of the Distance found by the said 22^d *Prop.* are determin'd by the Moon's Orbit, which has been before describ'd; therefore the Moon's Horizontal Parallaxes (which, by *Prop. 48. B. II.* are reciprocally as its Distances from the Center of the Earth) according to its Situation in respect of the Apogæum and Perigæum of its Orbit, that is, according to its mean Anomaly, are known and put into a Table.

But because such a Table of the Moon's Horizontal Parallaxes serves only for one Figure of the Orbit; therefore they construct one for the greatest Excentricity and another for the least,

least, and take the Moon's Horizontal Parallax for a given Time in the same Mean Ratio between the Parallaxes agreeable to the greatest and least Excentricity, because the Excentricity, agreeable to that Time, is a Mean between the greatest and the least Excentricity.

The Moon's Horizontal Parallax, thus found, is to be rectified by a certain *Æ*quation to be drawn from another Table, whose Foundation is as follows. It is plain from *Prop.* 2. that the Moon (*cæteris paribus*) is farther distant from the Earth in the Quadratures than in the Syzygies. One may find the Proportion between those Distances of the Moon from the Earth, from the Proportion of the Forces of the Sun which produce that Effect by Help of *Prop.* 20. and 25. Therefore a Table for *æquating* the Moon's Parallax consists of Particles to be subtracted from the Parallax, which it has in the Syzygies, as the Moon comes nearer to the Quadrature, where (*cæteris paribus*) the Parallax is the least, because the Moon is the farthest distant from the Earth: Therefore this *Æ*quation is Monthly, depending upon the Syzygies of the Sun and Moon.

But this Parallax of the Moon is also affected by the Annual *Æ*quation; as being less when the Earth is in the Perihelium, and greater when it is in the Aphelium, (*cæteris paribus*) because the whole Lunar System is in this Case contracted, but dilated in the other, by *Prop.* 17. This *Æ*quation depends upon the Earth's Mean Anomaly, and is constructed by Help of the 17th, 20th, and 25th Propositions.

PROPOSITION XXVIII.

TO find the Moon's Place, seen from any Place of the Earth's Surface, for a given Time, by Help of the Tables above describ'd.

Let the Time, reckon'd at the Place given, be (by *Prop. 32. B. II.*) reduc'd to the Time which is reckon'd under the Meridian of the Place for which the Tables are calculated; and when (by *Prop. 17. B. III.*) it is equated to it, let the Sun's Place be found.

At the Epocha mark'd in the Table, which goes before the given Time, mark the Mean Places of the Moon, of the Moon's Apogæum, and of the ascending Nodes; and collecting the Mean Motion of those three made from that Epocha to the Time given, add the two first to the aforesaid Places of the Moon and the Apogæum, that they may become the Mean Places of them for the Time given; and let the Motion of the Node made in the mean while be substracted from the radical Place (because they are mov'd *in antecedentia*) to have its Mean Place; and let the Moon's Mean Place be corrected by that Æquation depending upon the Earth's Mean Anomaly; Of which see *Seçt. III. Prop. 26.*

Moreover, let the Mean Place of the Apogæum be corrected by the three Æquations, of which we have spoken in the IVth *Seçt.* of the said 26th *Prop.* For the Sun Place being given, and the true Place of the Apogæum nearly, the Aspect of the Lunar Apogæum with the Sun is given, and from thence the Æquation that depends upon it. Likewise, because as well the Sun's true Place, as the Moon's true Place nearly is known, the Moon's Aspect to the Sun is also known, and from thence the Monthly Æqua-

Æquation of the Apogæum that depends upon it. And an Annual Æquation of the Apogæum agreeable to the Earth's Mean Anomaly (first necessary to determine the Sun's Place) will be found in the third Table of Æquation. And this Æquation is the true Annual one depending upon the Earth's Period about the Sun; for that first, which depends upon the Aspect of the Moon's Apogæum with the Sun, is by some improperly call'd Annual, since its Period is not finish'd in less than 400 Days; the Sun requiring that Time, that being gone from the Lunar Apogæum it may again follow the Apogæum which is carried *in consequentia*. So also the Excentricity of the Moon's Orbit, taken out of the Table from the Sun's Apogæum, is corrected by the Monthly and the Annual Æquation.

Having therefore found the true Place of the Moon's Apogæum, and the Moon's Mean Place being first corrected, by taking the first from the last, the Remainder is the Moon's Mean Distance from its Apogæum reckon'd or number'd *in consequentia*, or the Moon's Mean Anomaly; agreeable to which you will (in the Table of *Sect. VI. Prop. 26.*) find the *Prosthapheresis* for the Excentricity of the Orbit before found, by which the Moon's Mean Place, before corrected, being again rectified, will give the Moon's Place more correct, or *Equated*, as some call it.

From the Moon's Equated Place lately found, and the Sun's Place first of all determin'd, the Aspect of the Sun and Moon is now enough known, and the Variation that depends upon it, is known by the Table, of which we spoke in *Sect. VII. Prop. 26.* but even this is to be corrected by

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Book IV. of ASTRONOMY. 557

by the double Annual $\mathcal{A}$ equation, of which we spoke in the same Place. The Moon's equated Place is to be corrected by the Variation corrected in the abovemention'd Manner; and thus you will have to the greatest Nicety the Moon's Place in its Orbit, or in respect of its Apfis.

Now in order to reduce it to the Ecliptic, first of all we must determine the Places of the Nodes. And as the Sun's true Place, and the true Place nearly of the Nodes, or their mean Place will be given, so likewise will be given the Aspect of the Nodes with the Sun, and consequently its correspondent Equation from the Table, of which we made mention in the 8th *Sett. Prop.* 26. by which the Node's mean Place being corrected is again to be twice corrected by the Monthly and Annual $\mathcal{A}$ equation of which we also spoke there.

At the same Time, from the same Aspect of the Nodes to the Sun, may be gotten the Excess of the Inclination of the Limits above their least Inclination (or their Defect from the greatest) by which the least Inclination (or the greatest) is to be corrected, that it may become more exact: But it is again twice to be corrected, namely, according to the Moon's Aspect to the Sun, and the Earth's mean Anomaly; of which Tables of Equations we have spoken in *Sett.* 9. *Prop.* 26. If the most correct Place of the Moon in the Syzygies be requir'd, you may also have the Proportion of that Correction, which we have spoken of in *Sett.* 10. *Prop.* 26.

But because the true Places of the Node and the Moon's *Apogæum* in the Ecliptic, and the Inclination of the Moon's Orbit to it are known by the Resolution of a Rectangular Spherical
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558 *The* ELEMENTS Book IV.

Triangle, in which, besides the Right-Angle, one of the Angles, and one of the Sides next to the Right-Angle are given, the Distance of the Apogæum from the Node, computed in the Moon's Orbit, is found; *viz.* the Hypothénuse. And the Moon's Place in its Orbit reckon'd from the Apogæum is found: And therefore the Distance of the Moon from the Node, reckon'd in the Lunar Orbit, will be also known. And as the Inclination of the said Lunar Orb to the Ecliptic is also given; there will be found both the Moon's Latitude, and its Place in the Ecliptic, either by the Resolution of the rectangular Spherical Triangle, comprehended between the Lunar Orbit, the Ecliptic, and the Circle of Latitude, or sooner by the Tables of Latitude and Reduction, (of which see *Sett.* XI. and XII. *Prop.* 26. where the said Triangle is resolv'd) whose Use is enough known, and this Distance of the Moon, from the Node computed in the Lunar Orbit, is call'd the *Argument of Latitude*.

Therefore the true Place of the Moon is found, (that is, its Place seen from the Center of the Earth) as well in Longitude as in Latitude. But to find what Situation a *Phænomenon* of given Longitude and Latitude has in a given Horizon, that is, what Altitude and what Vertical it has at a given Time, is a Problem easy to be solv'd by the Doctrine of Spherics: For it is the Inverse of *Prop.* 26. and 27. *B.* II. Now if the Moon's apparent Place, or that in which it is seen, be desir'd, you must in the first Table, (see *Prop.* 27.) note the Parallax agreeable to the Excentricity of the Moon's Orbit and Mean Anomaly before known, which you must correct by the Monthly and Annual *Æquations*, of which

Book IV. of ASTRONOMY. 559

which we spoke in the said *Prop.* 27. From the Horizontal Parallax (by *Prop.* 47. B II.) find the Parallax agreeable to the Altitude before found, which being substracted from the first, and you will have the Moon's apparent Altitude: And there is no other Difference between the Moon's true and apparent Place, besides this Decrease of Altitude or Parallax, and the common one arising from the Refraction of the Rays passing thro' the Atmosphere, of which see *Prop.* 64. and 65. B. II. As for the Mutations of Longitude and Latitude which are owing to the Parallax and Refraction, you may gather them from *Prop.* 63. and 68. B. II. Therefore the Moon's Place is found, as well the true one, as that seen from any Point of the Earth's Surface, both in Longitude and Latitude. Q. E. F.

PROPOSITION XXIX.

TO give a short Account of such of the fore-mention'd Corrections concerning the Moon's Motion as have been hitherto taken Notice of by the Constructors of Tables, and such of them as have been neglected.

First, they neglect all the Annual Equations or Corrections depending upon the various Distance of the Earth from the Sun, which have been explain'd in *Prop.* 18. and 19. and consider only that which arises from the encreas'd and diminish'd Periodical Time of the Moon about the Earth, from the diminish'd or encreas'd Distance of the Earth from the Sun; that is, which depends upon the Earth's Mean Anomaly. Nay, tho' this Inequality was observ'd by *Tycho*, *Kepler*, and *Horrox*, yet they confounded it with the others: For by reason thereof they were compell'd (by the Nature of Things) to reduce the Appa-

560 *The* ELEMENTS Book IV.

Apparent to the Mean Time in reckoning the Moon's Places another Way than they us'd to do in reckoning the Places of the Sun and other Fix'd Stars; and especially *Tycho* quite left out that Part of the *Æ*quation of Time which depends upon the Earth's Mean Anomaly, upon which also this Inequality of the Moon depends. But *Kepler* and *Horox*, having carried the Matter farther for their equating of Time, make use of an additional Physical Part, when the true and Astronomical one may be taken away, and so on the contrary. The most ingenious *Dr. Halley* was the first that separated this Inequality (which *Mr. James Gregory* suspected to belong to the Moon, in the *Schol.* of *Prop.* 86. of his *Optics*) from the others, in a Treatise subjoin'd to his *Catalogue of the Southern Fix'd Stars*. But it is plain from the foremention'd *Prop.* 18. and 19. that all the Inequalities of the Moon want a double Annual Correction, to make them true and exact.

Lastly, all the Monthly *Æ*quations arising from the Aspect of the Sun and Moon are neglected, except the Variation only. 'Tis true, indeed, that some of them are almost compensated by others; as for Example, the Monthly Variation of the Limits, and the Monthly *Æ*quation of the Nodes, so take off and correct one another, that they may be both neglected in determining the Moon's Latitude; but the Monthly *Æ*quations of the Motion of the Apogæum, and of the Difference between the least and greatest Excentricity, and those of the Nodes and of the Inclination of the Moon's Orbit to the Ecliptic, are the Basis and Foundation of the other Inequalities, which depend upon the Aspect of the Sun and the Apogæum, or
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Book IV. of ASTRONOMY. 561

of the Sun and the Nodes; as may be fully gather'd from the 10th, 11th, 12th, 13th, 14th and 15th Propositions. Likewise the Monthly $\mathcal{A}$ equation between the New Moon and Full Moon (of which we spoke in *Prop. 16.*) is neglected.

And those $\mathcal{A}$ equations which affect the Moon's Motion, whether real or apparent, (of which we spoke in *Prop. 2.*) arising from the greater Distance of the Moon from the Earth, in the Quadratures than in the Syzygies, are altogether neglected: Neither is the Difference of the Moon's Distance from the Earth consider'd otherwise than as arising from the Excentricity of the Orbit; and depending upon the mean Anomaly; whence also the Monthly $\mathcal{A}$ equation of the Horizontal Parallaxes (*Prop. 27.*) is also neglected.

But those $\mathcal{A}$ equations which depend upon the Aspect of the Sun and the Lunar Apogæum (mention'd in *Prop. 11th*, and *13th.*) and also those which depend upon the Aspect of the Sun and the Nodes (treated of in *Prop. 14th*, and *15th.*) are consider'd by Astronomers. For they are far the greatest, and make up a great Part of the Inequalities of the Moon's Motion.

For the Construction of Tables therefore, besides the foregoing $\mathcal{A}$ equations depending upon the Aspect of the Sun with the Moon's Apogæum or Nodes, an Annual and a Monthly $\mathcal{A}$ equation (which we spoke of before) is to be consider'd. But the $\mathcal{A}$ equations of the Moon's Motion, in several Lunar Tables, have Titles so different from the true Periods of these Inequalities, that it is difficult enough to assert which of them the Constructors consider'd, and which of them they neglected. And sometimes also some Inequalities are mix'd and confounded with others:

As for Example, they do not purely deduce, from the Moon's Orbit given, the *Æ*quation of the Excentric (or of the Moon's Center) but as it were confusely mix it with the others; as with that Part of the Inequality of the Variation that depends upon the Time between the Quadratures, which arises from the Situation of the Moon in respect of its Apogæum; see *Schol. Prop.* 18. But as they made their Tables not from known Physical Causes and their Periods, but only by attending to Observations; it is no Wonder if they did not rightly distinguish the Inequalities from one another, and dispos'd them under foreign or less proper Titles.

SCHOLIUM.

I have thought fit to subjoin the Theory of the Moon made use of by Sir *Isaac Newton*, by which this incomparable Philosopher has compass'd this extremely difficult Matter, hitherto despair'd of by Astronomers; namely, by Calculation to define the Moon's Place even out of the Syzygies, nay, in the Quadratures themselves so nicely agreeable to its Place in the Heavens (as he has experienc'd it by several of the Moon's Places observ'd by the ingenious Mr. *Flamsteed*) as to differ from it (when the Difference is the greatest) scarce above two Minutes in her Syzygies, or above three in her Quadratures; but commonly so little that it may well enough be reckon'd only as a Defect of the Observation. In this Calculation, which we give in the Words of the Author, he does not wholly mention all the Inequalities, whose Causes are above explain'd, nor those which are as yet only suspected; but omitting those which he knew wou'd take off one another, and others of less Moment

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Book IV. of ASTRONOMY. 563

Moment, he only confines those to *Æquations* and *Tables*, that have the greatest Force and produce the most sensible Effects.

Sir Isaac Newton's Theory of the Moon.

The Royal Observatory at *Greenwich*, is to the West of the Meridian of *Paris* $2^{\circ}. 19'$, of *Uraniburg* $12^{\circ}. 51'. 30''$, and of *Dantzick* $18^{\circ}. 48'$.

I put down the Sun and Moon's Mean Motions from the Vernal *Æquinox* at the Meridian of *Greenwich* as follows: *viz.* the last Day of *December* 1680 *Old Style*, at Noon, the Sun's Mean Motion, was $9^{\text{fig.}}. 20^{\circ}. 34'. 46''$. That of the Sun's Apogæum $3^{\text{fig.}}. 7^{\circ}. 23'. 30''$. The Moon's Mean Motion $6^{\text{fig.}}. 1^{\circ}. 45'. 45''$. That of the Moon's Apogæum $8^{\text{fig.}}. 4^{\circ}. 28'. 5''$. That of the ascending Node of the Moon's Orbit $5^{\text{fig.}}. 24^{\circ}. 14'. 35''$. And *December*, the last Day, 1700 *Old Style*, at Noon, the Sun's Mean Motion was $9^{\text{fig.}}. 20^{\circ}. 43'. 50''$. That of the Sun's Apogæum $3^{\text{fig.}}. 7^{\circ}. 44'. 30''$. The Mean Motion of the Moon $10^{\text{fig.}}. 15^{\circ}. 19'. 50''$. Of the Moon's Apogæum $11^{\text{fig.}}. 8^{\circ}. 18'. 20''$. And of the ascending Node $4^{\text{fig.}}. 27^{\circ}. 24'. 20''$. For in Twenty *Julian* Years, or in 7305 Days, the Sun goes thro' $20^{\text{rev.}}. 0^{\text{fig.}}. 0^{\circ}. 9'. 4''$. The Motion of the Sun's Apogæum $21'. 00''$. The Moon's Motion $267^{\text{rev.}}. 4^{\text{fig.}}. 13^{\circ}. 34'. 5''$. The Motion of the Moon's Apogæum $2^{\text{rev.}}. 3^{\text{fig.}}. 3^{\circ}. 50'. 15''$. Of the Node $1^{\text{rev.}}. 0^{\text{fig.}}. 26^{\circ}. 50'. 15''$. All the foresaid Motions are from the Point of the Vernal *Æquinox*. And if from them be substracted the Precession, or Retrograde Motion of the *Æquinoctial* Point it self, which has mov'd in the mean Time in *antecedentia*, *viz.* $16'. 40''$: The Motions will remain in respect of the Fix'd Stars

564 *The* ELEMENTS Book IV.

in 20 *Julian* Years, viz. the Motion of the Sun, 19^{rev.} 11^{fig.} 29°. 52'. 24". That of the Sun's Apogæum 4'. 20". Of the Moon 247^{rev.} 4^{fig.} 13°. 17'. 25". Of the Moon's Apogæum 2^{rev.} 3^{fig.} 3°. 33' 35". Of the Moon's Node 1^{rev.} 0^{fig.} 27°. 6'. 55".

According to this Computation, the *Tropical Year* is 365^{days} 5^{hours} 48'. 57". And the *Sydereal Year*, 365^{days} 6^{hours} 9'. 14¹/₂".

The Mean Motions of the Luminaries above-mentioned have several Inequalities.

First, there are the Annual *Æquations* of the said Mean Motions of the Sun and Moon, and of the Apogæum and Node of the Moon. The Annual *Æquation* of the Mean Motion of the Sun depends upon the Excentricity of the Earth's Orbit about the Sun, which is 16¹/₂ of such Parts as the Mean Distance of the Sun from the Earth is 1000; and therefore is call'd the *Æquation of the Center*: And when greatest, it is 1°. 56'. 20". The greatest Annual *Æquation* of the Moon's Mean Motion is 11' 49". of its Apogæum 20'. and of the Node 9'. 30". And those four Annual *Æquations* are proportionable to each other: Therefore when any one of them is the greatest, the three others are the greatest; and when any one is diminish'd, the others are also diminish'd in the same Proportion: Whence if the Annual *Æquation* of the Sun's Center be given, you have the other three *Æquations* agreeable to it; therefore the Table of that alone is sufficient. For if the Annual *Æquation* of the Sun's Center, agreeable to any Time taken out of it, be call'd *P*, and $\frac{1}{10} P = Q$, $Q + \frac{1}{10} Q = R$, $\frac{1}{2} P = D$, $D + \frac{1}{10} D = E$, and $D - \frac{1}{10} D = 2 F$; the Annual *Æquation* of the Moon, agreeable to that Time, will

Book IV. of ASTRONOMY. 565

will be *R*, that of the Moon's Apogæum *E*, and that of the Node *F*. It is to be noted, that if the *Æ*quation of the Sun's Center is to be added, the forefaid *Æ*quation of the Moon is to be subtracted, that of the Lunar Apogæum to be added, and the *Æ*quation of the Node to be subtracted: And on the contrary, if the *Æ*quation of the Sun's Center is to be subtracted, the Moon's *Æ*quation is to be added, that of the Lunar Apogæum to be subtracted, and that of the Node to be added.

There is another *Æ*quation of the Moon's mean Motion, depending upon the Situation of the Moon's Apogæum in respect of the Sun, which is the greatest when the Apogæum of the Moon is in the *Octant* (or at half Right-angles) with the Sun, and none at all when it is come to the Syzygies or Quadratures. This *Æ*quation, when greatest, rises to $3'. 56''$. the Sun being in the Perigæum; but if the Sun be in the Apogæum, it is not above $3'. 34''$. In other Distances of the Sun from the Earth, this greatest *Æ*quation is reciprocally as the Cube of that Distance. But when the Moon's Apogæum is out of the Octants, the said *Æ*quation becomes less, and is to the greatest *Æ*quation (supposing the Distance of the Sun from the Earth to remain the same,) as the Sine of twice the Distance of the Moon's Apogæum from the next Syzygy or Quadrature, to the Radius. This is added to the Moon's Motion, whilst the Moon's Apogæum passes from the Sun's Quadrature to the Syzygy; but is taken from it, as the Apogæum goes from the Syzygy to the Quadrature.

Moreover, there is another *Æ*quation of the Moon's Motion depending upon the Aspect of the Nodes of the Moon's Orbit with the Sun; and

it is the greatest when the Nodes are in the Octants with the Sun, and vanishes when they come to the Syzygies or Quadratures. This *Æ*quation is proportional to the Sine of twice the Distance of the Node from the next Syzygy or Quadrature, and when greatest rises to $47''$. This is added to the Moon's Motion, when the Nodes pass from the Sun's Syzygies to his Quadratures; but is subtracted, when they pass from the Quadratures to the Syzygies.

From the Sun's true Place take away the mean Motion of the Moon's Apogæum equated, as was before shewn; the Remainder will be the Annual Argument of the said Apogæum. Then reckon the Moon's Excentricity and the second *Æ*quation of its Apogæum in the following Manner, [*which also obtains in the Computation of any other intermediate Æquations:*] Let T (Fig. XXVII.) represent the Earth; TS , a Right Line joining the Earth and Sun; $TACB$ a Right Line drawn from the Earth to the mean Place of the Moon's Apogæum equated as before; let the Angle STA , be the Annual Argument of the said Apogæum; TA the least Excentricity of the Lunar Orbit, and TB its greatest Excentricity. Bisect AB and C , and about the Center C , with the Interval CA , describe the Circle AFB ; let the Angle BCF be made equal to twice the Annual Argument: Then TF being drawn will be the Excentricity of the Lunar Orbit; and the Angle BTf the second *Æ*quation of the Lunar Apogæum. In order to determine them, let the mean Distance of the Moon from the Earth, or the Semi-diameter of the Lunar Orbit consist of 1000000 Parts; its greatest Excentricity TB , will consist of 66782 of those Parts, and the least TA , 43319 such Parts, so that

Book IV. of ASTRONOMY. 567

that the greatest *Æ*quation of its Orbit, *viz.* when the Apogæum is in the Syzygies, shall be $7^{\circ}.39'.30''$, or perhaps of $7^{\circ}.40'.00''$; (for there is Reason to suspect that that Equation varies according as the Apogæum is in ☿ or ♀;) but when it sticks in the Quadrature, the said greatest *Æ*quation must be $4^{\circ}.57'.56''$. and the greatest *Æ*quation of the Apogæum $12^{\circ}.15'.4''$.

Having upon these Principles constructed a Table of the *Æ*quations of the Moon's Apogæum, and the Excentricity of its Orbit for every Degree of the Annual Argument, whence the Excentricity *TF* and the Angle *BTF* (namely the second and chief *Æ*quation of the Apogæum) agreeable to a given Time, may be easily taken; to the Place of the Moon's Apogæum first equated as before, add the lately found *Æ*quation, if the Annual Argument be less than 90° , or greater than 180° , but less than 270° ; but if it be otherwise, let it be subtracted from it: And then the Sum or Difference will be the Place of the Moon's Apogæum equated a second Time; which being subtracted from the Moon's Place, equated a third Time, the Remainder will be the Moon's mean Anomaly agreeable to a given Time. Moreover, from that mean Anomaly of the Moon, and the Excentricity of the Orbit lately found, you will have (by Help of the Table of *Æ*quations of the Center of the Moon calculated for every Degree of the mean Anomaly, and some Excentricities, *viz.* 45000, 50000, 55000, 60000, and 65000) the *Prosthaphæresis* or *Æ*quation of the Center of the Moon, as commonly; which being subtracted in the first Semicircle or 180° of the mean Anomaly, and added in the last, to the Moon's Place, hitherto three

568 *The* ELEMENTS Book IV.

Times equated, you will have the Moon's Place equated a fourth Time.

The Moon's greatest Variation, namely, that which happens when the Moon is in the Octants with the Sun, is almost reciprocally as the Cube of the Distance of the Sun from the Earth: Let it be taken equal to $37'. 25''$. when the Sun is in the Perigæum, and $33'. 4''$. when it is in the Apogæum: And let the Difference of this Variation in the Octants be reciprocally as the Difference of the Cubes of the Distances of the Sun from the Earth, and from thence construct a Table of the aforesaid Variation of the Moon in the Sun's Octants (or the Logarithms of it) for every ten, or six, or five Degrees of the mean Anomaly: And for the Variation out of the Octants, as the Radius is to the Sine of twice the Distance of the Moon from the next Syzygy or Quadrature, so let the above-found Variation in the Octant be to the Variation agreeable to the given Aspect, which being added to the Moon's Place above-found in the first and third Quadrant (reckoning from the Sun) or subtracted from it in the second and fourth, will give the Moon's Place equated a fifth Time.

Again, as the Radius is to the Sine of the Sum of the Distances of the Moon from the Sun and of the Moon's Apogæum from the Sun's Apogæum, (or to the Sine of the Excess of that Sum above 360°) so is $2'. 10''$. to a sixth Equation of the Moon's Place, which is to be subtracted, if the aforesaid Sum or the said Excess be less than a Semicircle, and to be added if greater.

As the Radius is to the Sine of the Moon's Distance from the Sun, so also let $2'. 20''$. be to a seventh Equation. Take that away when the
the

Book IV. of ASTRONOMY. 569

the Moon's Light encreases, and (on the contrary) add it when the Light diminishes, and you'll have the Moon's Place equated a seventh Time, which is also its Place in its proper Orbit. It is to be observ'd, that the *Æquation* which is here express'd by the Mean Quantity $2'. 20''$. is not always of the same Magnitude, but encreases and diminishes according to the Situation of the Lunar Apogæum. For if the Lunar Apogæum be in Conjunction with the Sun's Apogæum, the foresaid *Æquation* becomes greater by $54''$; if it be opposite to it, it is as much less: And it varies between the greatest Quantity $3'. 14''$; and the least $1'. 26''$. And these Things hold good when the Lunar Apogæum is in the Sun's Syzygies; but when it sticks in the Sun's Quadrature, the foresaid *Æquation* is to be diminish'd about $50''$ or one whole Minute, when the Moon's Apogæum and the Sun's are in Conjunction; but if they are in Opposition, I can't determine (for Want of Observations) whether it is to be encreas'd or diminish'd. Neither can I affirm any Thing certain concerning the afore-said Encrease or Decrease of the *Æquation* by $2'. 20''$, for Want of Observations exact enough.

If the sixth and seventh *Æquation* are encreas'd or diminish'd in a reciprocal Ratio of the Distance of the Moon from the Earth, that is, in a direct Ratio of the Moon's Horizontal Parallax; they will become more exact. And that may be readily done, if Tables be first constructed for every Minute of the said Parallax, and every sixth or fifth Degree as well of the Argument of the sixth *Æquation*, for the sixth *Æquation*, as of the Distance of the Moon from the Sun; for the seventh.

From

From the Sun's true Place take away the mean Motion of the Moon's ascending Node equated as before; the Remainder will be the Annual Argument of the Node; whence its second *Æ*quation will be computed in the following Manner: Let *T* (*Fig. XXVII.*) represent the Earth as before; *TS* a Right Line joining the Earth and the Sun: Moreover, let *TACB* represent a Line drawn to the Place of the Moon's ascending Node equated as before, and *STA* the Annual Argument of the Node. Let *TA* be taken in the same Ratio to *AB*, as 56 to 3, or 18½ to 1. Biffect *BA* in *C*, and about the Center *C*, with the Interval *CA*, or *CB*, draw the Circle *AFB*, and let the Angle *BCF* be equal to twice the Argument of the Annual Node found as before; and the Angle *BTf* will be the second *Æ*quation of the ascending Node, to be added whilst the Node goes from the Sun's Quadrature to the Syzygy, and subtracted as the Node goes from the Syzygy to the Quadrature. And so you have the true Place of the Node of the Moon's Orbit: Whence, from Tables constructed after the common Manner, may be computed the *Moon's Latitude*, and the *Reduction of the Moon from its Orbit to the Ecliptic*, supposing the Inclination of the Moon's Orbit to the Plane of the Ecliptic to be 4°. 59'. 35". when the Nodes are in the Sun's Quadrature; and 5°. 17'. 20". when they are in the Syzygies. From the Longitude and Latitude lately found, and the given Obliquity of the Ecliptic of 23°. 29", the Moon's Right Ascension and Declination will be found.

I put down 57'. 30". for the Parallax of the Moon in the Syzygies, when it is in its mean Distance from the Earth; 33'. 32". 32". for the
 Horary

Book IV. of ASTRONOMY. 571

Horary Motion; and $31'. 30''$. for the apparent Diameter. But in the Moon's Quadratures at a mean Distance from the Earth, I make the Horizontal Parallax to be $59'. 40''$. the Horary Motion $32'. 12''. 2'''$. and the apparent Diameter $31'. 3''$. The mean Distance of the Moon's Center when it is in an Octant with the Sun, is distant from the Earth's Center about $60\frac{1}{2}$ Semidiameters of the Earth.

I set down $10''$ for the Sun's Horizontal Parallax; and $32'. 15''$. for its apparent Diameter in its mean Distance from the Earth.

The Earth's Atmosphere, by refracting and dissipating the Sun's Light, casts a Shadow just as if it was an opaque Body, as far as the Height of 40 or 50 Geographical Miles: (I call a Geographical Mile, the 60th Part of a Degree of a great Circle on the Earth's Surface:) This Shadow in an Eclipse of the Moon falling upon the Moon, encreases the Earth's Shadow. And Seconds in the Moon's Disc answer to Miles in the Earth's Atmosphere: Therefore the Semidiameter of the Earth's Shadow projected in the Moon's Disc, is to be encreas'd about 50 Seconds, (or what comes to the same) in a Lunar Eclipse the Horizontal Parallax is to be augmented in a Ratio of about 70 to 69.

If several Places of the Moon nicely observ'd (chiefly about the Quadratures) be compar'd with the Places of it calculated for the same Time in the Theory above; it will appear whether or no there are any sensible Æquations wanting to make it more perfect.

PROPOSITION XXX.

TO determine the Time of a Mean Conjunction or Opposition of the Sun and Moon next following a given Time.

A mean Conjunction is, when the mean Place of the Sun is the same with the mean Place of the Moon in the Ecliptic: And a mean Opposition is when the former is in Opposition to the latter, in the Tables of the mean Motion of the Moon from the Sun, get the mean Distance of the Moon from the Sun for the given Time (reduc'd to a Mean) which if it be none, or six Signs, then there will be either a mean Conjunction, or a mean Opposition; if it be less, note the Difference, (or the Arc which the Moon will run thro' in order to overtake the Sun again, if you want a mean Conjunction, or in order to be six Signs distant from it, if you want a mean Opposition,) and from the Table of the mean Motion of the Moon from the Sun, in Days, Hours, and Parts of Hours, gather the Time requisite for that Arc to be pass'd thro'; namely, by always subtracting from the Arc, yet to be run thro', the next lesser Arc found in the Table, by marking the Time required to run thro' the subtracted Arc, and gathering those Times into one Sum: This being added to the given Time, will make up the Time of the mean Syzygy requir'd.

If the Times on each Side the Syzygies be requir'd, to the afore-found Time must be added the Time of one whole Revolution of the Moon about the Sun; and you will find the Time of the next mean Syzygy. And this may be repeated as often as you will, and so any Number of Syzygies may be found.

PRO-

PROPOSITION XXXI.

TO determine the Time of the next true Conjunction or Opposition of the Sun and Moon for a given Time.

A true Conjunction of the Sun and Moon happens, when the Sun's true Place is the same with the Moon's true Place corrected and reduc'd to the Ecliptic; and a true Opposition, when the former is in Opposition to the latter. Having found (by the foregoing Proposition) the Time of the next mean Syzygy (which need not be exactly determin'd, it being sufficient nearly to do it) find (by *Prop. 18. B. III. and Prop. 28. B. IV.*) the true agreeable Places of the Sun and Moon, which, if either they coincide together or are opposite, the mean and true Syzygies will also coincide; but if they don't, mark the Distance of the Moon from the Sun, namely, that Arc by which the Moon either has not yet reach'd or has pass'd the Sun or its Opposition. But to find out the Space of Time, in which the Moon will go thro', or has gone thro' such an Arc, you must use the following Method: Find out the Places of the Sun and Moon for the Hour before or after that Point of Time for which you already have them; by which Means you will have the Sun and Moon's Horary Motions, and consequently their Difference, or the Horary Motion of the Moon from the Sun for the given Time; that is, the Space of Time (*viz.* an Hour) requir'd for the Moon to move from the Sun thro' a known Arc in the Manner before found. And therefore, supposing the Sun and Moon to move equably in that small Time that the true Syzygy may be distant from the mean Syzygy, you will have the Time requir'd for the Moon

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574 *The* ELEMENTS Book IV.

to move from the Sun the Length of the Arc of its Distance from the Sun at the Time of the mean Syzygy. That Time added to the Time of the mean Syzygy, if the Moon has not yet reach'd the Sun or its Opposition, or taken from it, if it has already pass'd it, will give the requir'd Time of the true Syzygy.

If the greatest Exactness be desir'd (as is done in the Ecliptical Syzygies) at the Time of the true Syzygies lately found, find again the Sun and Moon's Places, and if they happen to be just together, or precisely opposite, the Thing is done; if not, let that Time be again corrected as before; *viz.* from the (almost) true Time of the true Syzygy already found, find the (exactly) true Time after the same Manner as you found the (almost) true Time, from the before found Time of the Mean Syzygy.

Those that make Calculations don't endeavour to find the Horary Motion of the Moon from the Sun, agreeable to the mean Syzygy, after the Manner that we have taught; but take it out of the Tables, where the Horary Motions of the Sun and Moon are set down according to the Degree of Anomaly which they respectively have; and that not amiss in the Sun, but in the Moon, which is subject to so many Inequalities, we ought to proceed more cautiously: But a farther Correction takes off all Error, that may arise from hence. After the same Manner, the Time of any other Aspects of the Sun and Moon may be determin'd.

SECTION

SECTION VII.

Of the Moon's Eclipse.

HAVING in the third Book spoken of the Earth's true Motion about the Sun, to which the apparent Motion of the Sun about the Earth answers in all Things; and in this Fourth Book, treated of the various Motions of the Moon seen from the Earth; and of all those Things which appear in the Heavens by means of the Diurnal Rotation of the Earth about its own Axis, in the Second Book; from what has been before laid down, it is easy to deduce whatsoever happens to the Sun and Moon. But because the *Eclipses* of the *Luminaries* are some of the most remarkable Appearances in the Heavens, and Astronomy owes a great deal to their Contemplation and Calculation, and to the finding out of the Causes of them; after the Manner of others we will treat of them by themselves, especially since here occur several Speculations worthy to be fully discuss'd. To which End we must lay down some Lemma's for each of the Eclipses concerning the apparent Semidiameters of the Luminaries, the Shadows of the Earth and Moon, &c. which are commonly call'd the *Diagram of Hipparchus*, because by the Comparison of them (which is easy in a Diagram) *Hipparchus* attempted to define the Magnitudes and Distances of the Luminaries; as it was before done by *Aristarchus*, in his *Book of the Magnitudes and Distances of the Sun and Moon*, and afterwards by *Ptolemy*;

576 *The* ELEMENTS Book IV.

Ptolemy, in the 15th and 16th Chapters of the Fifth Book of his great Construction.

PROPOSITION XXXII.

TO find the Sun and Moon's apparent Semidiameters agreeable to a given Time.

Let the Sun and Moon's apparent Semidiameters be observ'd at any Time ; and since, from the Theories of the Sun and Moon above given, their Distances from the Earth, either at the Time of Observation, or at any other Time, are given ; their apparent Semidiameters will be also given : For the apparent Semidiameters of a Spherical Body are reciprocally as its Distances from the Eye.

Astronomers make Tables of the Sun and Moon's apparent Semidiameters, for every Degree (or every fifth Degree) of a Luminary's mean Anomaly. And moreover, as the Semidiameter of a Sphere, in the same Distance from the Center of the Earth, is greater as it is nearer the Zenith, because it is nearer to the Spectator ; and the Difference of it in the Horizon and in the Zenith, is sensible in the Moon, but not in the Sun : Therefore they also make Tables of the Encrease of the Moon's apparent Semidiameter for every (or every fifth) Degree of the Elevation of the Moon above the Horizon.

PRO-

PROPOSITION XXXIII.

IF an opaque Sphere be oppos'd to a lucid one, the whole or full Shadow (that is, the Space of the ambient Medium to which no Ray of the lucid Sphere can come at) will be cast towards the Part opposite to the lucid Sphere, included in the Superficies of a Right Cone, whose Axis is a Right Line joining the Centers of the Spheres, and Base a Circle in the opaque Sphere, dividing its enlighten'd from its darken'd Part: And the vertical Angle of the Triangle passing thro' the Axis is nearly equal to the Difference of the Angles, under which the lucid Sphere is seen from the Center of the opaque one, and the opaque one from the Center of the lucid one. And if the Cone of the Shadow be cut any how by a Plane parallel to the Plane of the Base, the Diameter of the Circle of the Shadow so generated, or made, will appear from the Center of the opaque Sphere, under an Angle nearly equal to the Interval of two Angles; one of which is the Aggregate of the Angles under which the opaque Sphere is seen from the Center of the lucid Sphere, and from the Circle of the Shadow; and the other, the Angle under which the lucid Sphere is seen from the Center of the opaque one. [Fig. XXVIII.]

Let there be two Spheres, whose Centers are *A* and *B*, one of which is lucid and the other opaque; draw the Right Line *AB*, thro' which let a Plane pass, whose common Sections with the Spheres are the Circles *DF*, *EG*; draw *DE*, *FG* Tangents to the two Circles on the same Side, at the Points *D* and *E*, *F* and *G*, and let them be produc'd till they meet at *C*, with *AB* also produc'd. It is plain, that the whole, or full Shadow, which either of the Spheres, suppos'd opaque, being enlighten'd by the lucid one, casts, is contain'd under a Conic Surface

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578 *The* ELEMENTS Book IV.

describ'd by the Rotation of the Right Line DC about the Axis AB ; that is, by the Surface of a Right Cone whose Axis is the Right Line ABC , and Base a Circle whose Diameter is the Right Line DF or EG . Among Conic Surfaces I reckon a Cylandric one made when the Spheres happen to be equal.

From A draw Right Lines touching the Circle GE in H and I , and let AH be produc'd till it meets with CD at K , and from B draw Tangents to the Circle DF in L and M . By *Prop. 32. Elem. 1.* the Angle ACD is equal to AKD , minus KAB : But (by reason of the great Distance of the Spheres that we suppose) the Angle AKD is nearly equal to ABL ; and therefore ACD is equal to ABL , minus KAB . And taking the Doubles of them, the Angle FCD is equal to the Angle MBL , minus IAH .

Let us suppose either of the Spheres, as for Example, EG , to be opaque, and let the Cone of the Shadow ECG be cut any how by a Plane parallel to the Plane of the Base, and the Section (by *Prop. 4. of B. I. Elem. Conic.*) will be a Circle, whose Diameter NO is parallel to the Right Line DF , or EG , and whose Center P is in the Axis of the Cone; from which draw PQ , PR Tangents to the Circle EG , and join the Right Lines BO , BN . By *Prop. 32. Elem. 1.* the Angle PBN is equal to BNE , minus BCN : But (if BN be almost equal to BP which I suppose) BNE is nearly equal to BPQ , and it has lately been shewn, that BCN is equal to ABL , minus BAH ; and therefore BPQ is equal to the Aggregate of the Angles BPQ and BAH , minus the Angle ABL . And taking the Doubles of them, the Angle OBN is equal to the Aggregate of the Angles QPR , HAI , minus the Angle $LB M$. Q. E. D. If

Book IV. of ASTRONOMY. 579

If EG be suppos'd a lucid Sphere, and DF an opaque one, the same Things would hold good, if ON and all the other Lines depending upon it be drawn on the other Side of the Sphere DF , in respect of B .

SCHOLIUM.

If the Sphere DF represents the Sun; EG the Earth, reckoning whatever is circumsus'd about it, and hinders the Passage of the Rays as Part of it; ECG the Shadow of the Earth; and NPO Part of the Moon's Orbit: The Angle ABL will be the Sun's apparent Semidiameter, seen from the Center of the Earth; BAH the Sun's Horizontal Parallax: BPQ the Moon's Horizontal Parallax at the Distance BP ; therefore BCE the Vertical Semi-angle of the Cone of Shadow is equal to the Difference of the apparent Semidiameter of the Sun and its Horizontal Parallax; and PBN the apparent Semidiameter of the Earth's Shadow, (thro' which the Moon is to pass) seen from the Center of the Earth, equal to the Sum of the Horizontal Parallaxes of the Sun and Moon, when the Sun's apparent Semidiameter is taken from it. And therefore, three of these four being given, the fourth will also be known: As for Example, the Sun's Parallax, and consequently (by Prop. 46. B. II.) its Distance will from hence be known: For the Difference of the Moon's Horizontal Parallax and the Sun's Semidiameter, known by Observation, being subtracted from the apparent Semidiameter of the Earth's Shadow, known by the Observation of an Eclipse of the Moon, at the known Distance of the Moon, leaves the Parallax of the Sun.

568 *The* ELEMENTS Book IV.

But if DF be the Sun, EG the Moon, and NPO Part of the Disc of the Earth, (that is, of the Plane of a great Circle of the Earth, to which a Right Line joining the Centers of the Sun and Earth is perpendicular) in which the Shadow falls; the Semidiameter of the Shadow is seen from B , the Center of the Moon, under the Angle PBN equal to the Sum of the apparent Semidiameters of the Moon seen from the Centers of the Sun and Earth, less'n'd by the Sun's apparent Semidiameter seen from the Moon's Center. And since, by reason of the immense Distance of the Sun from the Earth, in respect of the Distance between the Moon and the Earth, the Sun's Diameter seen from the Moon appears almost under the same Angle as when seen from the Earth, and the Moon's Semidiameter seen from the Sun is to be look'd upon as of no Magnitude, as being almost subquadruple of the Sun's Horizontal Parallax; the Semidiameter of the Moon's Shadow, receiv'd upon the Disc of the Earth, is seen from the Center of the Moon, under an Angle almost equal to the Difference of the Angles, under which the Semidiameters of the Sun and Moon appear from the Center of the Earth.

But if the Semidiameter of the Sun, seen from the Earth, be greater than the Semidiameter of the Moon seen from it, the apparent Semidiameter of the Shadow (since it is equal to a less, after a greater is taken away) becomes negative; that is, after the Shadow is terminated and ends in the Vertex of the Cone, the little Circle, made by the Section of the opposite Cone, to the Cone of the Shadow, is seen from the Moon, under an Angle equal to the Difference of the Angles, under which the Sun and

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Book IV. of ASTRONOMY. 581

and Moon are seen from the Center of the Earth. But this little Circle is instead of a Circle of the Shadow, for it is the most obscure Part of the *Penumbrous* Circle, and (like a full Shadow) equally dark; that is, all the Points thereof become enlighten'd, as the Disc of the Sun exceeds that of the Moon appearing from the Earth: For all the foregoing Things are true of the Right Line *ON* drawn beyond *C*.

PROPOSITION XXXIV.

I*F an opaque Sphere be expos'd to a lucid one, besides the total or full Shadow, there will be a Penumbra (that is, a Space of the ambient Medium, from all of whose Points some Rays are kept off by the opaque Sphere) falling the contrary Way to the lucid Sphere, included by the Surface of a Right Cone, whose Axis is the same with Axis of the full Shadow; and the Vertical Angle of the Triangle passing thro' the Axis is equal to the Sum of the Angles, under which the lucid Sphere is seen from the Center of the opaque one, and the opaque one from the Center of the lucid one. And if this penumbrous Cone be cut any how by a Plane parallel to the Plane of the Base, the Diameter of the penumbrous Circle so generated (taking in the small Circle of full Shadow at its Center, if there be any such) seen from the Center of the opaque Sphere, appears under an Angle equal to the Aggregate of three Angles; viz. those Two under which the opaque Sphere is seen from the Center of the lucid one, and from the Center of the penumbrous Circle, and the Third under which the lucid Sphere is seen from the Center of the opaque one.*
[Fig. XXIX.]

Things remaining as in the foregoing Prop. draw the Right Lines *DE*, *FG*, touching the
H h h 3 Circles

582 *The* ELEMENTS Book IV.

Circles on the alternate Sides, and crossing each other in the Point C of the Right Line AB . It is manifest, that besides the whole Shadow, there is also projected about it a Penumbra the contrary Way to the lucid Sphere, in respect of the opaque one, because all the Space contain'd under the Conic Surface, generated by the Right Line DE , produc'd beyond the opaque Sphere, and turn'd round the Axis AB , is such, that from any Point of it some Rays of the lucid Sphere are kept off by the Interposition of the opaque One, and therefore it is not so much enlighten'd as the Parts round about this Conic Space, from which no Rays are kept off. And those Parts of this Space which are nearest to the full Shadow, (*viz.* those about its Middle, from which all the Rays of the lucid Sphere are kept off) are more obscure than those which are more remote; as being enlighten'd by Rays flowing from a less Part of the lucid Sphere: And the Surface that circumscribes the Penumbra, is that of a Right Cone, whose Axis is the Right Line AB , extended beyond the opaque Sphere, and Base, a Circle describ'd on the Diameter DF , or EG .

After the same Manner as in Proposition preceding, from A draw Right Lines touching the Circle GE in H and I , of which let AH meet with DE produc'd at K : And from B let Right Lines be drawn touching the Circle DF in L and M . By *Prop. 32. Elem. 1.* the Angle ACD is equal to AKD and BAK together; but AKD is nearly equal to ABL ; and therefore ACD is equal to ABL and BAH together. And taking the Doubles of them, the Angle FCD is equal to the Angles MBL and IAH taken together.

Let

Book IV. of ASTRONOMY. 583

Let us suppose either of the Spheres, as for Example, EG to be opaque, and let the penumbrous Cone be cut by a Plane parallel to the Plane of the Base, and the Section will be a Circle, whose Diameter NO is parallel to the Right Line DF or EG , and whose Center P is in the Axis of the Cone; from which, let PQ , PK be drawn touching the Circle EG , and let the Right Lines BO , BN be drawn. By *Prop. 32. Elem.* 1. the Angle PBN is equal to BNC and BCN together: But (if BN be nearly equal to BP , which I suppose) BNC is nearly equal to BPQ ; and it has been lately shewn, that BCN is equal to ABL and BAH together; and therefore PBN is equal to the Aggregate of the three Angles, BPQ , BAH and ABL . And taking the Doubles of them, the Angle OBN is equal to the Aggregate of the three Angles, QPR , HAI , and $LB M$.

SCHOLIUM.

If the Sphere DF represents the Sun, EG the Earth, and NPO Part of the Lunar Orbit; the Semidiameter of the Penumbra, seen from the Center of the Earth, is equal to the Sum of the Horizontal Parallaxes of the Sun and Moon, and the Sun's apparent Semidiameter.

But if the Sphere DF represents the Sun, EG the Moon, NPO that Part of the Disc of the Earth in which the Penumbra falls; the Semidiameter of the Penumbra, seen from the Center of the Moon, appears under an Angle equal to the Aggregate of the apparent Semidiameters of the Moon, seen from the Centers of the Earth and Sun, and the apparent Semidiameter of the Sun, seen from the Center of the Moon; and therefore (for the Reasons alledg'd in the

584 The ELEMENTS Book IV.

Schol. of the foregoing Proposition) under an Angle nearly equal to the Aggregate of the Semi-diameters of the Luminaries, seen from the Center of the Earth.

PROPOSITION XXXV.

TO determine, whether a Full-Moon to come, will be eclipsed; and, if it be, to define its Species; that is, to shew whether it will be Partial or Total; whether it be cum mora or sine mora; that is, whether the whole Moon makes any Stay in the Earth's Shadow, or not. [Fig. XXX.]

By Prop. 31. find the Time of the true Full-Moon upon a given Month, for which, (by Prop. 18. B. II.) find the Sun's Place, and you will consequently have the Point opposite to it: Let it be express'd by O , (Fig. XXX.) where $C\Omega$ represents the Ecliptic. Find the Places of the Moon's Nodes for the same Time, (of which let Ω be the nearest to the Point O) and also the Inclination of the Moon's Orbit to the Ecliptic. Let the Circle of Latitude OP meeting the Moon's Orbit ΩP in the Point P , where the Moon is at the Time of the true Full-Moon, be suppos'd to be drawn. In the Heaven of the Moon the Circle of the Shadow, express'd by $R G$, is constantly about the Center O , being a Circle made by the Section of the Conic Shadow of the Earth, by a Position of the said Heaven parallel to the Base of the Cone, which if the Moon $M H$ does in part, or wholly enter, there will be an Eclipse; otherwise none. But the Moon comes nearest of all to O , and therefore, if it is immers'd in the Shadow, it is most of all immers'd, when its Center is in the Point L , which the Arc of the great Circle OL , perpendicular to the Moon's Way, shews: For the

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Point

Book IV. of ASTRONOMY. 585

Point L comes nearest to O than any other of the Way of the Moon. But it is plain, that if the Arc OL exceeds OR and LM , the apparent Semidiameters of the Shadow and of the Moon together, (as in the 1st Fig. XXX.) the Moon will go on it its Way ΩP , quite clear of the Shadow. If OL be equal to the Sum of the said Semidiameters, the Moon will indeed touch the Shadow, but neither enter it nor be eclipsed. If the Arc OL be less than the Aggregate of the Semidiameters of the Moon and the Shadow, and yet greater than their Distance, (as in the 2d Fig. XXX.) the Moon will be eclipsed; but the Eclipse will not be Total, but Partial, viz. of so many Digits as there are 12th Parts of the Diameter of the Moon's Disc immers'd in the Shadow. But if the foresaid Arc OL be equal to the said Interval of the said Semidiameters, there will indeed be a Total Eclipse, but one *sine mora*: For as soon as the whole Moon is immers'd into the Shadow, the Moon's Disc, touching the Circle of the Shadow within, will immediately begin to emerge as the Moon goes ever so little forward. But if the Arc OL be less than the foresaid Semidiameter of the Shadow and Interval of the Moon, then the whole Moon will be immers'd into the Shadow, some Part of the Shadow being extended beyond the Moon, (as in the 3d Fig. XXX.) whence the Moon's Eclipse will not only be Total, but last some time; that is, will be Total, *cum mora*, and that more or less, according to the Quantity of the Arc OL , the Thickness of the Shadow, and Velocity of the Moon.

Now the Way to find the Quantity of the Arc OL , is as follows; In the spherical Triangle ΩOP (Fig. XXXI) Rectangular at O , ΩO , the Distance

Distance of the Node from the Point opposite to the Sun, and the Angle at Ω of the Inclination of the Moon's Way to the Ecliptic, at the Time of Conjunction, are given, from whence may be found the Angle at P , and the Side OP . Again, in the Triangle POL , besides the Right Angle at L , you have given the Side and Angle lately found, from which may be discover'd OL . But you may readily find out OL from the Tables of the Moon's Latitude, by taking it equal to the Latitude for the Argument of Latitude equal to ΩO : For if you suppose $\omega\lambda$ equal to ΩO , it is plain, that the Arc $\omega\lambda$, perpendicular to the Ecliptic, is equal to the Arc OL ; for in the Triangles $OL\Omega$, $\omega\lambda\Omega$, having a common Angle at Ω , all Things are equal. Moreover, for the Time of the Full-Moon before found, the apparent Semidiameter of the Moon is (by *Prop.* 32.) given, and (by *Schol. Prop.* 33.) the apparent Semidiameter of the Shadow of the Earth, where the Moon goes thro' it; and therefore the Sum or Difference of the Semidiameters of the Shadow, and of the Moon, may be compar'd with the before-found Arc OL , and from it you may find out of what Kind the Moon's Eclipse will be, according to what has been above demonstrated.

By the Shadow of the Earth, the Total Shadow is commonly only meant; for that alone takes away all the Light of the Moon, because no Rays of the Sun can come at it: Neither does the Moon appear to us, Inhabitants of the Earth, to be eclips'd, unless that happens. But the Moon does truly begin to be eclips'd as soon as it enters into the Earth's Penumbra, because in that Case it is less enlighten'd, the opaque Earth intercepting several of the Sun's Rays. But the Effects of the Earth's Penumbra itself
upon

Book IV. of ASTRONOMY. 587

upon the Moon, are manifest to us Inhabitants of the Earth. Hence it happens, that when the Moon is about to be eclips'd, it first grows dull and pale, especially in that Part of it which is near the Shadow. For (as it hath been explain'd in the 18th *Prop. B. I.*) before the whole Sun be cover'd from any Part of the Moon; that is, before the Moon comes to the whole Shadow, Part of the Sun is intercepted from it by the Earth, and the greater Part of the Sun, by its Limb, which is the nearest to the total Shadow: But the whole Sun does illuminate the Moon more clearly than Part of it, and a greater Part of it more than a lesser. If therefore we speak of such an Eclipse of the Moon, instead of the whole Shadow of the Earth (of which alone we spoke above) you must take the Penumbra, whose Semidiameter will be found by *Schol. Prop. 34.*

COROLLARY.

Hence will the Moon's *Ecliptic Terms* (as they call 'em) be defin'd. For if OL be less than the Sum of the Semidiameters of the Shadow and of the Moon, there will at least be some Eclipse. Therefore in the Triangle ΩLO , having, besides the Right Angle at L , the Angle $L\Omega O$, and the Side OL equal to the said Sum of the Semidiameters, the Side ΩO will be found, namely, the Distance of the Moon from the Node reckon'd in the Ecliptic; which if it be such, at the Time of the true Full-Moon, the Moon will pass by the Shadow, without any Decrease of its Light: But if at the Moment of the said Full-Moon, the Moon's Distance from the Node be less than the above-found Ark ΩO ; the Moon will suffer some Eclipse: But by reason of the Smallness of the

the Angle at Ω and of the Sides of the Triangle ΩOP , OP differs very little from OL , and therefore the Ecliptical Terms may safely enough be plac'd in it, when at the time of the true Full-Moon, the Latitude is equal to the Sum of the Semidiameters of the Moon and of the Shadow.

L E M M A.

If, in the Surface of a Sphere, there be given BC, DE, Arcs of great Circles, which, in given Angles, stand upon ABD, which is also a great Circle, and also the Arc BD; to find the Angle A, which a great Circle, drawn thro' C and E, comprehends with BD. (Fig. XXXII.)

Let the great Circles BE , CD , be drawn. In the Triangle CBD , the Sides CB , BD , together with the Angle CBD , which they contain, being given, you have also the Side CD , and the Angle CDB : But the Angle BDE is given, therefore CDE will also be known. Moreover, in the Triangle CDE , the Sides CD , DE , and the Angle CDE being given, the Angle CED will also be known. Lastly, if in the Triangle ADE , the Angles ADE and AED , together with the Side DE be known, the Angle A will be found.

PROPOSITION XXXVI.

THE Inclination of the Moon's Orbit to the Ecliptic, and the Horary Motion of the Sun and Moon being given, to find the Inclination of the Moon's Way, from the Sun to the Ecliptic, or to the Circle of Latitude, agreeable to a given Distance of the Moon from the Node. [Fig. XXXIII.]

It has been shewn, in the foregoing Proposition, how the Inclination of the Moon's absolute Way, or of its Orbit to the Ecliptic, may be found for a given Time. But for calculating the Eclipses of the Luminaries, you must know how the Moon's Way from the Sun, or from a Point opposite to it, is at that time inclin'd to the Ecliptic; and the Motion of the Moon from the Sun is to be consider'd as if the Sun stood immoveable, or rather as if the Sun, after having mov'd a little Way, shou'd be drawn back to its old Place, and the Moon also as much, in Lines parallel to each other. In such a Case it appears, that the Way which the Moon, going from the Sun, enters into in the forementioned Sense, is different from the Moon's absolute Way, and differently inclines to the Ecliptic.

In Order to find this Inclination, let SC represent the Ecliptic, LB the Moon's absolute Way, $\oslash$ the Node. Let S be the Sun, or any other Point of the Ecliptic, having a given Distance from the Sun; L the Moon's Place, and ZA its Latitude. In a small Part of Time, (as for Example, an Hour or two) let the Sun be suppos'd to have mov'd from S to σ , and the Moon from L to λ . Now suppose that the Sun be drawn back from σ to S , and the Moon as much drawn back, viz. from L to λ , along the Right Line λI , equal and parallel to σS ; draw the

590 *The ELEMENTS Book IV.*

the Arc Ll , which will be the Way which the Moon, mov'd from the Sun, goes thro', tho' its absolute Way be the Circle LB . To determine that Way Ll , there are given LA , the Latitude of the Point L , given in the Moon's true Way, and la the Latitude of the Point l , which is the same as λa , the Latitude of the Point λ , also given in LB , the true and absolute Way of the Moon; because its Distance λL , from the given Point L , is given; namely, the Way of the Moon, gone thro' in a given Time; for Aa , the Difference of Longitude, is given, that is, the Excess of the given Arc Aa , above $a a$, or S ; and therefore (by the foregoing Lemma) you will have given the Angle between Ll , the Moon's Way from the Sun, and the Ecliptic; and the Angle Lla , comprehended between the said Way and the Circle of Latitude, al . $\angle E. F.$

COROLLARY.

By how much less the Motion of the Moon from the Sun is, by so much (*cæteris paribus*) the Angle between the Ecliptic and lL is greater; for λl remaining the same, it is plain that by how much $L\lambda$ is less, by so much the Angle λLl is greater: And by how much the Point L is nearer to the Node, by so much (*cæteris paribus*) will the Angle between lL and the Ecliptic be greater; for by how much L is nearer to Ω , by so much will λL , a small Part, considered as a Right Line, contain a greater Angle with ΩC , and therefore it will hold much more in lL : But the Angle $l\lambda L$ is also greater, because λl is every where parallel to ΩC .

SCHO-

SCHOLIUM.

Astronomers construct Tables of those Angles, which the Moon's Way from the Sun comprehends with the Ecliptic, according both to the various horary Motion of the Moon from the Sun, and to the Moon's Distance from the Node, that the Way of the Moon, through the Shadow of the Earth, or that of the Shadow of the Moon, over the Disc of the Earth, may be readily and easily determin'd. They calculate these Tables, for every Minute of a Degree of the Excess of the Moon's Motion, above the Solar, in one Hour: But as they serve chiefly for Eclipses, the Tables of them are only carried on so far as such a Distance from the Node, in which the Moon may run into the Earth's Shadow, or project its own Shadow on any Part of the Earth.

PROPOSITION XXXVII.

TO determine the Mean, or Middle Time, of a Lunar Eclipse, happening in a given Month. [Fig. XXXIV.]

Let ΩK represent the Ecliptic; ΩE the Way of the Moon from the Sun, found by the foregoing Prop. for the Mean Eclipse, (if you will have it as it would be, for the whole Time of the Eclipse, which may be done safe enough; otherwise you may work more exactly, by often finding that Way of the Moon from the Sun;) Ω the Node next to it; $DRCG$ the circular Section of the Shadow of the Earth, made by the Heaven of the Moon, which the Moon, in its Eclipse, runs into; whose Center, or rather Pole, is O . Round the same Pole, or Center O , describe another Circle AFB , the Circle of

592 *The ELEMENTS* Book IV.

of the Shadow, in such Manner that AC , the Distance between their Circumferences, may be equal to the Moon's apparent Semidiameter. Let these Circles meet with the Moon's Way in M, N, A, B . If $\odot E$ does not cut the Circle AFB , there will be no Eclipse, by the foregoing 35th *Prop.* Let OA, OB , Arcs of great Circles, be suppos'd to be drawn, meeting with the Circle DCG , at C and D ; and OP , a Circle of Latitude, meeting with the Moon's Way at P , as before. If through O be drawn ORF , an Arc of a great Circle, perpendicular to the Circle $\odot E$, meeting it at L ; the Arcs LB, LA , and the Arcs LM, LN , will (by the *El. of Spherics*) be equal. Therefore the Moon, (which we suppose to move equably at the Time of the Eclipse) takes up as much Time in going from A to L , as from L to B . Let Circles be suppos'd drawn round the Centers or Poles AB , at the Distances AC, BD , which will rightly represent the Moon (because AC is to OC , as the Semidiameter of the Moon to the Semidiameter of the Shadow) and will touch the Circle DCG , viz. at C and D . When the Moon's Center is in the Point A , the Moon's Eclipse begins, (because the Limb of the Moon touches the Limb of the Shadow at C ;) and that Center being come on to B , the Eclipse ends for the same Reason; and therefore the Middle of that Eclipse happens whilst the Moon's Center is in the Point L ; viz. nearly that in which when the Center is, the Moon will be most immers'd in the Shadow, as has been shewn in the foregoing *Prop.* 35. We shall likewise shew, that at the time in which the Moon's Center comes to the Point L , in Total Eclipses, will be the Middle of the whole Obscurity, when

Book IV. of ASTRONOMY. 593

when the whole Motion is immers'd in the Shadow; or, in a Word, it is the Middle of the Passage of the Center of the Moon from N to M , because NL is equal to LM . The Place of the Node, and the Inclination of the Limits, are look'd upon as unchangeable, not only by reason of the small Time of the Duration of the Eclipse, but because when the Nodes are in the Syzygies with the Sun, these are truly immovable, by *Prop. 14* and *15*. Therefore the Problem comes to this, *viz.* That the Time is to be determin'd, when the Moon's Center is found at L . For doing of which, you must (by *Prop. 31*.) find the Time of the true Opposition; that is, the Time when the Center of the Shadow will be at O , and the Moon at P ; unless these two coincide, which happens when in the Time of the true Opposition the Sun is in one of the Moon's Nodes, in which Case OL , PL , and OP , vanish: But the Magnitude of the Arc LP , has been found in the foregoing *Proposition*, by the Resolution of the Triangles QOP , OLP . Therefore the Time required for the Moon to run thro' PL , must be found, and not only PL , but PL beyond the Sun's Motion in the Ecliptic; (for whilst the Moon passes from the true Opposition to a Point of its Orbit, next to the Center of the Shadow, or from this Point to that true Opposition, the Sun, and consequently its opposite Point, which is the Center of the Shadow, goes on in the Ecliptic;) which is done by finding the Horary Motions of the Sun and Moon, agreeable to the Time of the Eclipses of the Sun and Moon, and from thence their Differences agreeable to the Time of the Moon's Horary Motion from the Sun, as in the 31st *Prop.* whence, by the Analogy there, you will find the Time requir'd for

the Moon to move from the Sun through the known Arc LP . By the Inspection of the Figures adapted to the respective Cases, it is plain, that the Time in which the Moon's Center is in the Point L , near to the Center of the Shadow, goes before the Time of the true Opposition, when the Motion is come to P , in the present Case; that is, when the Moon has just pass'd the ascending Node, which obtains also when it has just pass'd the descending Node: But when the Moon is not yet come to the Nodes, the Time of the Middle of the Eclipse is after the Time of the true Opposition: Whence it is evident, when the Time between the true Opposition and the Middle of the Eclipse is to be added to the Time of the true Opposition, and when it is to be subtracted from it, to make up the Time of the Middle of the Eclipse.

Astronomers commonly make Tables of the Time, in which the Arc LP is run thro', to be added or subtracted from the Time of the true Syzygy, to make the Time of the Middle of the Eclipse, or of the greatest Obscurity. This Time varies according to the different Velocity of the Motion of the Moon from the Sun, and the different Distance from the Node. They continue these Tables, as well as those of which we spoke before, in the *Schol. Prop.* 36. for every Degree of the ecliptic Limits.

PROPOSITION XXXVIII.

TO determine the Time of the Beginning and End of the Eclipse of the Moon that happens in a given Month, and also of the whole Immersion of the Moon into the Shadow; and from thence also to determine the Duration, as well of the whole Eclipse, as of the whole Darknefs. [Fig. XXXV.]

Book IV. of ASTRONOMY. 595

Things being as in the foregoing *Proposition*, (except that in the first *Fig. 35.* the Moon in its Eclipse, is supposed to tend towards the descending Node,) it appears that the Eclipse begins when the Center of the Moon is in *A*, the Moon's Limb touches the Limb of the Shadow at *G*, by *Prop. 4.* of the *II^d B. of Theod.* Therefore in the Triangle *ALO*, Rectangular at *L*, you have given, besides the Right Angle, the Hypotenuse *OA*, that is, the Sum of the Semidiameters of the Moon and of the Shadow, and *OL* the least Distance of the Center of the Shadow from the Center of the Moon, found by *Prop. 35.* Therefore the Arc *AL* will be found, and consequently the Time requir'd for the Moon (by the Excess of its Motion above the Sun's Motion) to go thro' it; that is, half the Duration of the whole Eclipse. But the Time of the Middle of the Eclipse is known by the foregoing *Proposition*; therefore its Beginning and End is known.

For determining the Beginning of the whole Immersion of the Moon into the Shadow of the Earth, Things remaining as before, observe the *II^d Fig. 35.* where the Moon's Eclipse happens, whilst the Moon goes towards the ascending Node, and there let the Circle *AFB* be within the Circle of the Shadow *CDG*; a Circle drawn with the Distance *AC*, about the Center *A*, will (by *Prop. 3. B. II. of Theod.*) touch *CGD* within, and therefore will rightly represent the Moon going on in its proper Way, as soon as it is all immers'd into the Shadow *GCD*. To find the Point of Time when the Center of the Moon is in *A*, you have given in the Rectangular Triangle *ALO*, the Side *OL* (by *Prop. 35.*) and *OA*, which, by Construction, is equal to the Difference of the Semidiameters of the Moon, and of the

596 *The* ELEMENTS Book IV.

Shadow; therefore the Arc AL , of the Moon's Way to be run thro' by the Moon's Motion, above that of the Sun, (from the Time that the whole Moon is immers'd into the Shadow, until the Middle of the Eclipse) will be known. Whence, by knowing the Motion of the Moon from the Sun, agreeable to a certain Time, (*viz.* an Hour,) at the Time of the Eclipse, you will know what Time must be taken up for the Moon (by the Excess of its Motion above that of the Sun) to run thro' the Arc AL ; (that is half the Duration of the whole Darkness. But by the foregoing *Proposition*, the Moment of the Middle of the Eclipse is known, whence the Beginning and End of the total Immersion of the Moon, into the Shadow of the Earth, will be known. *Q.E.F.*

SCHOLIUM.

In the trigonometrical Resolution of the Triangles OLP , ALO , BLO , as also of the Figures $Ll^{\wedge}$, and $AL^{\wedge}a$, in the foregoing *Proposition*, they may be look'd upon as Rectilineal, (which also often happens) by reason of the Shortness of the Sides: For so small a Portion of a spherical Surface may safely be taken for a Plane.

PROPOSITION XXXIX.

TO define the Quantity of an Eclipse happening at a given Month, as well in its Middle, as at any other Time. [Fig. XXXVI.]

Things being as before, in the Figure representing the Moon's Eclipse happening a little after it has pass'd the descending Node, we must find the Quantity of the Middle of the partial Eclipse. Let the Disc of the Moon in the Middle of the Eclipse, drawn about the Center C , (already often determin'd) cut the Right Line
OR

Book IV. of ASTRONOMY. 597

OR at the Point *M*. Having then *OR* given, (in Parts of a great Circle in the Celestial Sphere) and by *Prop.* 35. having found the Arc *OL*, which is the least Distance of the Center of the Moon from the Center of the Shadow in the same Parts; you will have (by 4. *Enc. Dat.*) *LR*, the Difference of those Arcs: But *LM*, the Moon's Semidiameter is given; whence (by 3 *Dat.*) *MR*, the Sum of them is also given in the same Parts; and therefore (by 1 *Dat.*) the Ratio to the Moon's whole Diameter *MH*, also known in Parts of a Circle: And *MR* is Part of the Diameter of the Moon darkened in the Middle of the Eclipse; therefore the Ratio of the darkened Part of the Diameter to the whole Diameter is given, and consequently the Number of Digits, and Parts of Digits, of the darkened Diameter.

After the same Manner may be found the apparent Quantity of an Eclipse for a given Time, either before or after the true Full-Moon. Having, by the foregoing Method, found the Moon's horary Motion from the Sun, the Arc of the Moon's Distance from the Point *P* will be determin'd, in which Point the true Full-Moon happens. Let it be *PS*; and so the Place of the Moon's Center for the given Time is *S*. Draw the Arc *OS*, meeting with the Moon's Disc at *M* and *H*, and the Circle *CDG* at *R*. In the Triangle $\triangle O P$, besides the Right Angle at *O*, you have the Side $\triangle O$, and the Angle $\triangle O P$; whence you will find the Side *OP* (already defin'd) and the Angle $\triangle O P$. Again, in the Triangle *OPS*, you have the two Sides *PO*, *PS*, and *OPS*, the Angle which they contain; whence you will also have the Arc *OS*, and consequently the Difference between it and the known

598 *The* ELEMENTS Book IV.

Arc SM , viz. OM : But OR is given, and consequently MR the Difference of OR , OM ; and all these in Parts of a Circle as before; and therefore, as in the foregoing Case, the Quantity of the Eclipse in Digits, and Parts of Digits.

If MR exceeds the Moon's whole Diameter (*Fig. XXXVII.*) the Eclipse is said to be of so many *Digits*, as MR contains 12th Parts of the Moon's Diameter; but improperly, because, strictly speaking, the Moon cannot be eclipsed more Digits than its Diameter contains; but MR is made up of the Moon's Semidiameter MS and SR , the least Distance of the Center S from CDG , the Margin of the Shadow, which (by *Elem. Spheric.*) is an Arc of a great Circle, joining the Centers O and S .

COROLLARY 1.

Hence may easily be determin'd the Places of the Earth, from which a given Phasis of the Moon's Eclipse may be seen. For since by this *Proposition*, the Place of the eclips'd Moon for that Phasis is given, and the Hour in a given Place of the Earth, you will (by *Corol. 36. B. II.*) have that Place of the Earth, to which the Moon is Vertical, and all the Places, which are distant from it, less than a Quarter of a Circle, will see it; those which are farther distant, will not see it, and those, which are just at the Distance of a Quadrant, will see it in the Horizon.

COROLLARY 2.

By Help of the three last Propositions, you may exactly draw a Scheme of the Moon's Eclipse upon a Plane. For since so small a Part of the Heaven of the Moon, as the Shadow (or even the

Book IV. of ASTRONOMY. 599

the Penumbra) of the Earth, together with the Moon touching it at the Beginning of the Eclipse, may safely be taken for a Plane: Having drawn Right Lines, which cross one another in an Angle equal to that which the Way of the Moon contains with the Ecliptic, representing those Circles, and the next Node by their Concurrence; in the Right Line which represents the Ecliptic, take a Point distant from the common Concurrence of the Right Lines, by so many equal Parts of any Kind, as are the Parts (for Example, Minutes) of the Circle, which measure the Distance of the Center of the Shadow, or Point opposite to the Sun from the Node; and about that Center describe a Circle representing the Earth's Shadow, with a Semidiameter of as many of the equal Parts taken before, as the Semidiameter of the Shadow contains Minutes in the Heavens; then drawing Right Lines from that Center, perpendicular to the Ecliptic, and to the Moon's Way, and in the Moon's Way you will have the Points of the true Full-Moon, and of the Middle of the Eclipse. And if about the same Center, both within and without this Circle of the Shadow, you draw two other Circles distant from the first, by the Space of so many equal Parts taken before, as the Moon's apparent Semidiameter contains Minutes; the Intersections of those Circles, with the Right Line representing the Moon's Way, will, in the Scheme, shew the Points in which the Moon's Center is, at the Beginning and End of the Eclipse, and of the total Obscurity. Moreover, since the horary Motion of the Moon from the Sun, and the Number of Hours in a given Place, at the Time of the true Full-Moon, and also of the Beginning, Middle, and End of the Eclipse,

by the foregoing Proposition are given; the Right Line, which represents the Moon's Way, may be divided into two equal Parts, mark'd by Numbers, which denote the Hours, reckon'd in a given Place of the Earth, where the Moon is in the like Points of the Heaven. And the Phasis of the eclipsed Moon, happening at any Time, will appear very visible, by this Scheme, if from a Point in the Moon's Way, (mark'd with the Number of the given Time) for a Center, you draw a Circle, with an Interval representing the Moon's apparent Semidiameter: For that Part of this Circle, falling within the Circle representing the Shadow, will represent the eclipsed Part of the Moon; and the Part which falls without the Circle of the Shadow, (if any Part does fall without it) will shew the enlighten'd Part: Therefore the eclipsed Digits manifestly appear, as also the Ratio between the whole Disc of the Moon, and its enlighten'd Part (if it has any;) viz. the Difference between the two Segments of the given Circles, which is evident from the Dimension of the Circle.



SECTION

SECTION VIII.

Of the Sun's Eclipse,

HAVING thus spoken of the Lunar Eclipse, we come now to the Solar one. And first, we will speak of it universally, and consider the Moon's Shadow falling upon the Earth, and how it might truly and properly be called an Eclipse of the Earth: For in this Eclipse, the Earth does the same as the Moon in the Lunar Eclipse described above, and the Moon does in this Case, what the Earth did in that. Afterwards we will consider this Phænomenon of the Eclipse of the Earth, as caused by the Moon's covering some Part of the Sun from an Inhabitant of the Earth, immers'd in the Moon's Shadow, and looking up to the Heavens; and so the Sun seems to be eclips'd, for that Reason; whence this Phænomenon has been call'd an Eclipse of the Sun.

PROPOSITION XL. LEMMA.

IF the apparent Distance between the Sun and the Moon, seen from the Center of the Earth, be equal to the Moon's Horizontal Parallax; and you encrease it by the Sun's Horizontal Parallax, it will be equal to the Distance between the Center of the Earth, and the Center of the Moon's Shadow, receiv'd upon the Disc of the Earth, seen from a Point where a Right Line, joining the Centers of the Sun and Earth, meets with the Moon's Heaven: And proportionally, if the Distance of the Sun and Moon be greater or less. [Fig. XXXVIII.]

Let

Let the Earth's Center be T , the Sun's S , TS a Right Line joining the Centers of the Sun and Earth, DT a Perpendicular to the Disc of the Earth, meeting the Moon's Heaven at M . Let ΩL be the Moon's Orbit, in which L is the Moon; ΩM the Ecliptic; LM the apparent Distance of the Sun and Moon, seen from T , equal to the Moon's Horizontal Parallax. Let SL be drawn and produc'd, till it meets with the Earth's Disc DT at V , where the Center of the Moon's Shadow, receiv'd upon the Disc of the Earth, is. I say the Angle TMV is equal to STL , which is the Sum of the apparent Distance of the Sun and Moon, and the Sun's Horizontal Parallax. Draw LR parallel to MT , and join MR . Because the Angle TMV is equal to the Angles TMR and RMV taken together, and the Angle TMR (by *Prop. 29. El. 1.*) is equal to LRM , that is, (upon account of the Nearness of the Points R and T) LTM and RMV (for the same Reason) is equal to RLV , or (by *Prop. 29. Elem. 1.*) to LSM ; TMV will be equal to the Angles LTM and LSM taken together. Moreover, because the Angle LTM is suppos'd equal to the Moon's Horizontal Parallax, and L the Moon; the Distance of the Points L and M will be equal to the Earth's Semidiameter; (for the Moon's Horizontal Parallax is the Angle, under which the Earth's Semidiameter is seen, at a Distance equal to the Distance of the Earth and the Moon) therefore the Angle LSM is nearly the Sun's Horizontal Parallax, (*viz.* the Angle under which the Earth's Semidiameter appears at the Distance of the Sun from the Earth, that is, almost at the Distance of the Sun and Moon) and the Angle LTM is the Distance of the Sun and Moon, seen from the Center of the Earth.

And

Book IV. of ASTRONOMY. 603

And therefore the Angle $\mathcal{T}MV$ is equal to the Distance of the Sun and Moon, seen from the Center of the Earth, together with the Sun's Horizontal Parallax.

After the same Manner, if the Arc LM be equal to half, or one and a half, or twice the Moon's Horizontal Parallax; the Angle $\mathcal{T}MV$ will be equal to $L\mathcal{T}M$, and a half, or one and a half, or twice the Sun's Horizontal Parallax.
Q. E. D.

PROPOSITION XLI.

TO determine whether the New-Moon of a Month shall be eclipsed, that is, whether in any Part of the Earth the Sun will appear to be partially or totally eclips'd; and if there is to be an Eclipse, to determine its Kind. [Fig. XXXIX.]

Let DCG represent the Earth, in which let P and the latent Point opposite to it be the Poles of the Diurnal Revolution. Let the Hemisphere, in the Figure, be that which is enlighten'd by the Sun; and so the Circle DCG , whose Center is $\mathcal{T}$, will divide the enlighten'd from the darken'd Hemisphere, and the Plane thereof is the Disc of the Earth. Let the Shadow of the Moon, caus'd by the Sun, be suppos'd to fall on the Plane of the Disc of the Earth: The Figure of the Shadow so receiv'd will be circular, viz. HM having V for its Center; because the penumbrous Cone is a right one, and its Axis is nearly perpendicular to the Plane of the Disc; for we speak of such a Case where V is but very little distant from $\mathcal{T}$, because it is only in that Case that the Moon's Shadow can reach the Earth; or, that some Species of an Eclipse of the Sun can happen. In the Center of it there is a little Circle altogether umbrous, if the Moon's
Total

604 *The* ELEMENTS Book IV.

Total Shadow reaches so far: The remaining Crown round about is penumbrous, those Parts of it being darkeſt which are neareſt to the aforeſaid ſmall Circle, by *Prop.* 34. It is plain, that if the Penumbra does not touch the Earth, there will not be an Eclipse in any Part of the Earth: But if ſome Part of the Earth be immers'd in the Penumbra, it will (properly ſpeaking, as we ſaid before, of the Moon) ſuffer ſome Eclipse; or the Sun, by reaſon of the Interpoſition of the Moon, will appear to be eclips'd, to the Inhabitants of that Tract of Land. But it is plain, that the Shadow reaches the Earth, if the Circle *HM* cuts the Circle *DCG*; otherwiſe it does not, becauſe *DCG* is a great Circle of the Earth.

Now, to know if, on the New-Moon of a given Month, the Circle *HM* will cut the Circle *DCG*, you muſt ſuppoſe that Hemisphere of the Earth which is enlighten'd by the Sun, and the penumbrous Circle to be ſeen from an Eye plac'd above, in that Point of a Right Line joining the Sun's and the Earth's Centers, where it meets with the Heaven of the Moon; in which Caſe that Hemisphere, tho' it be convex, will appear as the Plane of the Diſc, as it happens when we look at the Moon, the Diſtance of the Eye naturally taking off the Convexity. The Ecliptic, upon the ſame Account, will appear like a Right Line, *viz.* $\Omega T K$, when the Eye happens to be in its Plane: And becauſe the Eye is but a little way off from the Plane of the Moon's Orbit, the Moon's Orbit, receiv'd upon the Diſc, or rather the Way of the Center of the Circle of the Shadow; that is, the Line which it deſcribes in that Plane of the Diſc of the Earth, will alſo be a Right Line; *viz.* ΩI . From *T*, the Center of the Earth, draw *TV* perpendicular to ΩI , and

Book IV. of ASTRONOMY. 605

and V will be the nearest of all the Points of $\Omega\zeta$ to the Center of the Earth. Therefore if the penumbrous Circle HM does at all reach CDG , the Earth's Disc, it will reach it when its Center is in V : For in any other Point these Circles are farther from each other. And for the same Reason, if HM reaches CDG , it will most of all run into it when the Center of the penumbrous Circle is come to V . But it is plain, that the Circle HM does not touch DCG , if when the Center of HM is come to V , VT exceeds TR and VM taken together: But (by the foregoing Proposition) when the Place of the Eye is suppos'd as above, TV appears under an Angle equal to the apparent Distance of the Sun and the Moon, seen from the Center of the Earth, augmented by an Angle which is to the Sun's Horizontal Parallax, as the said Distance of the Luminaries, to the Moon's Horizontal Parallax; and TR appears to the same Eye under an Angle equal to the Moon's Horizontal Parallax; and VM (by *Schol. Prop. 34.*) is seen from it under an Angle equal to the Aggregate of the Semidiameters of the Sun and Moon, seen from the Center of the Earth: If therefore, when the Center of the penumbrous Circle is come to V , the Distance of the Luminaries, seen from the Center of the Earth, augmented by an Angle which is to the Sun's Horizontal Parallax, as the said Distance of the Luminaries is to the Moon's Horizontal Parallax, be greater than the Aggregate of the Moon's Horizontal Parallax, and the Semidiameters of the Luminaries; it is plain, that in such a Case there will not be an Eclipse of the Sun in any Part of the Earth. And since in the New-Moons, in which there is a Suspicion of an Eclipse, or in those in which HM touches DCG , the ap-

606 *The* ELEMENTS Book IV.

parent Distance of the Luminaries is sesquialter (or as $\frac{1}{2}$ to 1) to the Moon's Horizontal Parallax, (namely TV is sesquialter to TR) when VM , the Aggregate of the apparent mean Semidiameters of the Luminaries, is as it were subduple of TR , the Moon's Mean Horizontal Parallax; there will be no Eclipse on the whole Earth, if the Center of the Circle HM , being come to V ; the Distance of the Luminaries, seen from the Center of the Earth, encreas'd by one and a half of the Sun's Horizontal Parallax, exceeds the Sum of the Sun's and Moon's Semidiameters, and the Moon's Horizontal Parallax; that is, if the Distance of the Luminaries, seen from the Center of the Earth, exceeds the Sum of the Semidiameters of the Luminaries, and the Moon's Horizontal Parallax, once and a half the Sun's Parallax being subducted from it.

And if that Distance be equal to this Sum, the Penumbra just touches the Earth, but will not fall upon it, neither will the Sun appear eclips'd to an Inhabitant of the Earth.

But if the Distance be less, that is, if when the Center of the Penumbra [*Fig. XL.*] is in V , the Distance of the Luminaries, seen from the Center of the Earth, wants of the Sum of their Semidiameters and the Moon's Horizontal Parallax, by the Quantity of once and a half the Sun's Parallax; that is, if TV be less than TR and VM together, or the Circle HM cuts the Circle DCG (namely, in V and Z) and at the same Time the little umbrous Circle, in the Center of the Circle HM , either not touches at all, or does but just touch the Circle DCG ; Part of the Sun will appear eclips'd to those that inhabit that Part of the enlighten'd Hemisphere, which is taken in by the Segment $VRZM$ common to both Circles.

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Book IV. of ASTRONOMY. 607

But because the Semidiameter of this little umbrous Circle, to an Eye posited as before, appears under an Angle equal to the Semidiameter of the Moon, seen from the Earth's Center, the Sun's apparent Semidiameter being taken from it, by *Schol. Prop* 33. it is plain, that this Circle (tho' at *V*) will not reach the Disc *CDG*, if the Distance of the Luminaries, seen from the Center of the Earth, together with the Sun's Horizontal Parallax (for in that Case *TV* does not much exceed *RT*) does exceed the Moon's Horizontal Parallax, and the Excess of the Moon's apparent Semidiameter above the Sun's apparent Semidiameter, or is equal to them; that is, if the said Distance of the Luminaries exceeds the Semidiameter and Horizontal Parallax of the Moon, taking from it the Semidiameter and Horizontal Parallax of the Sun; or if it be equal to them: But if it wants of that Sum, then the whole Shadow will fall upon the Earth, and make a Total Eclipse of the Sun to the Inhabitants of that Tract of the Earth which it falls upon; but without any sensible *Mora* or Duration, because this little Circle is very small, since the Moon's apparent Diameter does but a little exceed the Sun's apparent Diameter, and so is commonly look'd upon as the Center of the Penumbra. Nay, sometimes this Circle quite vanishes, when the Sun's apparent Diameter is greater than the Moon's apparent Diameter. Now in either Case, some Place of the Earth has a total (or a central, tho' not total) Eclipse, when the Center of the Penumbra being at the least Distance from the Center, the apparent Distance of the Luminaries is equal to, or less than the Sum of their Horizontal Parallaxes.

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But the whole Penumbra of the Moon darkens the Earth, first when TV (*Fig. XLI.*) the least Distance of the Centers of the Earth and penum-
brous Circle, that is, (by the foregoing Propo-
sition) the least apparent Distance of the Lumi-
naries, half the Sun's Horizontal Parallax being
subtracted from it, (for VM is nearly equal to
half FR ,) is equal to the Difference of TR , HV ,
or to the Excess of the Moon's Horizontal Paral-
lax above the Sum of the Semidiameters of the
Luminaries. And if that Distance is less than this
Excess, the whole Penumbra falls upon the Earth,
and there remains a Tract of Land round about,
where there is no Eclipse of the Sun.

Tho' the Moon's Shadow or Penumbra, re-
ceiv'd upon the Plane of the Disc, be a Circle,
yet on the spheric Surface of the Earth (where
it really falls) it is bounded by a Line not at all
Circular, (unless it be when the Axis of the Cone
of the Shadow is extended thro' the Center of the
Earth,) and even not lying in the same Plane;
(of which Kind of Lines *P. Courfier* wrote a small
Treatise in the Year 1663.) which is also true
of the Shadow of the Earth receiv'd on the Moon's
spherical Surface. But in treating of the Moon's
Eclipse, there was no Need of taking Notice of
this, because its spherical Surface does not differ
from a flat Disc to an Observer on the Earth.
But in the Earth, our Habitation, the Thing is
otherwise: For all the Spectators in the Earth
which have an Eclipse of the Sun of the same
Quantity in the same Moment of Time, lie un-
der such a Line. But the Determination of it
does not belong to this Place, nor indeed proper-
ly to Astronomy.

But the nearest Distance between the Centers
of the Luminaries near the New-Moon seen
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Book IV. of ASTRONOMY. 609

from the Earth is to be found the same Way as the nearest Distance between the Centers of the Moon, and of the Shadow of the Earth near the Full-Moon, was found in *Prop. 35*; viz. by finding (by *Prop. 31.*) the Time of the true Conjunction, happening on a given Month, viz. when the Moon is found in N , the Sun being at S , [*Fig. XLII.*] SN being drawn for a Circle of Latitude, and finding the Places of the Moon's Nodes for the same Time, (of which let Ω be nearest to S) as also the Inclination of the Moon's Orbit to the Ecliptic; upon which Account, in the Spherical Triangle ΩSN rectangular at S , you have given ΩS , the Distance of the Node and the Sun, and the Angle Ω , which is the Inclination of the Lunar Orbit to the Ecliptic; whence you will find the Side SN and the Angle N . Again, in the Triangle SLN , (which, by reason of its Smallness, may be look'd upon as rectilinear) besides the Right Angle at L , (for SL , by Construction, is perpendicular to the Moon's Orbit, and so is the nearest Distance of the Luminaries, namely, when the Moon is in L) you have given the Side SN lately found, and the Angle N ; whence SL , the least Distance of the Luminaries, will be found, and also the Side NL , which will be of Use afterwards: But you will readily find the Arc SL from the Tables of the Moon's Latitude, by taking it equal to the Moon's Latitude, when the Argument of Latitude is equal to the Arc ΩS , being the Distance of the Sun from the Node, the Reason of which we have shewn in the same Case, *Prop. 35.*

COROLLARY.

Hence will be defin'd the Terms for all Manner of Eclipses of the Sun, that happen in any

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Part of the Earth. For if TV be less than the Sum of the Semidiameters of the Earth's Disc, and the Moon's Penumbra, there will only be a partial Eclipse in some Part of the Earth. If TV be less than TR the Semidiameter of the Disc, there will somewhere be a total Eclipse of the Sun; at least, a central one. Therefore in the rectilineal Triangle ΩVT , which is describ'd in the Disc of the Earth, having given, besides the Right Angle at V , the Angle $V\Omega T$, and the Side VT , equal to the said Sum of the Semidiameters, you will find the Side ΩV ; namely, the Distance of the Center of the Moon's Penumbra from the nearest Node, and of the Moon from the opposite Node; which being such, at the Time of the true New-Moon, the Penumbra will go by the Earth without touching it: But if, at the said Moment of the New-Moon, the Distance from the nearest Node be less than the above found Arc ΩV , the Moon's Penumbra will involve some Tract of the Earth, and the Sun will appear to be somewise eclipsed to the Inhabitants of that Tract. After the same Manner will the Terms be defin'd for a central Eclipse of the Sun, happening somewhere in the Earth, by taking the Semidiameter of the Disc itself, instead of the Sum of the Semidiameters of the Disc and Penumbra, as was before shewn. But by reason of the Smallness of the Angle at Ω , and of the Sides of the Triangle VNT , TN differs but very little from TV : Whence these will nearly be the Terms for the Sun's Eclipse, whether Total or of any Kind happening in any Place; namely, the Distances of the Moon from the nearest Node, when at the Time of the true New-Moon, the Latitude is equal either to the Sum of the Semidiameters of
the

Book IV. of ASTRONOMY. 611

the Luminaries and the Moon's Horizontal Parallax diminish'd by once and a half the Sun's Horizontal Parallax, or to the Difference of the Sun's and Moon's Horizontal Parallaxes.

PROPOSITION XLII.

TO define the Time of the Beginning, Middle, and End, as well of every Eclipse, or lessening of Light, upon account of the Moon's Penumbra; as of a total Eclipse of the Sun, happening in any Part of the Globe of the Earth, upon a given Month; and thence to determine their Duration. [Fig. XLIII.]

Let DCG be the Disc of the Earth describ'd about the Center T , seen from a Point of the Right Line joining the Centers of the Earth and Sun, where it meets with the Heaven of the Moon. Let the common Intersection of its Disc and the Plane of the Ecliptic be the Right Line ΩK ; and TN perpendicular to it will be the common Section of the Planes of the Circle of Latitude, and of the Disc of the Earth, in which the Point N is, where the common Intersection of the Moon's Orbit, and the Plane of the Disc, or the Projection of the Moon's Orbit on the Plane of the Disc, cuts the said Circle of Latitude. When we suppose the Eye seeing this Eclipse of the Earth plac'd in a Right Line, which joins the Centers of the Sun and the Earth, it is plain, that it will move together with the Sun; and therefore the Center of the Moon's Penumbra will appear to describe upon the Disc of the Earth that Way from the Sun, of which we have amply treated in *Prop. 36*. Therefore let the Right Line $KN\Omega$ be conceived to be drawn thro' N , making with TN Angles on each Side equal to those which the Moon's Way from the Sun

612 *The ELEMENTS* Book IV.

makes with the Circle of Latitude at the Time of the true New-Moon; and let this Way of the Moon from the Sun, defin'd for the New-Moon or Middle of the Eclipse, be suppos'd to continue during the whole Eclipse; which indeed nearly happens: But if we wou'd proceed more exactly, we must draw it a-new in other Distances of the Moon from the Node. From T draw TV perpendicular to $l\Omega$; and V will be the Point, to which, when the Center of the Moon's Penumbra is come, it will be the least distant of any from the Center of the Disc, and run upon the Earth most of any; as has been shewn in the foregoing Proposition. Moreover (from *Prop. 3. El. 3.*) V is also the Middle of that Part of the Right Line Ωl , which lies in the Circle DCG , or any other concentric to it; and therefore it is such, that the Center of the Penumbra is found in it in the Middle of any Eclipse of the Earth, which lasts as long as the Center of the Penumbra is in running thro' the Right Line AB , supposing TA to exceed TR by the Excess AC , equal to the Semidiameter of the penumbrous Circle; and also such as the Center of the Penumbra may be found in it in the middle Time, that the Center of the Penumbra, going thro' the Right Line EF , produces a total and central Eclipse of the Sun in some Part of the Earth. Therefore the Problem comes to this, that the Point of Time be determin'd when the Center of the Penumbra is found in V . For doing of which, find (by *Prop. 31.*) the Time of the true Conjunction, which is in N ; and since in the Triangle TVN , besides the Right Angle at V , you have the Side TN , and (by *Prop. 36.*) the Angle TNV , you will find NV in the same Parts in which TN or TR is given; that is, it will be known what Angle

Book IV. of ASTRONOMY. 613

gle at the Eye is subtended by NV . But from the given Horary Motion of the Sun and Moon, the Moon's Horary Motion from the Sun is given, or the Arc which the Moon, going from the Sun, runs thro' in the Space of an Hour, that is, the Time necessary for the Moon to move from the Sun, thro' a given Arc; therefore the Time will be known which is requir'd for the Moon to move from the Sun thro' an Arc subtending an Angle, which the Right Line NV subtends, that is, that the Center of the Lunar Penumbra may run thro' the Right Line NV upon the Disc of the Earth; that is, lastly, the Time between the Moment of the true Conjunction, and the Moment of the Middle of the Eclipse, in respect of the whole Earth; therefore the first being given, the last will also be given: For the Moment of the Middle of the Sun's Eclipse, in respect of the whole Earth, and also that of the Middle of the Moon's Eclipse, goes before the true Conjunction, when the Sun and the Moon have just pass'd either of the Nodes, but follows it when the Luminaries are not yet come to the Nodes. To find this Time more readily, we must make use of the Tables of the Time between the true Syzygy and the greatest Approximation of the Centers of the Disc and of the Shadow, of which we spoke in *Prop. 37*.

Now having found the Time of the Middle of the Eclipse, it will be easy to define the Beginning and the End of every Sort of Eclipse, or the Moments when the Moon's Penumbra first comes at, and last of all leaves the Disc of the Earth. For in the rectangular Triangle TVA , you have given the Sides TV , TA , the former of them before found, and the latter, made up of the Semidiameters of the Disc and of the Penumbra

614 *The* ELEMENTS Book IV.

before determin'd; and therefore AV will be given, and thence as before, from the Moon's Hourly Motion from the Sun, you will find the Time requir'd for the Penumbra's Center to run over it; that is, half the Duration of every Kind of Eclipse; and as the Moment of the Middle of the Eclipse is given, the Beginning and End of it are known. The *Semi-mora*, or half the Duration of the whole Penumbra in the Disc of the Earth, will likewise be found, if instead of TA , the Sum of the Semidiameters of the Disc and of the Penumbra, you take their Difference, just after the same Manner as it is done in the total Immerision of the Moon in the Earth's Shadow in total Eclipses of the Moon.

But the *Semi-mora* of the Center of the Penumbra between the Earth's Disc (or of the whole Shadow, which, as it is always slender, sometimes none at all, is look'd upon as a Center) is found by investigating EV from the given Lines TE , TV , and the Time requir'd for the Center of the Moon's Shadow to go over it, as before. But because the Moment of the Middle of the Eclipse, and half the Duration of the Center within the Disc are known, the Time will also be known when the whole Shadow comes at or leaves the Earth; that is, when the Sun's total Eclipse begins or ends to an Inhabitant of the Earth.

C O R O L L A R Y.

Hence you have all the Angles of the rectilineal Triangles ATN , ETN , VTN , FTN , and BTN , and from hence also such a Scheme of the Eclipse of the Disc of the Earth, as this which we speak of, may be exactly described upon a Plane; as was done of the Moon's Eclipse,

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Book IV. of ASTRONOMY. 615

Corol. 2. Prop. 39: For the 43d Figure is the very Scheme requir'd; therefore the Scheme may be rightly drawn. And as it is known what Parts of the Right Line $\delta\delta l$, the Center of the Moon's Shadow, will go thro' in the Space of an Hour, and what a Clock it is in a given Place, when the said Center is at N or V , or any other Point given in $\delta\delta l$; it is plain that if we ascribe to the Points of this Line, rightly drawn, the Numbers by which the Hours are express'd in a given Place, the Scheme requir'd will be perfect for a given Place.

PROPOSITION XLIII.

TO determine the Quantity of a Solar Eclipse in respect of a Spectator upon the Earth within the Penumbrous Circle, which is a Section of the Moon's Penumbra by the Disc of the Earth, when the said Penumbra is in a given Place. [Fig. XLIV.].

Let EFG be a Circle separating the enlightened Hemisphere of the Earth, from the darken'd one, whose Plane therefore is the same with the Plane of the Disc mentioned before. Let $KT\delta$ represent the Ecliptic on this Plane; $lN\delta$ the Way of the Center of the Moon's Penumbra; N the Point, at which, when the Center of the Penumbra is come, there is a true New-Moon. Let A be the Center of this Penumbra, at a certain Point of Time, when the Shadow goes thro' the Right Line $l\delta$, and let the Penumbrous Circle be $M\mathcal{Q}X$; AN will be that Part of the Journey of the Penumbra, which will be perform'd between that Point of Time and the Moment of the true Conjunction, which will happen when that Center will be come to N . Let us suppose the Observer plac'd within this Circle as at O , or ra-

616 *The* ELEMENTS Book IV.

ther in a Point of the enlighten'd Hemispherical Surface, which is shewn by a Right Line let fall from O perpendicular to the Plane of the Disc; then we are to find what Part of the Sun's Diameter is cover'd by the Moon, from the foresaid Observer. Let the Right Line AO be drawn and produc'd, till it meets with the Circle $M\mathcal{Q}X$ at M ; I say that that Part of the Sun's Diameter, which is cover'd by the Moon, is to the whole Diameter as OM to AM . For if the Observer was at M , the Moon wou'd cover no Part at all of the Sun's Diameter from him; but as the Observer goes gradually towards A , the Center of the Penumbra, a greater and greater Part of the Sun's Diameter is cover'd from him by the Interposition of the Moon, till at last, when he is come to the Center A , the whole Sun (that is, its whole Diameter) is cover'd by it, if the Sun's apparent Diameter does not exceed the Moon's apparent Diameter, (which I here suppose, and which commonly happens in their mean Magnitudes) and the Vertex of the total Shadow of the Moon reaches as far as the Earth, or the penumbrous Circle $M\mathcal{Q}X$ has its Center perfectly and totally umbrous: And therefore if the Sun's Diameter be divided into twelve Parts, from the given Ratio between AM and OM , which is known from the given Situation of the Place between the Circle $M\mathcal{Q}X$, you will have the Digits, and Parts of Digits contain'd in the darkened Part of the Sun's Diameter, in respect to the Observer.

But if, (in the 2d and 44th Fig.) the little Circle BC , which is either wholly umbrous, or not altogether, but more obscure than the rest, and that equally be concentric to the Circle $MX\mathcal{Q}$; that is, if the Center or Point of the Moon's

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Book IV. of ASTRONOMY. 617

Moon's Shadow be either extended beyond the Spectator, or falls short of him, instead of AM you must in the first Case take BM , the Difference of the Semidiameters AM and AB ; in the latter, CM their Sum, that is, the Sun's whole Diameter is to its Parts, cover'd by the Moon at O in the first Case, as BM to OM , and in the latter Case, as CM to OM . For in the first Case, the Spectator at B will see the whole Sun cover'd by the Moon; in the latter, the Spectator, plac'd at A , will see a bright Circle, which is the Remainder of the Sun's Disc, encompassing the dark Moon, like a Crown, whose apparent Latitude is equal to the Semidiameter of the Sun, when the Semidiameter of the Moon is taken from it, which is the same as the Semidiameter AB , seen from the Moon; and the Spectator must be at C , to see the Margin of the Moon touch the inner Disc of the Sun. Therefore the Quantity of the Sun's Eclipse is determin'd in every Case. Q. E. F.

PROPOSITION XLIV.

TO give an Account of the chief Phænomena of a Solar Eclipse, and to describe the Places of the Earth where they will happen, whilst the Shadow of the Moon, in a New-Moon eclipsing the Sun, goes along the Earth. [Fig. XLV.]

Let CGD represent the Globe of the Earth, revolving from West to East by its Diurnal Motion upon the Pole P and the other Pole which is hid, or turning from C upwards towards G . Let the Hemisphere of this Globe, seen in the Scheme, be that which is enlighten'd by the Sun, therefore the Sun will directly and perpendicularly shine on the Circle CDG , whose Pole is S . Thro' the Pole P and the Point S , let PSR , an
immovable

618 *The* ELEMENTS Book IV.

immoveable Circle be suppos'd to be drawn without the Earth's Surface, and also KST another, *viz.* the common Section of the Plane of the Ecliptic with the Surface of the Earth; ZSX another, at Right Angles to this, both these being alike fix'd, between which the Earth performs its Diurnal Revolution. It is certain that it is Noon at every Place of the Earth, when it is come to that Half of the Circle PSR , which is here visible, because the Sun is in the Plane of it. Let AB represent the Way of the Center of the Moon's Penumbra, describing the Track ENF on the Surface of the Earth: Tho' the Hemisphere of the Earth which is enlighten'd by the Sun, and consequently has Day, be suppos'd to be rais'd up above the Surface of the Scheme, yet the Way AB is conceiv'd to be gone thro' by the Center of the Penumbra, on the Plane of the Disc, upon which the said Hemisphere stands. Moreover, tho' PSR , KST , ZSX , be Circles in the said enlighten'd Hemisphere, crossing one another in S , the Point directly under the Sun, they also represent the Right Lines upon the Disc of the Earth, directly under these Circles, *viz.* those which are the orthographical Projection of these Circles, when the Eye is suppos'd at a very distant Point of the Right Line, which joins the Centers of the Earth and Sun.

It is plain, that when the Center of the Moon's Penumbra is come to A (namely, when the penumbrous Circle touches the Disc of the Earth in C) the Eclipse of the Sun will begin to be to an Inhabitant at C : Now he that is at C , by the Diurnal Revolution of the Earth, begins to enter the enlighten'd Hemisphere; that is, the Sun rises to him; therefore to the Inhabitants of the

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Book IV. of ASTRONOMY. 619

the Earth at *C*, which is the first Place the Penumbra reaches to, the rising Sun will first of all appear to be eclipsed in its Upper or Western Limb or Edge. When the Center of the Penumbra it self comes to the Earth at *E*, an Inhabitant at *E* sees the rising Sun totally eclips'd, because he is under the Center of the Penumbra. He that lives upon that Point of the Earth, where the whole Penumbra, receiv'd within the Disc of the Earth, (if that happens) touches it inwardly, will perceive an Eclipse of the Sun to end at the lower Limb of the rising Sun: So he which is in that Point, where the Penumbra does last of all touch the Disc inwardly, as it is ready to go out of it, will perceive the Eclipse of the Sun to begin, as the Sun sets. We have not express'd those Two Cases in the Figure, lest we should make a Confusion.

When the Center of the penumbrous Circle is come to *ZSX*, which is a Right Line in the Disc perpendicular to the Ecliptic, or when the Axis of the Moon's Penumbra at *N*, comes to the Circle *ZSX*, which answers to the afore-said Right Line, it being the Diameter of it; that Point of the Earth's Surface, which is under the Point *N* of the Circle *ZSX*, has a Central Eclipse of the Sun, and to a Spectator at *N* or any other Point of the Circle *ZSX* the Sun appears in the highest Point above the Horizon. For since, by Supposition, the Circle *ZX* goes thro' the Zenith of the Spectator, it will be perpendicular to this Horizon: But it is also perpendicular to the Ecliptic at *S*; and therefore (by *Elem. Spheric.*) is a whole Quadrant distant from the said Concurrence of the Ecliptic and the Horizon. But half the Ecliptic (because it is a great Circle) is above any Horizon, whence *ZX* meets

meets with the Ecliptic in a Point distant from the East Point, by a whole Quadrant, or the highest Point. Therefore an Inhabitant of the Earth, which is then at the Point of the Earth's Surface at *N*, sees the Sun centrally eclips'd in the 90th Degree: But he that lives from *N* towards *Z* or *X*, but in the Circle *Z S X*, does indeed see the Sun eclips'd at its 90th Degree, if he dwells within the Limits of the penumbrous Circle; but more or less according to his greater or less Distance from *N*, as has been shewn in the foregoing Proposition. Likewise when the Penumbra comes to the immoveable Circle *P S R* at *M*, an Inhabitant of the Earth, which is under it, sees the Sun eclipsed just at Noon: But those, that live from *M* towards either Pole under the said Meridian *P R*, see the Sun at Noon in a Partial Eclipse, if they are within the Limits of the Penumbra. And he, that is removed from *M* towards the North Pole *P*, sees the Sun's Southern Part eclipsed, and so on the contrary; which also happens in the first Case, and universally in all Cases. At last the Center of the Penumbra being come to *F*, the End of the Disc, the Spectator, which at the same Time is at that Point, sees the setting Sun (because being ready to change Day for Night, he goes out of the enlighten'd, into the darken'd Hemisphere) centrally eclips'd. But he that, being at *D*, receives the last Stroke of the Moon's Penumbra, sees the setting Sun as it were contiguous to the Moon in its upper Limb, and the End of the Eclipse, be it of what Kind it will.

P R O-

PROPOSITION XLV.

TO determine the Place of the Earth (that is, its Longitude and Latitude) where the rising Sun appears to be eclips'd in its upper Limb. [Fig. XLVI.].

Things being as before, the Longitude and Latitude of the Point C is to be determin'd. Let the great Circle SC , which in this, as well as in the foregoing Figure, appears as a Right Line, be drawn thro' the Point of the Earth S , to which the Sun is perpendicular, and thro' the Point C . Draw also PC , PZ , Arcs of great Circles. In the Spherical Triangle PSZ , you have three Sides given, *viz.* PS , the Distance of the Sun from the enlighten'd Pole, SZ , a Quadrant, and PZ , the Distance of Z , the Pole of the Ecliptic (because the Arc SZ is perpendicular to the Ecliptic KST and extended to a Quadrant) from P the Pole of the Æquator; therefore the Spherical Angle between the Arcs SP , SZ , is known, being the same with the rectilineal Angle between the Right Lines SP , SZ , drawn, in the Plane of the Disc; as generally the Angle in the Surface of a Sphere, is equal to a rectilineal Angle, which the common Sections of the Planes of Circles, comprehending a Spherical Angle, make on a Plane perpendicular to both of them. Moreover (by *Corol. Prop. 42.*) the rectilineal Angle NSA and ZSC on the other Side of it are given, and therefore also PSC the Sum or Difference of it and of ZSP , or the Spherical Angle on the Surface of the Earth, comprehended by the Circles SP , SC : Now in the Spherical Triangle PCS , besides the Angle PSC lately found, you have the Sides SP , SC , of which the last is a Quadrant, and the first the Distance

622 *The* ELEMENTS Book IV.

Distance of the Place C from the Pole P , that is, the Complement of its Latitude; therefore the Angle SPC will also be given, in this Triangle. But (by *Prop.* 42.) the Time is given when the Moon's Penumbra first touches the Disc of the Earth at C , reckon'd in some determinate Place: Let this Place be H , whose Meridian is PH ; therefore the Angle SPH will be given, *viz.* that by which the Sun has already pass'd the Meridian, or is short of it. Therefore HPC , the Sum or Difference of the Angles CPS , HPS , will be given, whereof HPC is the Difference of the Meridians of the Place C , and the given Place H . Therefore the Longitude and Latitude of the Place is found. Q. E. F.

You may after the same Manner find a Place, where the Setting Sun is eclips'd in its upper Limb, which the Moon's Penumbra leaves last of any.

PROPOSITION XLVI.

TO determine what Place in the Earth is under the Center of the Moon's Penumbra at a given Hour, in a given Place, or in which the Sun is centrally eclips'd at a given Time. [Fig. XLVII.]

Things remaining as before, since the Place, where the Sun appears certainly eclips'd, is requir'd, and the Time of such an Eclipse is given, as well as the Time of the true Conjunction, by *Prop.* 31. this Interval is given, and therefore from the Sun and Moon's Horary Motion, the Moon's Motion from the Sun, agreeable to that Interval, is also given. Whence, in the rectilineal Triangle SNA , describ'd in the Earth's Disc, you have given (by *Prop.* 36.) the Angle SNA equal to the Angle comprehended between the Circle of Latitude, and the Way of the Shadow,

Book IV. of ASTRONOMY. 623

dow, the Side SN being the Sine of the Moon's Latitude in the true Conjunction, and AN the Motion of the Center of the Penumbra agreeable to the Interval between the propos'd Time and the Time of the true Conjunction; therefore the Side SA , and the Angle ASN will be given, or its Complement to two Right Angles: But ZSP is given (as was shewn in the foregoing Proposition) therefore you will also have the rectilineal Angle PSA , and the spherical Angle upon the Earth's Surface PSA , which is equal to it. You have also the two Sides SA, SP , of the spherical Triangle PSA , this last being equal to the Sun's Distance from the Pole P , and the first will be found after the following Manner: To the Right Lines SC, SA , drawn on the Plane of the Disc, answer upon the spherical Surface, two Arcs, beginning from the Point S , which is directly under the Sun, of which SC is a Quadrant, and SA is an Arc, whose Right Sine is the Right Line SA ; therefore the Arc SA , being the Side of the Triangle PSA , is that whose Sine is to the Radius, as the given Right Line SA is to the given one SC ; and therefore the Arc SA is known. Whence in the Triangle PSA you will know PA , the Complement of the requir'd Latitude of the Place A , and also the Angle SPA . But since the Time is given, as reckon'd in the determinate Place H , the Angle SPH is given; therefore you will also have the Angle HPA , which is the Difference of the Meridians of the determin'd Place H and of the requir'd Place A .

You may, after the same Manner, find the Place of the Earth, where the Sun's central Eclipse happens in the 90th Degree of the Ecliptic from the East. We have demonstrated in the 44th Proposition, that that happens to him, who

by

624 *The ELEMENTS* Book IV.

by the Earth's Diurnal Revolution is brought to *N*, when the Center of the Moon's Penumbra falls on that Point: And we have shewn in *Prop.* 31. the Time number'd in the Place *H*, when this happens, (for it is the same with the Time of the true Conjunction of the Luminaries;) therefore this is only a particular Case of the foregoing Problem.

SCHOLIUM.

Hence therefore is defin'd that Tract of Earth, which the Center of the Moon's Penumbra goes thro' in the Solar Eclipse of a given Month; that is, all the Places which have successively a central Eclipse of the Sun. If (by *Prop.* 46.) you would know in what Place of the Earth the rising Sun is centrally eclips'd, and then another Place where it is centrally eclips'd half an Hour, or an Hour after, and so on till you come to the Place where the Sun is eclipsed after the same Manner as it sets; those Places, join'd by a Line or Lines on the Earth's Surface, will shew that requir'd Tract. If on either Side about this Line, you draw as it were a Zone, whose Breadth is equal to the Diameter of the Moon's Penumbra; the whole Tract of the Earth involv'd in the Penumbra which can have any Eclipse of the Sun, will be taken in. In that Tract may be mark'd the Places where the chief Phases are, as where the Sun is eclipsed in the 90th Degree, and where in the Meridian, &c.

PRO-

Book IV. of ASTRONOMY. 625

PROPOSITION XLVII.

TO determine the Place in the Earth's Surface, where an Eclipse of the Sun is either Total or Central, or where a given Part of its Diameter, towards a given Pole, will be hid by the Moon just at Noon. [Fig. XLVIII.]

Things remaining as before, it appears, that that Point of the Earth which comes to *M*, together with the Center of the Moon's Penumbra, will have an Eclipse of the Sun just at Noon. In order to find this, observe, that in the rectilineal Triangle *NMS*, describ'd in the Disc as before, you have given (by Prop. 36.) the Angle *SNM*, (namely, that which the Way of the Moon from the Sun contains with the Circle of Latitude) *NS* the Distance of the Centers of the Disc and Penumbra, at the true Conjunction, and the Angle *NSM* equal to the Spherical Angle *ZSP* in the Superficies of the Sphere contain'd at the Sun between *SZ* and *PSR*, which are secondary Circles of the Ecliptick and the Æquator; whence you will have the Right Line *NM*, and therefore the Time requir'd for the Center of the Penumbra to go thro' it in its Motion from the Sun: But the Time reckon'd at a given Place, in which that Center is in that Point *N*, is given, namely, the same with the Time of the true Conjunction of the Luminaries; therefore also the Time number'd in the said Place, when the Center of the Penumbra is in *M*, will be given. Find therefore, by the foregoing Proposition, that Place of the Earth where at that Time the Sun will be seen in a central Eclipse; and it will be the same in which the Sun is centrally eclips'd at Noon.

626 *The* ELEMENTS Book IV.

Suppose that about the Center M , in the Plane of the Disc, there is describ'd a penumbrous Circle, cutting the Right Line PSR at C and D ; then, as the Sun's whole apparent Diameter is to the given Part of it, which is not eclips'd in the Place requir'd, so let the Semidiameter of the Penumbra be to MO , and let MO be taken towards the Pole opposite to that, towards which the eclips'd Part of the Sun is, (for Example, if the Sun's eclips'd Part lies towards the hidden Pole in *Fig. 49.* let O be taken towards P , the Pole in Sight;) then the Point on the Earth's Surface which a Right Line drawn from O , perpendicular to the Plane of the Disc, shews, will be the Place requir'd, in which the Sun just at Noon, in a given Part, is eclips'd towards the Pole opposite to P ; as it is plain enough from the 43d and 44th Propositions. If instead of the umbrous Center M , either the whole Circle be totally umbrous, or there is no total Shadow, but a Circle equally enlighten'd, (as in *Schol. of Prop. 33.*) instead of the Semidiameter MC , you must take the Difference or the Sum of the Semidiameters of the Circle CD , and another within and concentric to it; as it has been shewn in the said 43d *Prop.* To find the Latitude of this Place O , (for the Longitude is known from knowing the Hour in a given Place, when it is Noon to an Inhabitant at O) observe that the Right Line SM is given, namely, a Side in the Triangle SNM , also MO , each of them (and therefore their Sum or Difference) in Parts of SZ , the Semidiameter of the Disc of the Earth; therefore you will also have the Arc, which is over it in the Surface of the Earth, of which SO is the Sine: But the Arc SP , viz. the Distance of the Sun from the Pole P , is given; and there-

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Book IV. of ASTRONOMY. 627

therefore the Arc PO will be known, and its Complement to the Quadrant of a Circle, is the Latitude of the Place O , which was requir'd.
Q. E. F.

If the Point O falls out of the Disc of the Earth $KZTX$, the Problem is impossible. After the same Manner may be determin'd the Place where the Eclipse of the Sun is in its 90th Degree, or of its Rising or Setting, is of a given Quantity, or where the Sun upon a given Time, reckon'd in a given Place, is eclipsed a determinate Number of Digits. In the last Case there are innumerable Places, but in each of the former Cases, there are, at most, but two, which answer to what is requir'd.

PROPOSITION XLVIII.

TO determine whether there will be a visible Eclipse in a given Place of the Earth, on the New-Moon of a given Month; and, if there is one, to determine the Time of the greatest Obscurity, and the Quantity of the Eclipse. [Fig. XLIX.]

We have hitherto consider'd the Sun's Eclipse in respect of the whole Earth, (or rather, properly speaking, the Eclipse of the Earth,) as we did the Moon's before: Now we must consider it as it is seen from a given Place of the Earth's Surface. Since the Time that *Kepler* gave Astronomers the Moon's Shadow on the Earth's Disc, to be viewed from the Lunar Heaven, they have with Emulation endeavoured to deduce an Eclipse of the Sun, visible in a given Place, and all the Phænomena attending it, from the same Scheme of the Disc of the Earth, over which the Moon's Penumbra passes. But as this is done in a less Geometrical Manner, (and a Trigonometrical Calculation can scarce

628 *The* ELEMENTS Book IV.

be applied to it) and certainly a less natural one, and (as Mr. *De la Hire*, who makes use of it, confesses) something more tedious; we, according to the Example of *Kepler* himself, in the *Rudolph. Tables* (who certainly knew how far his Invention wou'd reach) shall follow another Way, shewn by *Ptolemy* (in the 6th Book of his *Almagest*.) and his excellent Interpreter *Regiomontanus*, that it may also be known to the Reader; at the same Time not forgetting to commend the Method given by the ingenious Mr. *Flamsteed*, in his *Doctrine of the Sphere*, by which the Solar Eclipse, or the Moon's Appulses to the fix'd Stars in a given Place, is graphically and handsomely protracted, the Places in the other Hemisphere of the Earth, turn'd from the Sun or Star, being orthographically projected on its great Circle.

Let *CN* represent the Ecliptic, *S* the Sun's, and *L* the Moon's true Place at the Time of the true Conjunction; find (by *Prop. 63. B. II.*) the Sun's and Moon's Parallaxes of Longitude and Latitude, by which their Places in the Ecliptic, or Distances from the Ecliptic, may be corrected; and this for the said Time, tho' not very accurately: Then, from the true foregoing Places, or Places seen from the Center of the Earth, *S* and *A* seen from a given Place in the Surface of the Earth (or simply seen, as they call it) will be had. We don't make the Sun's Place differ from that mention'd before, to avoid Confusion in the Scheme, as well because the Sun's Parallax is so small, that it may well be overlook'd in this Scheme, and also chiefly, because in taking the Moon's Parallax from the Sun (as it has been shewn in the said 63d *Prop. of Book II.*) the Line *NC* may be taken for the
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Book IV. of ASTRONOMY. 629

apparent Way of the Sun, or for the Ecliptic. Let the Moon's apparent Place A be reduc'd to the Ecliptic in C , and let its Latitude be AC . Now to determine the Point, in which the Center of the Moon will appear to the propos'd Spectator, to be at the nearest Distance from the Center of the Sun, that is, when the greatest Obscurity (if any will be) shall happen; find the Situation of the Moon's apparent Way from the Sun, by seeking again, from what has been laid down, (for a Time little enough distant from that in which the Moon appears in A) the Places of the Sun and Moon, as they appear to the given Observer, and imagining the Sun to be drawn back to its old Place S , and the Moon at the same Time drawn back as much. The Place of the Moon so found let be K , which, being reduc'd to the Ecliptic, will shew its Longitude at D , and its Latitude KD . Now the Arcs CA , DK , and CD , being given, you may (by *Lemma of Prop. 36.*) determine the Situation AN of the Moon's apparent Way, in respect to the Ecliptic; namely N , the Point of Concurrence with the Ecliptic, and the Angle ANC . From the Sun's Place S , let fall SM , perpendicularly upon AN , and the Point M is the nearest to S , of all the Points in AN : To find which, observe that in the Triangle NSM , besides the Right Angle at M , the Angle N and the Hypotenuse NS , each lately found, are given: Whence MS and MN will be known; therefore also MA , because the Arc AN is known. Now because M is a Point of the Lunar Way, which is the farthest distant of any, from the Sun S ; it is plain, if a propos'd Spectator is to have any Eclipse of the Sun, that it will happen when the Moon's Center appears to be in M , and that

then also the greatest Part of the Sun's Disc will be hid by the Moon; and it is also plain, that if the before-found SM be greater, or equal to the Sun and Moon's apparent Semidiameters SR and MH together, then a propos'd Observer will have no Eclipse; if SM be less than SR and MH taken together, then there will at least be some Eclipse. But SM is found for the New-Moon of a given Month, and (in the same Parts, *viz.* Parts of a Circle) the apparent Diameters of the Sun and Moon are above determin'd; whence it may be defin'd, whether the New-Moon of a given Month will be eclipsed or no.

If there is to be an Eclipse, in order to determine its Quantity, we must take Notice that [*Fig. L.*] SR and MH , taken together, are equal to MS and RH together: If therefore, from the known Sum of the Semidiameters of the Sun and Moon be taken the known Arc SM , there will be left RH , Part of the Sun's Diameter, cover'd from the Moon at the Time of the greatest Obscurity, which is easily reduc'd to Digits and Parts of Digits, because we already have the Sun's whole Diameter, and the cover'd Part of it in such Digits.

But it is plain from Observation, that the Moon's apparent Diameter is less (or perhaps the Sun's apparent Diameter is greater) when there is an Eclipse of the Sun, than at the same Distance from the Eye, when there is no Eclipse, (for Reasons shewn in *B. I. Prop. 19.*) and that that Difference is about the 60th Part of the whole Diameter: We must therefore have Regard to this Correction, when we define the Quantity and Phases of the Eclipse.

Lastly,

Book IV. of ASTRONOMY. 631

Lastly, to determine the Time that the Moon's Center will be found in M , observe that the Time is given (because it is taken) wherein the Moon by its apparent Motion from the Sun (compounded partly of the Moon's true Motion, partly of its apparent Motion arising from the Parallax) goes thro' the known Part AK of the apparent Way AN , or rather the known Arc CD of the Ecliptic, and that besides there is given the Arc AM , namely, the Sum or Difference of the known Arcs AN and MN ; whence by Analogy you will have the Time necessary for the Moon, by its apparent Motion from the Sun, to go thro' AM : But the Time is given in which the Moon is seen in A ; therefore the Time will also be known when it will appear in M , where it hides the Sun most of all from a given Observer. *Q. E. F.*

For the Time found, in which the Moon is in M , find again the Place of the Luminaries, to know whether all is rightly perform'd, or whether there is Need of a farther Correction.

SCHOLIUM. [Fig. XLIX.]

In the foregoing Operations we have suppos'd the Moon's Motion from the Sun in the apparent Way AN to be equable, which is not true; for tho' the Moon's Motion from the Sun in its true Way LB , during the whole Time of the Eclipse, differs but little from an equable one, yet since there is a Parallax of its apparent Motion in its apparent Way (as has been shewn at large in the Definitions of the Parallaxes (*Prop. 45. B. II.*) for that Reason it differs a little from an equable one. Therefore for the Determination of the apparent Way AK , you must take the Time after the Moon has been in A , if C is

632 *The ELEMENTS* Book IV.

after S ; but the Time before, if C be before S , according to the Order of the Signs, and, as far as we can conjecture, almost equal to that which the Moon must take up to move from A to M ; which will be nearly equal to that which is requir'd for the Moon, by its Motion from the Sun along the Ecliptic, to run thro' the known Arc CS ; for they only differ by the apparent Motion arising from the Parallax.

If from S be drawn SV , perpendicular to the Ecliptic, meeting the Moon's apparent Way at V , V will be the Point where the Moon's Center is, at the Time of an apparent Conjunction. But AV may be thus found: In the Triangle NSV , besides the Right Angle NSV , there are given the Side NS and the Angle N ; wherefore the Side NV will be known; and since the Arc NA is also known, VA will likewise be found; therefore the Time necessary for going thro' it will be known, and therefore the very Moment of the apparent Conjunction. But because the Points M and V are very little distant from each other, therefore the Time of the apparent Conjunction is commonly taken for the Moment of the greatest Darknefs.

PROPOSITION XLIX.

TO determine the Time of the Beginning and End of a Solar Eclipse, happening upon a given Month, and seen in a given Place, and shew what its whole Duration will be. [Fig. LI.]

Things being as before, let us suppose XR to be the Moon, going along its Way AN , and HR to be the Sun at R : It is requir'd to find the Time, when they will be in this Position. Let S and L , the Centers of the Luminaries, be join'd by SL , the Arc of a great Circle. In the

Book IV. of ASTRONOMY. 633

the Triangle SML , besides the Right Angle at M , you have given the Sides SM , SL , the first of them determin'd in the foregoing Proposition, and the last, the Aggregate of SR , LR , the Semidiameters of the Luminaries; therefore the Arc ML will be known, which is in Appearance to be run thro' by the Moon's Motion from the Sun between the Beginning of the Eclipse, and the Moment of the greatest Obscurity (known by the foregoing Proposition) happening when the Center of the Moon comes to M : Therefore also you will know the Quantity of its correspondent one SE , in the Ecliptic, reckon'd backwards from S . In order to determine the Time requir'd for this, at the Time assum'd (which is an Hour or Two, according as the Duration of the Eclipse is great or small, which will be known from the Greatness or Smallness of the Arc SM ; and here again all will be so much more exact, as the Time assum'd comes nearer to the half Duration of the Eclipse) before the known Moment of the greatest Obscurity, find the apparent Places of the Luminaries, which being reduc'd to the Ecliptic (the Sun's Place being come to S) will make the computed Distance CS , in the Ecliptic. Knowing then the Time (because it is taken) that CS , a known Arc of the Ecliptic, is run thro' by the Motion of the Moon from the Sun; you will also know the Time, in which another known one, ES , is gone thro', by the same Motion of the Moon from the Sun; that is, the Time between the Moment of the greatest Obscurity, and the Beginning of the Eclipse: But the former is determin'd by the foregoing Proposition; therefore this latter will be also known.

If

If the Moon's apparent Motion was equable, there would be as much Time requir'd from the Moment of the greatest Obscurity, to the End of the Eclipse, as from the Beginning to the said Moment, because the Right Line MP is equal to LM , supposing the Moon HT to touch the Sun RH , at the End of the Eclipse; and the Time of the greatest Obscurity wou'd coincide with the Time of the Middle of the Eclipse, as it happens in the Moon's Eclipse, where there is no Parallax of the Moon from the Shadow of the Earth; for they touch one another. But because here it happens otherwise, as has been shewn in the foregoing Proposition, in order to find the Time of the End of the Eclipse, you must repeat the same Operations, finding what a Portion of its Way the Moon goes thro' in a Time taken after the greatest Obscurity, and finding, by Analogy, how much Time is requir'd for the Arc PM (equal to the before-found one LM) to be mov'd thro' by the Moon's apparent Motion from the Sun. For this, being added to the before-found Moment of the greatest Obscurity, will give the Moment of the End of the Solar Eclipse, seen from a propos'd Place: And knowing the Time of the Beginning and End, you will know the whole Duration. *Q. E. F.*

SCHOLIUM.

Whilst the greatest Part of the foremention'd Triangles is resolv'd by Trigonometry, they may be look'd upon as Plane: For by Reason of the Smallness of their Sides, the Part of the spherical Surface, which they take up, differs little from a Plane, as was said before in the like Case, *Prop. 38.* And this gives a compendious Way of Calculating Eclipses of the Sun, which is very

valuable,

Book IV. of ASTRONOMY. 635

valuable, because the Way of doing it by the Calculation of the Parallaxes, which must be so often found, is very tedious; but we must be cautious how we use this short Method.

COROLLARY.

Hence also may we exactly draw a Scheme of a Solar Eclipse for a given Place. For since so small a Part of the Heavens, as comprehends the Sun, (reckoning the Space it takes up from the Beginning to the End of the Eclipse) may be look'd upon as a Plane, the Ecliptic CN , together with the Sun's apparent Way AN (which sometimes, by reason of the Parallax, will sensibly vary from a great Circle, or a Right Line) and the Points S , M , L , and P , where the Sun's and the Moon's Centers are at the said Times, may be rightly protracted after the common Method: And describing Circles about the Points S and M , whose Diameters are proportional to the apparent Diameters of the Luminaries at that Time, and are taken in such Parts, as the Portions of the Ecliptic, and the apparent Way, were before taken, you will have the Sun's Phasis, at the Time of the greatest Obscurity. And as (by *Prop. 36. B. II.*) you have the Angle which the Ecliptic comprehends with the Sun's Vertical, and the Situation of the Vertical, in respect to the Meridian; if the Scheme, above drawn, be inclined to the Plane of the Meridian, and to the Horizon, in the due Angles already found, the Scheme of the greatest Obscurity will be perfect. Moreover, if the Line AN , representing the Moon's apparent Way, be divided into such Parts as the Moon appears to go thro' in Hours, and Parts of Hours (all the while supposing

636 *The* ELEMENTS Book IV.

supposing the Sun at Rest) and you number them with the Hours, as they are reckon'd in a given Place, (which may be easily done, because the Times of the greatest Obscurity, of the Beginning and of the End, which are set against the Points *M*, *L*, and *P*, are known from the foregoing Propositions;) you will have an exact Scheme: And a Phasis, agreeable to a given Hour, may immediately be drawn, if about the Center, which has the given Hour set down against it, you draw a Circle with a Diameter equal to the Moon's apparent Diameter; for it will cut off a Segment from the Circle about the Center *S*, which represents the Sun, similar and similarly posited to the Ecliptic, with that of the Sun, which the Moon at that Time covers.

The Tables necessary for calculating the Eclipses (besides those that we have described in the 26th and 27th Propositions) their Foundation, Description and Order, (as far as our Design will permit) are to be had in their proper Places; namely, that of the Luminaries apparent Diameters in *Prop.* 32. that of the Inclination of the Moon's Way from the Sun to the Ecliptic, in the *Schol.* of *Prop.* 36. and that of the Time between the true Syzygy, and the Middle of the Eclipse, in *Prop.* 37.

PRO-

PROPOSITION L.

TO give the Reason why the Lunar Tables, hitherto calculated, shew the Moon's Place in the Eclipses of the Luminaries, generally more accurately in the Syzygies than in any other Aspects, especially than in the Quadratures.

Besides that, in order to determine the Lunations, the Syzygies of the Sun and Moon have been more accurately observ'd by Astronomers, than any other Phases, and the Moon's Place, in respect of the Sun, has been most nicely observ'd in Eclipses of the Luminaries, and thence a Theory has been particularly made and fitted for finding the Moon's Place, in those Cases, by Calculation; all the Monthly Inequalities, whether those that are to be fill'd up in the Syzygies with the Sun, or those that affect the Motion of the Moon and its Orbit, cease in the Syzygies; and have sensible Effects in the other Aspects. But all of them (except the Variation alone) are neglected by the Constructors of Lunar Tables, as has been shewn in the 29th Prop. whence we see the Reason why the Moon's Place, taken from those Tables, is more exact in the Syzygies, than in any other Aspect with the Sun. For all these Inequalities become greater, by how much the Moon is farther remov'd from the Syzygies, so that they are greatest when the Moon is in the Quadratures: For the Apes of the Lunar Orbit, which, absolutely speaking, go forward, do in the Moon's Quadratures either go back, or at least go forward more slowly, by Prop. 8. Likewise the Nodes, whose absolute Motion is regressive, go back slower, when the Moon is in Quadrature with the Sun, by Prop. 14. And the Excentricity of

638 *The ELEMENTS* Book IV.

of the Lunar Orbit is less, and its Inclination to the Ecliptic greater (by the 12th and 15th Propositions) when the Moon is in Quadrature with the Sun, than in any other Aspect, *cæteris paribus*; whence the Lunar Tables, hitherto constructed, are very far from determining the Moon's true Place in the Sun's Quadrature. Again, since there is no Account in the Tables, of the Moon's greater Distance from the Earth in the Quadratures, than in the Syzygies, (of which we have treated in the 2d *Prop.* and its Corollaries,) and of the Moon's Parallax and apparent Place, which depended upon it; these last are falsely taken from the Tables.

Moreover, since many Inequalities of the Moon depend upon the Aspect of the Nodes of the Moon's Orbit with the Sun, and tho' there is Notice taken of them, in some Tables that have been hitherto constructed, yet this is not done with the greatest Exactness: Therefore the Moon's Place will, even from these Tables, be had more accurately, when those Inequalities cease; but they cease when the Nodes are in the Syzygies; for (by *Prop. 14.*) the Nodes are then at rest, and the Inclination of the Limits is then the greatest, by *Prop. 15.* Besides, in the Eclipses of the Luminaries, the Nodes are at a stand, in, or near the Syzygies; hence the Reason is plain why in these, the Moon's Place is more accurately found in the Tables: For the Errors of the Tables in the various Regress of the Nodes, and the various Inclination of the Lunar Orbit to the *Magnus Orbis*, according to the various Aspect of the Nodes to the Sun, (which are certainly so, in some Measure) have no Place in this Case. For at the Time of the Eclipses, there is a Cessation of the Causes that
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produce those Inequalities which are not exactly set down in the Tables, and therefore the Effects of the Errors of the Tables in the Moon's Place, for either Eclipse (and for a whole Month before and after either Eclipse) by being wrong defin'd, also cease. Therefore since in the Eclipses of the Luminaries, both the forementioned Cases obtain, namely, the Moon itself, and the Nodes of the Moon's Orbit also are in the Syzygies, the Moon's Place, at that Time, taken out of the Tables (as wanting fewer *Æquations*, because it is liable but to few Inequalities) will be more accurate and exact, than in any other Situation or Aspect of the Moon, or its Nodes, to the Sun, where there are Inequalities, of which there is no Account, or at least exact one in the Tables.



SECTION IX.

Of the Motions of the Satellites revolving about other Primary Planets, besides the Earth.

PROPOSITION LI.

THE Distance of a Primary Planet from the Sun remaining the same, if several Secondary Planets revolve about a Primary one in similar excentric Orbits, and similarly inclin'd to the Plane, in which the Primary one revolves about the Sun, all the Angular Errors in the Motion of the Satellites, seen from the Center of the Primary Planet, and generated in any Revolution of the Satellites, are directly as the Squares of the Periodical Times respectively.

Because all Bodies have the same accelerating Force towards the Center of the Sun, or of a Primary Planet, at the same Distance from it, it is plain that several Secondary Planets, revolving about a Primary one, are affected the same Way, as one Secondary Planet, whose Orbit is dilated and contracted, as much as the Orbit of those different Secondary Planets are distant from each other. But in this Case it is plain (from Prop. 18.) that the said Angular Errors seen from the Center of the Primary Planet, and generated in every Revolution, are directly as the Squares of the Periodical Times. Whence the Truth of the Proposition is evident. Q. E. D.

COROLLARY.

Since all the Errors abovemention'd, generated in the different Times of the different Revolutions,

volutions, are as the Squares of the said Times ; those that are generated in the same and a determinate Time, will be as the Times of the Revolutions themselves, that is, as the Periodical Times ; viz. taking away on either Side one of the compounding Ratio's, the simple Ratio of the Periodical Times.

But because some of the afore'said Errors, viz. the Motion of the Ap'ses and Nodes, is sometimes *in antecedentia*, sometimes *in consequentia* ; therefore their mean Motions, made in any remarkable Time, towards the same Parts, will, in the present Case, be as the Periodical Times of the Satellites.

PROPOSITION LII.

IF two Primary Planets revolve about the Sun at different Distances, and one Secondary Planet revolves about each Primary one, and their Orbits be equal, similar and similarly inclined to the Orbits of their Primary Planets ; the Sun's Forces and their Effects, that disturb the Motion of the Satellites, that is, all the angular Errors, respectively seen from the Centers of their Primary Planets, are reciprocally in a Duplicate Ratio of the Periodical Times of the Primary Planets about the Sun.

For, because the accelerating Force of any Primary Planets whatever about the Sun, is the same, at the same Distance, it is plain that those Secondary Planets which revolve in equal Orbits, about different Primary Planets are affected the same Way as one Secondary Planet revolving about its Primary one, which revolves about the Sun, in an Orbit so excentric, that its Distances from the Sun, in divers Points of such an Orbit, differ as much from one another, as the Distances of the two

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642 *The* ELEMENTS Book IV.

Primary Planets in the present Case. But it is plain (by *Prop. 19.*) that in such a Case, the said Errors, both in Longitude and Latitude, seen from the Center of the Primary Planet, are inversly as the Cubes of the Distances of the Primary Planet from the Sun. And therefore the said angular Errors of Secondary Planets, revolving in equal similar and similarly inclin'd Orbits about two Primary Planets, mov'd about the Sun, at different Distances, seen from the Centers of the respective Primary Planets are inversly as the Cubes of the Distances of the Primary ones from the Sun. But, by the 40th and 42d Propositions, the Squares of the Periodical Times of the Primary Planets, about the Sun, are as the Cubes of the mean Distances of them from the Sun: And therefore all the angular Errors of the Secondary Planets respectively, seen from the Centers of their Primary ones, are inversly as the Squares of the Periodical Times of the Primary Planets about the Sun. Q. E. D.

PROPOSITION LIII.

THE angular Errors of the Satellites, generated in every one of their Revolutions about their respective Primary Planets, and seen from their Centers, are in a Ratio, compounded of the direct duplicate Ratio of the Periodical Times of the Satellites, about their respective Primary Planets, and of the duplicate reciprocal Ratio of the Periodical Times of the Primary Planets, about the Sun.

For, by *Prop. 51.* if the Distances of the Primary Planets from the Sun were equal, the fore-said Errors wou'd be in a direct duplicate Ratio of the Periodical Times of the Satellites, about their Primary Planets respectively: And by *Prop. 52.* if the Distances of the Satellites, from their Primary Planets were equal, the said Errors wou'd

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Book IV. of ASTRONOMY. 643

wou'd be in a reciprocal Duplicate Ratio of the Periodical Times of the Primary Planets about the Sun: And therefore, if neither the Orbits of the Primary Planets about the Sun are equal, nor those of the Satellites about the Primary Planets; the Ratio of the foresaid angular Errors, generated in every respective Revolution of the Satellites, and seen from the Centers of their Primary Planets, is compounded of the foresaid Ratio's, *viz.* the direct Duplicate Ratio of the Periodical Times of the Satellites about their respective Primary Planets, and the Duplicate reciprocal Ratio of the Periodical Times of the Primary Planets about the Sun. Q. E. D.

SCHOLIUM.

From the same Reasoning (by *Prop. 52.* and the Corollaries of *Prop. 51.*) you may gather, that the mean Motions of the Apes and Nodes of Two Satellites, are in a Ratio compounded of the direct Ratio of the Periodical Times of the Satellites about their respective Primary Planets, and the reciprocal Duplicate Ratio of the Periodical Times of the Primary ones about the Sun.

PROPOSITION LIV.

TO determine the unequal Motions of Satellites revolving about any other Planets.

From the before determin'd unequal Motions of the Moon, seen from the Center of the Earth, the analogous Motions of any Satellite may, by the Help of the premis'd Theorems, be determin'd, if the Form of the Orbits of the Satellites and the Inclination of the Orbits to the Plane, in which the respective Primary Planets are carried about the Sun, do not much differ

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644 *The* ELEMENTS Book IV.

from the Form of the Lunar Orbit, and the Inclination of its Plane to the Plane of the Ecliptic, as it happens in the Satellites of Jupiter and Saturn: For their small Difference does not sensibly change the Motions, as it has been taken notice of in the 11th and 14th Propositions.

For Example, let the Motions of the outermost of the Circumjovial Planets be to be determin'd: It is plain, from the Scholium of the foregoing Proposition, that the regressive mean Motion of the Nodes of the said Satellite is to the mean Motion of the Moon's Nodes, in a Ratio compounded of the Ratio of the Periodical Time of the said Satellite about Jupiter, to the Periodical Time of the Moon about the Earth, and in the Duplicate Ratio of the Periodical Time of the Earth about the Sun, to the Periodical Time of Jupiter about the Sun; and therefore it is given, since the said Periodical Times are known by Observation.

Likewise, from the mean progressive Motion of the Apfes of the Moon, may be found the mean progressive Motion of the Apfes of a Satellite.

After the same Manner, from the foregoing Proposition, the Variation of the said Satellite is to the Moon's Variation above found, in a Ratio compounded of the Duplicate Ratio of the Periodical Time of that Satellite about Jupiter, to the Periodical Time of the Moon about the Earth, and of the Duplicate Ratio of the Periodical Time of the Earth about the Sun, to the Periodical Time of Jupiter about the Sun; that is, (by the said foregoing *Prop.*) as the whole Motions of the Nodes in the Periodical Times of the Satellite and the Moon to one another.

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Book IV. of ASTRONOMY. 645

And after the same Manner, the greatest *Æ*quations of the Nodes and the Apfes of a Satellite are to the greatest *Æ*quations of the Nodes and of the Apfes of the Moon respectively, as the Motions of the Nodes and Apfes of a Satellite, during the Time of one Revolution of the former *Æ*quations, to the Motions of the Nodes and Apfes of the Moon, during the Time of one Revolution of the latter *Æ*quations.

All these Things likewise hold true of any other Satellite. But having once found the said Inequalities in one of the Satellites, the analogous Inequalities in any other, revolving about the same Primary Planet, are more easily determin'd, by *Prop. 51.* and its Corollaries; for there will be no Need of making use of an inverse Duplicate Ratio of the Periodical Time of the Primary Planet about the Sun, since the Primary Planet remains the same.

PROPOSITION LV.

TO correct the Motions of the Apfes of the Satellites of *Jupiter* and *Saturn* found by the foregoing Proposition.

Because the Apfis of the Moon, in the Syzygies with the Sun, goes forward, and also the Apfis of a Satellite, by *Prop. 11.* but slower by reason of the short Periodical Time of the Earth about the Sun, in comparifon of the Periodical Time of Saturn or Jupiter about it; it is plain that the Moon's Apfes stick longer in the Syzygy of the Sun and near it, than the Apfis of a Satellite, because it does more closely follow the Sun, which is carried *in consequentia* faster than either Apfis. And on the other hand each Apfis of a Satellite goes back, when in the Sun's Quadrature,

646 *The ELEMENTS Book IV.*

drature, but that of the Moon faster ; therefore it remains a shorter Time in the Sun's Quadrature, which it meets. And since, for the first Cause, the Apfis of a Satellite goes forward a less Time than the Apfis of the Moon, and, by the second Cause, it is longer a going back than the Apfis of the Moon, and the absolute Progress of each arises from a greater Progress than Regress ; it is plain that those two Causes conspiring diminish the progressive Motion of the Apfes of a Satellite, more than in proportion to the other Causes. Therefore, upon this Foundation, we must build the Correction of the before found Motion of the Apfes, and diminish the said Motion according to the Permanency of this Cause and its Effects on each of the Satellites of Jupiter and Saturn.

SCHOLIUM.

These Inequalities which affect the Motion of a Satellite revolving about any Primary Planet, being determin'd, it will be easy (according to the foregoing Methods about the Moon) to find, by Calculation, the Places of those Satellites for a given Time, as also their Immersions into the Shadow of their Primary Planet, their Emerfions out of it, the Journey of the Shadow of a Satellite over the Disc of his Primary Planet, and other things concerning their Motions, as they are seen from their proper Primary Planets. And the Form of the Tables for determining their Motions, seen from thence, will be the same as that of the Moon's Tables above described ; from which, together with the Tables of the Primary Planets, (describ'd in the foregoing Book) one may readily find the Phænomena of those Satellites. But because the Eclipses of the Satellites,

Book IV. of ASTRONOMY. 647

lites, as they run into the Shadow of their Primary Planet, are very remarkable Phænomena to be observ'd by the Inhabitants of the Earth; Astronomers have constructed Tables for the immediate Calculation of them, whose Foundation and Form appear easy and simple to any one that understands the foregoing Propositions; and therefore we shall only give a few Examples to illustrate them over and above; which yet do not properly belong to these *Elements*, because they are only Physical and Geometrical, and not Arithmetical.

But yet we ought to take notice, that Regard is to be had to the different Distance of the Earth (the Place of the Observer) from these Phænomena of the Immerfion and Emerfion of the Satellites of Saturn and Jupiter: For such a Phænomenon is fooner feen when the Earth is nearer to it, than when further from it. Let *S* [*Fig. LII.*] represent the Sun; *C D E F* the *Magnus Orbis*, or the Way of the Earth; *G H*, the Way of Jupiter, ☿ Jupiter casting his Shadow ☿ *V*; *B A K* the Orbit of a Satellite of Jupiter. Let an Inhabitant of the Earth at *C*, fee the Emerfion of the Satellite out of the Shadow of Jupiter at *A*; for if the Satellite be one of the nearest to Jupiter, or the Earth be after the Oppofition, or before the Conjunction of Jupiter with the Sun, then the Immerfion at *B* will not be feen by reason of the Interpofition of Jupiter's Body: If the Spectator fhould remain in the fame Place, after fome Time the like Emerfion of the fame Satellite wou'd again appear, and another fuch Emerfion of it after the fame Space of Time, and fo on. But as the Earth goes from *C* to *D*, it is plain that there is Need of a longer Time for the fame

648 *The* ELEMENTS Book IV.

Appearance to reach to it at *D*; namely, as much Time more as is needful for the Light (which, as it were, brings the Phænomenon) to go thro' the Distance *CD*. For the same Reason, the Immersion of the Satellite at *B*, seen by an Inhabitant of the Earth at *E*, will, after a determinate Time, be again seen, and so on; but the Earth, as it were, meeting the Phænomenon, as it goes from *E* to *F*, a Spectator will sooner see the Phænomenon when he is at *F*, than when he was at *E*. And this proceeds from the different Distance of the Earth from the Bodies whose Phænomena are observ'd, to which we said that Regard was to be had: For by reason of this different Distance, they are seen sooner or later, as also all the Inequalities which really happen in the Motion of the Bodies themselves. And from comparing such Phænomena, seen sooner or later with the Earth's annual Orbit, we, on the contrary, discover the successive Motion of Light, and from it find out its Velocity by Observations.



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SECTION X.

Of the Motions of the Satellites about their own Centers respectively.

PROPOSITION LVI.

TO describe the Motion of the Moon about its own Center, as it is carried at the same Time round the Earth.

For an Example of the Motion of the Satellites about their own Centers, we shall give an Instance in our Moon, to whose Motion 'tis reasonable to believe the Motions of the Satellites of other Planets to be analogous; for we cannot make minute Observations of those small Bodies, which are at so immense a Distance. The Motion of the Moon is, in all things, like the Motion of the Earth about its own Center, whilst it is carried round the Sun, excepting the Inclination of its Axis and the Period of its Revolution. For as the Moon moves round the Earth, it moves also about its own Axis: This Axis is every where parallel to itself, therefore not perpendicular to the Plane of the Moon's Orbit, (for this Orbit is continually chang'd by reason of the Action of the Sun which disturbs it) but inclined to it, and almost at right Angles with the fix'd Plane of the Ecliptic. The Moon's Motion about this its own Axis (as also that of the Earth about its Axis, and of the other Primary Planets about their Axes, shewn in *Prop. 30. B. I.*) is uniform and equable, always of the same Tenour; and its Period is the same with the Periodical Month,

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650 *The* ELEMENTS Book IV.

different from what happens in the Earth and the other Primary Planets carried about the Sun, every one of which performs several Revolutions about its own Axis, whilst it goes once thro' its Orbit.

PROPOSITION LVII.

THE Plane of the same Lunar Meridian (or of a Circle made by the Section of a Plane along the Axis of its Rotation) being produc'd, does always nearly pass thro' that Focus of the Elliptic Orbit (describ'd about the Earth) in which the Earth is not. [Fig. LIII.]

Let $ABCD$ be the Ellipse, whose Periphery is describ'd by the Moon moving about the Earth: Let the Earth T be plac'd in one of its Foci, to which a Ray or Radius (which we have call'd a *Vector*) being drawn, describes Areas proportionable to the Times in which they are describ'd. And by *Prop. 5. B. III.* the Right Lines AF , BF , being drawn from any two Points of an Ellipsis to one of its Foci F , (namely, that in which the Earth is not) the Angle AFB is to four Right ones nearly as the Time in which the Moon runs thro' the Arc AB , to the whole Time of the Moon's Period. If AM represents the Plane of the Lunar Meridian, which (the Moon's Center being any where, as for Example, at A) being produc'd goes thro' F ; I say that the same Plane, being likewise produc'd, will go thro' F , in whatsoever Point of the Elliptic Orbit the Moon is. For, because the Rotation of the Moon is uniform about its own Axis, and it also has the same Period as the Moon has about the Earth; it is evident that the Plane of the Meridian which in the Moon's Situation A was AM , when the Moon's Center

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Book IV. of ASTRONOMY. 651

ter is got to any other Point (as B) will have such a Situation BE , that the Line Bm being taken parallel to AM , the Angle mBE will be to four Right Angles as the Time in which the Moon is carried thro' the Arc AB to the Moon's whole Periodical Time: Therefore (by *Prop. 11. El. 5.*) mBE is to four Right Angles as AFB is to four Right Angles; whence (by *Prop. 9. El. 5.*) the Angle mBE is equal to AFB . And since AF and Bm are parallel, BE and BF will be on a Right Line; that is, in a Situation of the Moon B taken at Pleasure, the Plane of the same Meridian, which, produc'd, did in the former Situation A go thro' F , produc'd in this does, also go thro' F . Therefore in what Point soever of the Orbit the Moon's Center is found, the Plane of the same Meridian will go thro' F , the Focus in which the Earth is not: Q. E. D.

PROPOSITION LVIII.

TO describe the Motions which appear in the Moon's Face, seen from the Earth, arising from the Moon's Motion about its own Center.

First, it is plain that nearly the same Face of the Moon is always turn'd towards the Earth, and that nearly the same Lunar Spots are seen by an Observer on the Earth. For since the produc'd Plane of the same Meridian goes thro' the empty Focus F of the Lunar Orbit, and the Lunar Orbit is not very excentric, that is, F is not very far from T , or the Angle FBT (when greatest) is not very great, it is plain that nearly the same Face of the Moon is always turn'd to the Earth.

But if that Face of the Moon which is turned towards the Earth be nicely consider'd, by looking at the Spots, so as to draw the Figure of

of it by the Help of a Telescope, it will appear that we don't precisely see the same Face. And since the Plane AM of the same Meridian is not directed to the Earth $\mathcal{T}$, but to the other Focus F , it is plain that the Hemisphere of the Moon when it is at A , seen from the Earth $\mathcal{T}$, is something different from that which is seen from it when the Moon is come to B , some Part of the Moon's Hemisphere which is drawn towards D , and was hid from the Eye in $\mathcal{T}$ when the Moon was at A , being now discover'd, and Part of the Hemisphere towards C , which was then seen, being now out of Sight. And this apparent Motion of the Moon from the Earth, by which some of its Spots in the Hemisphere turn'd from the Earth, come in Sight, and some of those in the Hemisphere towards the Earth get out of Sight, is properly enough call'd, the *Libration of the Moon*, and that which happens for the reason just given is call'd, the *Libration in Longitude*, arising from the Moon's Motion in Longitude, or along the Zodiac. This Libration is twice restor'd in every Periodical Month; namely, when the Moon is in the Apogæum Cor Perigæum D : For in either Case, the Plane of the same Meridian, which being produc'd falls on F , falls also on $\mathcal{T}$. And this Libration is affected by all the Inequalities of the Motion in Longitude, and, amongst others, by that Inequality, by which the Moon, in Quadrature with the Sun, is remov'd further from the Earth, of which Mention has been made in the Corollary of *Prop. 2*.

Moreover, because the Axis of the Moon's Rotation is not perpendicular to the Orbit which the Moon describes about the Earth, but inclin'd to it, it is plain that sometimes one Pole of
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Book IV. of ASTRONOMY. 653

the Moon, sometimes another, points towards the Earth, (for the same Reason that sometimes one Pole of the Earth, and sometimes another is turn'd towards the Sun) and therefore sometimes we see from the Earth the Lunar Spots which are near one Pole, and sometimes those which are near the other, and consequently we see another Libration of the Moon from the Earth; which, as it depends upon the Situation of the Moon, in respect of the Nodes of its Orbit with the Ecliptic, (for its Axis, as has been observ'd, is nearly perpendicular to the Plane of the Ecliptic) is rightly call'd, the *Libration in Latitude*. This is perform'd in the Space of a Periodical Month, or rather as the Moon returns to the same Situation in respect of the Nodes.

Again, there is a third Kind of Libration of the Moon, by which, tho' no other Part of it be turn'd to the Earth than what is occasion'd by the two foregoing Causes (namely, the Libration in Longitude, and the Libration in Latitude) yet another Part is enlighten'd by the Sun. For since the Axis of the Motion is nearly perpendicular to the Plane of the Ecliptic, when the Moon goes farthest from the Ecliptic towards the South (or is in the Southern Limit) the Moon's North Pole, and also some Part of the Lunar Globe beyond that Pole, are enlighten'd by the Sun, when at the same time the South Pole of it, and some Parts on this Side of it (that is, on the Side towards the Sun) are in Darkness; if therefore in this Case it happens that the Sun is on the same Side as the Moon's Southern Limit, the Moon, going from the Conjunction with the Sun to the ascending Node, will appear by little and little to draw away its

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654 *The* ELEMENTS Book IV.

Northern Parts and the Spots near the North Pole, into the darken'd Hemisphere, when at the same time, on the opposite Side, other Spots about the South Pole emerge out of Darkneſs into Light: And the contrary will happen half a Month after, when the New Moon deſcends from the Northern Limit; that is, the more Northern Parts of the Moon will by little and little break out into Light, at the ſame time that the moſt Southern Parts hide themſelves in Darkneſs. It is plain that this Libration, or rather theſe Effects of the Libration in Latitude, depend upon the Sun's Light, and therefore on the Synodical Month.

SCHOLIUM.

From this Propoſition it appears how, manifold this Libration of the Lunar Globe, ſeen from the Earth, is; which is produc'd by its moſt ſimple Motion about its own Axis, deſcrib'd in *Prop. 56.* for the foregoing Kinds of Librations, according to the various Situation of the Lunar Apſes, of the Nodes and of the Sun (upon all which they depend) are variously mix'd and combin'd, and make that moſt intricate Libration of the Moon obſerv'd by Aſtronomers.

And that all theſe things obtain likewise in the Satellites of Jupiter and Saturn is very probable, becauſe they go round thoſe Primary Planets, and together with them, are carried about the Sun, juſt as the Moon and Earth; and alſo that Jupiter's and Saturn's Satellites have that very remarkable Phænomenon, in reſpect of their Primary ones, which has ever been obſerved of the Moon, namely, that it always turns the ſame Face to the Earth, as was ſaid at the Be-

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Beginning of the last *Prop.* For it is observ'd of the outward Satellite of Saturn, that it can only be seen, when it is on the West Side of this Planet; and it is always hid, when on the East: It is likely that it happens for this Reason, that the greatest Part of its Surface is darker (that is, does not copiously enough reflect the Rays of Light every way) than the other. Because this Satellite, like our Moon, may have very ample Plains or Vallies, which being the darkest Part, when turn'd towards us, can't be seen upon account of the small Quantity of the Light reflected. And as this Saturnian Moon is found to be darken'd, when it is in the Eastern Part of its Orbit, and never in the other; it is a plain Sign that the same Face of the Moon always looks towards Saturn, that Phænomenon being a Consequence of this Phænomenon, as is evident. And now since this same thing is true both of that very Moon, and of our Moon, *viz.* that the same Face is always turn'd to the Primary Planet, who can doubt of its obtaining in the other Moons about Jupiter and Saturn, tho' we have not Observations to evince the Truth of it?



SECTION XI.

Of the Magnitude and Density of the Satellites.

IN endeavouring to find out the Magnitude of the Satellites, we shall also suppose that they, like the Primary Planets (about which they revolve) and all other great Bodies in the World, at first (by reason of the Gravitation of the Parts towards the Whole, and the Center of the Whole) when their Matter was fluid, were of a Spherical Figure. This abundantly appears to be true in the Moon's Satellite of our Earth, from its Monthly Phases, Eclipses, and all the other Arguments which can be devis'd, as may be seen in all Writers of Natural Philosophy and Astronomy: This also appears in the Satellites of Jupiter and Saturn; for whatever Face they turn to a Terrestrial Observer (for they turn all their Faces successively to the Earth) they appear to have a circular Disc, which only happens in Spherical Figures.

Tho' the Figure of the Planets be Spherical, considering any whole Planet, yet some Inequalities (and great ones if compar'd with those on our Earth) are found on their Surfaces, arising from particular Causes near their Surfaces, exerting their Forces on such Parts as are already hard and solid; such as are found in our Earth and in the Moon, but are not look'd upon as considerable, in respect of their Globes, and therefore do not take off any thing from their Roundness.

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Book IV. of ASTRONOMY. 657

In investigating the Magnitude and Density of the Satellites, we shall take our Moon for an Example, and compare them with it.

PROPOSITION LIX.

TO define the Moon's Magnitude. [Fig. LIV.]

Because from the foregoing Propositions we know both the Moon's Distance, and the Angle under which it appears, that is, (ABC representing the Moon, L its Center, and O the Eye) the Right Line OL and the Angle AOB , and therefore the Half of it AOL ; in the rectangular Triangle LAO , whose Right Angle is A , LA the Moon's Semidiameter will be found: Or more readily thus: The Diameter of the Moon is to the Diameter of the Earth, as the known apparent Semidiameter of the Moon, to the Moon's known Horizontal Parallax, as it is plain. Now the Moon is to the Earth (by *Prop. 18. El. 12.*) in a Triplicate Ratio of its Diameter, to the Diameter of the Earth, and therefore it is defin'd. *Q. E. F.*

The Magnitude of the Satellites of *Jupiter* and *Saturn* will be likewise defin'd, from knowing their Distance, and the Angle which each of them subtends.

PROPOSITION LX.

TO define the Quantity of Matter in the Moon, in respect of the Quantity of Matter in the Earth. [Fig. LV.]

Let S denote the Sun; ABC the *Magnus Orbis*, in whose Circumference the common Center of Gravity of the Earth T , and of the Moon L is carried, as has been explained in *Book I.* Let A represent that Center at the known Time

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658 *The* ELEMENTS Book IV.

of some New-Moon; and *B* the same Center near the known Time of a Quadrature (the next, for Example.) And from these two Times given, you will (by *Prop.* 18. *B.* III.) have the apparent Places of the Sun, seen from the Points *A* and *B*: For (by *Prop.* 63. *B.* I.) the common Center of Gravity of the Earth and Moon, goes thro' the *Magnus Orbis* in such Manner, that the Areas, cut off by Radii drawn to the Sun, are proportionable to the Times. Observe the Places of the Sun and Moon from the Earth at *t*, and then you will know the Angles *BtS*, *ASt*; but *ASB* was known before, therefore *BSt* will be known. Whence in the Triangle *BSt*, the Angles *BtS*, *BSt*, being given, and the Side *SB* (which is a Line of the *Magnus Orbis*, drawn from the Sun, and given in Position, whose Quantity is known from the Sun's Theory and Parallax in Diameters of the Earth) the Side *tB* will also be had in such known Measures; but *tI*, the Distance of the Moon from the Earth, is known from the Theory of the Moon; therefore (by *Dat.* 1.) the Ratio between *tI* and *tB* is known, and therefore that between *Bt* and *Bl*. Moreover, it is well known from *Mechanics*, that there is the same Ratio of the Gravity of the Moon towards the Sun, at a vast Distance, to the Gravity of the Earth towards it; that is, (by *Corol. Prop.* 47. *B.* I.) of the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, as of the Right Line *Bt*, to the Right Line *Bl*; therefore the Ratio of the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, is defin'd.

Q. E. F.

To make this Observation, we take one of the Times at the Syzygy of the Sun and Moon, and

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Book IV. of ASTRONOMY. 659

and the other near the Quadrature, that the Angle BSt may be the greater.

The same Thing may also, according to the same Principles, be gather'd from the different apparent Diameter of the Sun, in the New-Moon and the Full-Moon; for it is (*cæteris paribus*) less at the Time of the New-Moon, and greater at Full-Moon.

SCHOLIUM.

The forefaid Inequalities in the Motion of the Earth, arising from hence, that not the Earth it self, but the common Center of Gravity of the Earth and Moon, is carried in the *Manus Orbis*, and describes Areas (by Rays drawn to the Sun) proportionable to the Times, is also to be observ'd in the other Planets. For this Monthly Parallax of *Mars*, when *Achronical*, is twice, and that of *Venus*, in Conjunction with the Sun, and Retrograde, three times greater than the Sun's, (because in such a Case, *Mars* is twice, and *Venus* three times nearer the Earth than the Sun;) but in the other Planets it is almost insensible. There will likewise be the same Inequalities in the Motions of *Jupiter* and *Saturn*, (which have Satellites) especially when all, or most of the Satellites, are on one Side of the Primary Planet: But these Primary Planets are so very great, in respect of their Satellites, that this Inequality becomes less sensible.

PROPOSITION LXI.

TO define the Ratio of the Moon's Density to the Earth's Density.

It has been universally demonstrated in the 49th Prop. B. III. that the Ratio of the Density of two Bodies is compounded of the direct Ratio

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660 *The* ELEMENTS Book IV.

of the Quantity of Matter in them and the inverse Ratio of their Magnitudes ; that is (in the present Case) the Density of the Moon is to the Density of the Earth, in a Ratio compounded of the Ratio of the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, and the Ratio of the Magnitude of the Earth, to the Magnitude of the Moon : But the two compounding Ratio's are given, by the 59th and 60th Propositions ; therefore the Ratio compounded of them is given, namely, the Density of the Moon, to the Density of the Earth.
Q. E. F.

PROPOSITION LXII.

TO define the Ratio of the accelerating Gravity towards the Moon at its Surface, to the accelerating Gravity towards the Earth at its Surface.

It is plain from what has been before shewn, that at like Distances from the Centers, if they be taken without them, the accelerating Gravities of any Bodies, howsoever unequal, are as the Quantities of Matter in the said Bodies ; and at unequal Distances from the Centers, the accelerating Gravities of equal Bodies are inversly, as the Squares of the said Distances ; and therefore, if neither the Distances nor Bodies are equal, the Ratio's of the accelerating Gravities towards them, is compounded of the direct Ratio's of the Bodies themselves, towards which the heavy Bodies tend, and the inverse Ratio of the Squares of the Distances from their Centers, that is, (in the present Case) the accelerating Gravity towards the Moon at its Surface, is to the accelerating Gravity at the Earth's Surface, in a Ratio compounded of the Ratio of the Mass (or Quantity of Matter) of the Moon to the Mass

Book IV. of ASTRONOMY. 661

Mass of the Earth, and the Duplicate Ratio of the Semidiameter of the Earth, to the Semidiameter of the Moon, (for the Distances of the Surfaces from their Centers, are the Semidiameters of the Spheres) now the former of the compounding Ratios is given, by *Prop. 60.* and the latter of them, by *Prop. 69.* and therefore the compounded Ratio will be known, namely, the Ratio of the accelerating Gravity at the Surface of the Moon, to the accelerating Gravity at the Surface of the Earth.

By the same Observations and Reasoning may be defin'd the Masses and Densities of the Satellites of *Jupiter* and *Saturn*, in respect of their Primary Planets: And since their Masses and Densities, in proportion to those Qualities in the Earth, are defin'd, in the 48th and 49th Propositions of *B. III.* we have defin'd the Mass, Density and accelerating Gravity near the Surface of the Planets contain'd in the Solar System.



SECTION XII.

Of the Figure of the Satellites, and the Primary Planets, arising from their mutual Gravity.

ALtho' the Mountains and Vallies upon the Earth, and the Eminences and Cavities of the Moon (which appear to be such, from their Shadows projected, according to the different Situation of the Sun) do not hinder those Bodies from being round, as has been said before; yet there are some universal Deviations or Differences from a Spherical Figure, which are not to be omitted; as for Example, the prolate Spheroidical Figure, which has, in *Prop. 31. B. I.* been shewn to belong to all fluid Bodies turning round their own Axes. But by reason of the Slowness of the Rotation in the Lunar Globe, which is not perform'd in less than a periodical Month, the Deviation from a Spherical Figure, arising from it, will be almost insensible, if it be compar'd with the like Deviation in the Earth, as will be shewn in the following Proposition.

For another Reason the Moon is of an oblong Spheroidical Figure, whose Axis, produc'd, goes thro' the Center of the Earth. For since the Causes of this Figure of it are remarkable, and its Effects no less so, it is fit for us to consider the Matter further, especially since it will be a Means to explain the Phænomenon of the Tide, which is the most remarkable Thing of those, which, happening on the Earth, are produc'd by Celestial Bodies, as we shall shew it to be.

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PROPOSITION LXIII.

TO define the Moon's prolate Spheroidical Figure.

This Problem may be solv'd after the same Manner, as in *Prop. 53. B. III.* where the Figure of the other Planets is determined: For we have the Ratio between the Diameters of the Earth and Moon, about their own Axes, by *Prop. 56.* therefore we have (by *Prop. 26. B. I.*) the Ratio of the Centrifugal Force at the Æquator of the Moon, to the like Force in the Æquator of the Earth: But the Ratio between the Centrifugal Force at the Æquator of the Earth, and the Gravity at the Earth's Surface, is given, (by *Prop. 52. B. III.*) and (by *Prop. 42.*) you have the Ratio of the Gravity at the Surface of the Earth, to the Gravity at the Surface of the Moon; therefore the Ratio compounded of those Three will be given; namely, the Ratio of the Centrifugal Force in the Moon's Æquator, to the Gravity towards the Center of the Moon, which obtains on its Surface, which (in the said *Prop. 52. B. III.*) has been shewn to be the same as the Ratio of the Excess of the Moon's Æquatorial Diameter above its Axis. Therefore (by *Dat. 6.*) we shall have the Ratio between the Moon's Axis and any Diameter of it, which is perpendicular to the Axis; that is, between the Axis of an Ellipse, by whose Rotation about the shorter Axis, a prolate Spheroid like the Moon is generated.

The aforesaid Ratio comes so near the Ratio of Equality, that the prolate Spheroidical Figure of the Moon, arising from it, differs very little from a Spherical one.

PROPOSITION LXIV.

I*F the Globe of the Earth was every where covered with a deep Sea, it would nearly put on an oblong Spheroidical Figure, whose Axis produc'd wou'd pass thro' the Moon.* [Fig. LVI.]

Let a Circle, whose Center is T , represent the Earth, and a Circle, whose Center is L , the Moon: It is plain, that if neither the Moon nor any other Body acted on the Earth, if the Earth was every where cover'd with a Fluid, it wou'd be spherical, except so much as its Figure wou'd differ from a Sphere, upon account of the Reasons shewn in *Prop. 31. B. I.* which Deviation is not to be taken Notice of here. But because all the Parts of the Earth gravitate towards the Moon, and the Gravity towards the Moon (as well as towards any other Body) is reciprocally as the Square of the Distance from its Center; if (as it has been explain'd at large in the 60th *Prop.* of *B. I.* and in *Prop. 1. B. IV.*) the Right Line LT is put for the accelerating Gravity of a Body plac'd in the Earth's Center towards the Moon, and E (in the first *Fig. LVI.*) be any Particle of the marine Fluid, or of the Sea, and if in the Right Line LE (or LE produced) LK be taken equal to LT , and LF be taken such as to be to LK in a duplicate Ratio of LK to LE ; LF will express the Gravity of a Body placed at E towards L , which (as is commonly known) is divided into Forces, which are as FG , and GL , in the Directions of those Lines: But if from that Force, by which the Body, posited at E , is urg'd, you take away a Force which is as TL , by which the Center of the Earth, together with the whole Earth, is urg'd towards the Moon; there will be left
Forces

Forces as FG and GT , in the Directions FG and GT , by which a Body at E is urg'd, besides the Force of the proper Gravity, by which E tends towards the Center of the Earth. In the second *Fig.* LVI. things remaining as before, let C be a Point of the Earth, to which the Moon is in the Zenith, and A one, to which the Moon is in the Nadir, and B and D the Points round about (or rather a Circle) to which the Moon is in the Horizon. It is plain that the Point G is very much distant from T , when the Point E is either in C or in A : In the first Case, G will become M , and in the latter Case, N . But when the Point E is found in the Circle BD , the Point G nearly coincides with T , and in the Parts of the Fluid in the said Circle BD , there is no Force left besides that of their own Gravity, and the Force FG in the Direction FG ; but FG in this Case becomes BT or DT , the Point F falling in with K . Since therefore the Particles of the Fluid at C and A , besides their own Gravity towards the Center T , are urg'd by Forces, as MT and NT , that is, those at C , with the Force MT in a Direction from M towards T , that is, from C towards L ; and those at A , with the Force NT in a Direction from N towards T , that is, from A towards M ; they will not gravitate so much towards the Center T , as if the Force, arising from the Moon, was away: But the Particles at C are more attracted towards the Moon, than the whole Earth, which is look'd upon as plac'd in the Center T ; and the Particles at A are less drawn towards the Moon, than the whole Earth at T , and therefore being as it were left behind after the Earth, are affected after the same Manner, as if they were drawn to contrary Parts towards M : But the Particles

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in the Circle BD , whose Inhabitants have the Moon in the Horizon, have more Gravity towards T ; for besides their proper Gravity, another, as BT or DT , arising from the Force of the Moon, is superadded in the Direction from B or D towards T . But in the intermediate Places between A or C , and B or D , the Particles partake of both these Conditions, of which the first or the last prevails, as they are nearer to the first, or to the last Places; namely, by how much nearer the Parts of the Terrestrial Fluid are to the Points C and A , by so much less heavy are they, because the Action of the Moon (that is, a Force at GT) diminishes the Force of their own Gravity towards T , and by how much nearer they are to the Points B and D , by so much heavier are they, the same Action of the Moon encreasing their proper Gravity in that Case by a Force, as FG . But as the Globe $ABCD$ is suppos'd every where cover'd with a pretty deep Fluid, that is, such a one whose Particles easily yield to each other, and whose proper Motion is very free; it is plain that those Parts of the Fluid, which are at A and C , will be thrust out by the Parts which are at B and D , namely, the Lighter by the Heavier, according to the Laws of Hydrostatics. Therefore the Fluid at A and C , and near those Points, will stand up or be rais'd, but in the Circle BD , and near it, it will subside, till the greater Quantity and Height of the one is compensated by the greater Gravity of the other; so that the Earth, if every where cover'd with a deep Fluid, will form itself as it were into an oblong Spheroid, whose Axis is AC a Right Line, which, produc'd, will pass thro' the Moon. Q. E. D.

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SCHOLIUM.

For the same Reason the Terrestrial Fluid will be nearly of an oblong Spheroidal Figure, whose Axis, produc'd, goes thro' the Sun: For if, in the foregoing Reasoning, the Body *L* does not signify the Moon, but the Sun, the Proposition is plain. But the Axis of this Spheroid does not so much exceed the Diameters which are perpendicular to it, as it does in the Spheroid that the Moon produces. For, because the Elevation of the Fluid at *C*, under the Moon *L*, arises from hence, that it gravitates more towards the Moon than the Earth itself (which is to be look'd upon as plac'd at *T*;) because it is nearer to it; but its Elevation at the Point *A* arises from this, that the Earth at *T* gravitates more towards the Moon than the Fluid at *A* as being nearer to it; the Fluid at *C* drawn by the Moon (to use *Kepler's* Expression) leaves the Earth behind it, but the Fluid at *A* is, as it were, left by the Earth: If this is occasion'd by the Sun's Action, the Effects won't be so sensible, because the Semidiameter *TC* or *TA* of the Earth almost vanishes, in respect of the immense Distance between the Earth and the Sun, which, in respect of the Distance between the Earth and the Moon (which scarce exceeds thirty Diameters of the Earth) is sensible enough; and therefore the Gravity of the Fluid at *C* is but insensibly greater towards the Sun, than at *T*, and the Gravity of that Part of the Fluid which is at *A*, insensibly less than the Gravity of the Part of it which is at *T*; whence the Spheroid, thus produc'd by the Sun, will be but little rais'd above a Sphere at its Vertices *C* and *A*, and a little depress'd below the Surface of a Sphere in the Circle

668 *The* ELEMENTS Book IV.

cle *BD*. Yet this Elevation of the Terrestrial Fluid in *C* and *A*, and the Depression of it in the Circle *BD*, will be something sensible, upon account of the immense Forces of the immense Body of the Sun, tho' at a vast Distance.

PROPOSITION LXV.

THE Flowing and Ebbing of the Sea are owing to the oblong Spheroidical Figures of the Fluid that covers the Globe of the Earth, whose Axes, produc'd, go thro' the Sun and Moon.

¶ *J. Kepler* was the first (that I know of) who discover'd the true Cause of the Tide, and he explains it largely in his *Introduction to the Physics of the Heavens*, given in his *Commentaries on the Motion of the Planet Mars*, where after he has shewn the Gravity or Gravitation of all Bodies towards one another, he thus writes: *The Orb of the attracting Power, which is in the Moon, is extended as far as the Earth, and draws the Waters under the Torrid Zone, acting upon Places where it is vertical, insensibly on included Seas, or Bays, but sensibly on the Ocean, whose Beds are large, and the Waters have the Liberty of Reciprocation, that is, of Rising and Falling; and in the 70th Page of his Lunar Astronomy, — But the Cause of the Tides of the Sea appear to be the Bodies of the Sun and Moon drawing the Waters of the Sea. Afterwards that incomparable Philosopher, Sir Isaac Newton, improv'd the Hint, and wrote so amply upon this Subject, as to make the Theory of the Tides his own, by shewing that the Waters of the Sea rise under the Moon, and the Place opposite to it: For Kepler believ'd that the Impetus, occasion'd by the Presence of the Moon, does by the Absence of the Moon occasion another Impetus; till the Moon, returning,*

Book IV. of ASTRONOMY. 669

returning, stops, and moderates the Force of that Impetus, and carries it round with its Motion. Therefore this Spheroidical Figure, which stands out above the Sphere (like two Mountains, the one under the Moon, and the other in the Place opposite to it) together with the Moon (which it follows) being carried round by the Diurnal Motion, (or rather, according to the Truth of the Matter, as the Earth turns towards the East, it leaves those Eminences of Water, which being carried by their own Motion, slowly towards the East, are, as it were, at rest) in its Journey makes the Water swell twice, and sink twice, in the Space of 25 Hours, in which Time the Moon, being gone from the Meridian of any Place, returns to it again. But because (by the *Schol.* of the foregoing *Prop.*) the Terrestrial Fluid rises also in the Places where the Sun is in the Zenith or Nadir, tho' less than when the Moon is in those Places; in the Conjunction and Opposition of the Luminaries, the said Eminences of Water will also be join'd, and the greatest Elevation will happen, when both the Luminaries are (at the Conjunction) in or near the Zenith, or (at the Opposition) the one in the Zenith, and the other in the Nadir; also the greatest Depression will happen when the Luminaries are both in the Horizon, because the Water is then raised by both the Forces, and now depressed by them. In the Quadratures of the Luminaries the Sun raises the Water where the Moon depresses it; and from the Differences of the Forces arises the least Elevation of the Water, and also the least Depression. Between the Syzygies and Quadratures of the Luminaries, the Effects appear to be intermediate, as the Causes requiré they should.

If

670. *The* ELEMENTS Book IV.

If the Moon be in the *Æquinoctial* Circle, the two opposite Eminences of Water will be in the *Terrestrial Æquator*, and each of them, describing with its Motion that Circle of the Earth, will be more agitated, and therefore will be rais'd up higher against the Shore, when it reaches it; besides, that since the Earth's Diameter is greater than any other Right Line, drawn thro' the Center of the Earth, and terminated on either Side by its Surface, the Water will, by the Forces of the Luminaries, be rais'd (*cæteris paribus*) to a greater Height in that Circle, than any where else. And therefore, when the two Luminaries are in Conjunction or Opposition in the *Æquator*, they join their Forces to raise the Water in the *Terrestrial Æquator*, which happens in the Syzygies near the *Æquinox*; that is, such as happen on the Months of *March* and *September*; the Tides, then produced, will be the greatest and the most agitated, and the Tides, which in the said Months happen in the Quadratures, will be the least of all, and the least agitated, as being the Effects of the Difference of the Forces of the Moon, in its greatest Declination from the *Æquator*; (namely, when the Moon is in Quadrature to the Sun in the *Æquator*, and therefore in the Solstitial Points of the Zodiac) and therefore produces the least and least agitated Tide, and so, being the Occasion that the Sun, which has no Declination, produces its greatest and most agitated Tides; the Contrary of which happens about the Solstices, when the least Tides happen whilst the Luminaries are in the Syzygies, (I mean the least of those that do happen in the Syzygies) and the greatest (taken in the same Sense) in the Quadratures.

Again,

Book IV. of ASTRONOMY. 671

Again, the Tides (*cæteris paribus*) are greatest when the Luminaries are in their Perigæum, and least when in their Apogæum; and as that happens to the Moon at the Time of every Lunation, but to the Sun only in Winter; this, join'd with the former Causes, occasions the greatest Tides to happen in the Syzygies, and the least in the next Quadratures before the Vernal, and after the Autumnal Æquinox.

We have hitherto consider'd the Properties of the Tides, as they affect the whole Globe of the Earth: Now we must consider them according as they happen in Habitations, or Places of different Latitudes. In order to which, let *BEKAQH* [Fig. LVI.] represent the Earth, whose Center is *T*, the Poles *A* and *B*, the one the North Pole, and the other the South Pole, *E* & the Æquator, and the Circles *CD*, *FG*, two Parallels to it, the last being on the North Side, and the first on the South Side of it. Let the Fluid, circumfus'd about the Earth, form it self into an oblong Spheroid pointing to the Moon, whose Axis *KH*, being produc'd beyond *H*, will go thro' the Moon, and has been shewn in the foregoing *Prop.* the Right Line *TH*, or *TK*, will be the greatest Height of the Water (reckoning from the Center) and *TM* or *TN* the least, namely, the Height of the Water that is found in any Point of the Circle *NM*; as for *Example*, *X* or *T* where it meets with the Parallels *CD* and *FG*. The Right Lines *TC*, *TF*, *TD* and *TG*, will shew the Height of the Water in the respective Points *C*, *F*, *D*, and *G*; and the Circle *PO* being drawn parallel to the Circle *NM*, the Right Line *TO* will be equal to the Height of the Water in the Points *O*, *V*, *S*, *R* and *P*, where the said Circle *PO* meets with the Æquator and the respective

tive Parallels. Things being thus laid down, let us consider the Place, which, by its Diurnal Motion, describes the Parallel CD : It is plain, that when this Place is in D , the Height of the Sea TD is the greatest; that is, that it is high Tide or Flood, when the Moon L is in the Meridian; but afterwards that in the same Place the Height of the Water is the least, when the Place by its Revolution is come to X , as has been shewn before; and again, that it is Flood when the Place is come to C . But because TD is greater than TC (as being nearer to the longest Line TH) in the present Case, when the Moon declines to the elevated Pole, the Height of the Sea will be greater when the Moon is in that Part of the Meridian above the Horizon, than when it is in the Part of the Meridian which is below it. Likewise TG is less than TF , being nearer to TM ; that is, in the Place, which by its Diurnal Motion describes the Parallel FG , the greatest Height of Water, which happens when the Moon, declining towards the depress'd Pole B , is in that Part of the Meridian which is above the Horizon, is less than the greatest Height, which happens whilst the Moon is come to the other Half of the Meridian.

Moreover, the Difference of the said Tide, which is alternately higher or lower (in a Place situated out of the Æquator, as the Moon comes to the visible or hidden Part of the Meridian) will be greater, if it be join'd with some of the Causes before explain'd, which produce the same Kind of Effects, as for *Example*, at the Time of the Solstices; for then each Luminary very much declines from the Æquator, on which Declination it has been shewn that the Alteration of the Tides depend. And the Effects will yet be encreas'd, if the ascending Node of the Moon's

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Book IV. of ASTRONOMY. 673

Moon's Orbit be in the Vernal Æquinox; for then the Moon being in Conjunction with the Sun, besides a Declination equal to the Sun's greatest Declination, does then (being in the Limit) decline still more to the North or South by the Quantity of its greatest Latitude.

Things would be exactly thus, if the Globe of the Earth was cover'd with Water of a sufficient Depth; but by reason of great and small Islands which stop the Tide, and the Streights between them, also the Shelves and Shallows, along which the Tides are to be propagated, the Variety of this Phænomenon is almost infinite. But when you know the Situation of them in respect of each other, and just Observations of the Tide are exactly had, you may find out the Reasons of all the particular Phænomena of the Tide, because they naturally flow from what has been laid down. But as these are not to our Purpose, and even what we have hitherto shewn, are only as Lemma's for determining the Figure of the Moon, we shall not enter upon the Detail of such Particulars.

PROPOSITION LXVI. LEMMA.

IF the Moon's Globe was every where cover'd with a deep Sea like our Sea, it wou'd be indued with the Figure of an oblong Spheroid, whose Axis, produc'd with, wou'd pass thro' the Earth: And the Height of the Lunar Fluid, in the Parts of the Moon which are towards us, and those that are turn'd from us, above a Sphere inscrib'd in the Spheroid, wou'd be to the analogous Height of our Sea in the Parts which are under the Moon, and those which are diametrically opposite to them, above a Sphere inscrib'd in the Spheroid, in a Ratio compounded of the Ratio of the accelerating Gravity of the Moon towards the Earth, to the ac-

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674 *The* ELEMENTS Book IV.

celerating Gravity of the Earth towards the Moon, and the Ratio of the Moon's Diameter, to the Earth's Diameter.

That the Lunar Fluid wou'd rise up into a Spheroidical Figure, whose Axis wou'd pass thro' the Earth, is plain from the same Arguments, which we made use of in the 64th *Prop.* to prove that the Terrestrial Fluid wou'd put on such a Figure. For since it has been sufficiently shewn in the first Book, that all Bodies are heavy, we may very well ask with *Kepler*, in his *Lunar Astronomy*, *Why we may not say that the Earth draws the Lunar Waters, as well as the Moon draws the Terrestrial Waters?* Moreover, if the accelerating Gravity of the Earth, towards the Moon, was equal to the accelerating Gravity of the Moon towards the Earth, (that is, if there was the same Quantity of Matter in the Moon as there is in the Earth;) it is plain that the Fluids of the same Density, with which these two Globes are covered, will put on similar Spheroidical Figures. For there cannot be any Reason thought of, why the Fluid of one Globe shou'd rise into a more acute Spheroid, than that of the other: For, excepting the Mass alone, in the present Case, all Things are the same on both Sides. And when all Things necessary for producing every Kind of Equality are suppos'd, except the Mass, instead of all Kind of Equality, a Likeness or Similarity will be produc'd. Let the two Schemes [*Fig. LVIII.*] shew the similar Spheroidical Figures, in which the Moon (whose Center is *L*) and the Earth (whose Center is *T*) wou'd raise alternate Fluids of equal Masses or Density; and *LB*, *TA*, the Axes of those Figures, will lie in *directum*, by *Prop. 64.* If the Spheres *FD*, *EC* cutting the Axes in *C* and *D*,
be

Book IV. of ASTRONOMY. 675

be suppos'd to be inscrib'd in these Spheroids, by reason of the Similarity of the Spheroidical Figures in the present Case, BL will be to LF , as TA to TE , and by dividing, BD to LF , as AC to TE , and again, BD to AC , as LF to TE ; that is, if the accelerating Gravity of the Earth towards the Moon, and of the Moon towards the Earth was equal, the Elevation of the Lunar Fluid, in the nearest and the remotest Parts above the inscrib'd Sphere, wou'd be to the analogous Elevation of the Terrestrial Fluid in the Places under the Moon, and those that are opposite to it, as the Diameter of the Moon, to the Diameter of the Earth. Again, if the Earth and the Moon had equal Diameters, the Heights above the inscrib'd Spheres, to which Fluids of equal Density wou'd rise, wou'd be as the accelerating Gravities in the attracting Bodies themselves; that is, the Height of the Spheroid, to which the Lunar Fluid is rais'd above the Sphere, in the Parts nearest to, and farthest remov'd from the Earth, wou'd be to the like, or similar Height of the watery Spheroid above the inscrib'd Sphere, to which the Terrestrial Fluid is rais'd, as the Moon's accelerating Gravity towards the Earth, to the accelerating Gravity of the Earth towards the Moon. And therefore, if neither the accelerating Gravity of the Earth towards the Moon be equal to the accelerating Gravity of the Moon towards the Earth, nor the Diameter of the Moon equal to the Diameter of the Earth; yet the Height of the Lunar Fluid above the Sphere (inscrib'd in a Spheroid) on those Parts of the Moon which are turn'd towards us, and those which are turn'd from us, will be to the analogous Height, to which our Sea, in those Places that are under the Moon,

676 *The* ELEMENTS Book IV.

and such Places as are diametrically opposite to them, does rise above the Sphere inscrib'd in the Spheroid, in a Ratio compounded of the Ratio of the Moon's Diameter, to the Diameter of the Earth, and the Ratio of the accelerating Gravity of the Moon towards the Earth, to the accelerating Gravity of the Earth towards the Moon.
Q. E. D.

PROPOSITION LXVII.

TO define the oblong Spheroidical Figure of the Moon.

In former Propositions it has been shewn, that the Moon was once every where cover'd with a Fluid (which appears from its Spherical Figure) and that that Fluid has been of an oblong Spheroidical-Figure, whose Axis goes thro' the Earth. But because also the same Face of the Moon is always turn'd towards the Earth, (which happens upon account of the Moon's Period about its own Axis, being equal to the Moon's Period about the Earth, as was shewn in the 68th Prop.) the Situation of the Eminences of that Spheroid, beyond the Globe of the Moon, will always be the same on the Moon's Surface. Then as the Fluid, or soft Matter of the Moon, became hard by Degrees, the Eminences of this Spheroid on the Moon's Surface, being at rest, also became hard, and the Moon from soft, becoming hard, retains this oblong Spheroidical Figure; otherwise than it happens in the Earth, in which the Fluid Eminences of the Spheroid, above the inscrib'd Square, upon account of the swift Motion of the Earth about its own Axis, are in a perpetual Rotation about the Earth, and preserve their Fluidity.

To determine of what Kind this Spheroidical Figure is, we must observe that in Prop. preced.
it

Book IV. of ASTRONOMY. 677

has been demonstrated that the Excess of the Axis of an oblong Spheroid, above the Diameter perpendicular to it, is to the Excess of the greatest Height of our Sea, occasion'd by the Forces of the Moon alone above the least Height of it, in a Ratio compounded of the Ratio of the accelerating Gravity of the Moon towards the Earth, to the accelerating Gravity of the Earth towards the Moon, and the Ratio of the Moon's Diameter to the Diameter of the Earth: But each compounding Ratio is given, the first from the 62d *Prop.* and the last from *Prop* 59; therefore the compounded Ratio is given, *viz.* the Ratio of the Excess of the Axis of the Moon's Spheroid above the Diameter which is perpendicular to it, to the Excess of the greatest Height of our Sea, occasioned by the Force of the Moon alone above the least. But this Excess is given by Observation; therefore the other also, *viz.* the Excess of the Axis of the Lunar Spheroid above the Diameter perpendicular to it: But the Moon's Diameter is given, by *Prop.* 59; therefore also the required Axis of the oblong Spheroid. And therefore also the Moon's oblong Spheroidical Figure is defin'd. Q. E. F.

The Figure of the Moon is compounded of the prolate Spheroid, defin'd in *Prop.* 63. and of the oblong one determin'd in this 67th Proposition. But if its Surface was cover'd with a Fluid, there would besides be a third oblong Spheroidical Figure, made by the disturbing Force of the Sun, whose Axis produc'd would go thro' the Sun, and therefore whose Eminences, in one Rotation of the Moon about its own Axis, would go over the whole Compass of the Moon, like our Tide. The highest Elevation

678 *The* ELEMENTS Book IV.

of this Fluid, above the defin'd Figure of the Moon, wou'd be to the Elevation of our Sea, produc'd by the Sun's Force only, as the Diameter of the Moon to the Diameter of the Earth, and therefore known.

What has been said of the Earth, and of the Moon its Satellite, is also true of Jupiter and Saturn and their Satellites, if those great Bodies either now are, or have formerly been Fluid: For the mutual Gravity of Bodies, the Endeavour to recede from the Axis of Motion in Bodies; moving circular, and the Nature of a Fluid, are the same at Jupiter and Saturn, as at the Earth.

SECTION XIII.

Of Saturn's Ring and its Phases.

HAVING shewn and demonstrated the Geometrical Method, by which the Reasons of what has been observ'd of the Secondary Planets and their Phases are explain'd, and their Quantities are determin'd; the same must be done in respect of the Ring or Arch which encompasses Saturn, because it may be look'd upon as a Satellite, and therefore ought here to be spoken of, after our having explained other things concerning Satellites.

PROPOSITION LXVIII.

TO define the Magnitude, Figure, and Situation of the Ring which encompasses Saturn.

Saturn's Ring is thin and plane; and does not where cohere to his Globe; [*Fig. LIX.*] because

Book IV. of ASTRONOMY. 679

cause we can see the Fix'd Stars thro' the Space GCB , between the Ring and Saturn.

The Semidiameter of the Ring is to the Semidiameter of Saturn, as Two and a Half to One, and the Interstice between the Globe of Saturn, and CGH , the inner Limb of the Ring, is equal to CD , the Breadth of the Ring. Its two Surfaces, viz. $GDFHC$ seen here, and the other hidden one parallel to it, are Planes, but yet not polish'd like a Speculum, but reflecting the Light of the Sun that falls upon them every way. The Thickness of the Ring, or the Distance between the two said Planes, is sensible, tho' not very great: It either does not at all, or does but very little reflect the Sun's Rays falling upon it; or perhaps, like a Speculum, reflects the Light diverging from one Point, and not coming at the Earth.

This Ring, (which, by reason of its Gravity towards the Center of Saturn, is like an Arch every where sustain'd at an equal Distance from Saturn) if it has any Revolution, has it about its own Axis, or a Right Line drawn from the Center, perpendicular to its parallel Surfaces; and therefore, by reason of its Uniformity, appears, to a distant Observer, to be at rest. It is very probable that Saturn itself revolves about the said Axis upon the Poles P, p ; so that the Ring, in the produc'd Plane of Saturn's *Æquator*, encompasses this Planet at a Distance from its Surface, something greater than the fourth Part of its Diameter.

All these things appear to an Eye assisted with a good Telescope, and are measur'd by the Help of a Micrometer; that is, are compar'd with the Globe of Saturn.

680 *The ELEMENTS* Book IV.

Saturn, encompass'd with his Ring, is carried about the Sun in its Orbit, after the same Manner as the Earth in its Orbit, which we have describ'd in the 30th *Prop.* of *Book I.* viz. the Ring, and so perhaps the Æquator of Saturn, are carried in the Plane of the Orbit, the Axis always being parallel to itself; and the Plane of the Ring is always inclin'd in the same Angle, namely, of 23° Degrees, to the fix'd Plane of the Ecliptic, and therefore always in the same Angle to the Orbit of Saturn.

About the Year of our Lord 1659, the common Interfection of the Ecliptic, and of the Plane of Saturn's Ring, was observ'd by *Christian Hugen*s, to be in the 20½ Degree of Virgo, and the 20½ Degree of Pisces: For at that Time, when Saturn was in those Points, it appear'd without Handles or its Ring; in the 20½ Degree of Gemini and Sagittarius, namely, the most remov'd Points from the former, the Handles appear'd the most open, and drawn asunder, because the Circle of the Ring, being then seen with very little Obliquity, puts on the Figure of a broad Ellipsis. But because the said Points at 20½ Degrees of ♍, and of ♋, are not very far distant from the Points where the common Interfection of the Terrestrial Æquator, with the said Plane of the Ecliptic, going thro' the Sun, meets with the Ecliptic, and these two Planes are inclin'd in equal Angles to the Plane of the Ecliptic; these two Planes, namely, that of the Æquator, and that of Saturn's Ring, will be nearly parallel to each other. And therefore if this Ring be obliquely look'd upon from the Earth, and for this Reason appears Elliptical, the greater Axis of this Ellipse will be nearly parallel to the Celestial Æquator.

It

Book IV. of ASTRONOMY. 681

It is likely that the said Right Line (which is the common Interfection of the Plane of the Ring of Saturn, and the Plane of the Ecliptic) is carried *in antecedentia*, by an angular Motion. If this Ring be connected magnetically with Saturn's Æquator, or by any other imperceptible Manner (for it is plain from Observation that it touches it no where,) so that the Planes of them may always coincide in one; Saturn's Æquinoctial Point will go backward: For this Ring is so much redundant Matter about Saturn's Æquator; whence (by *Prop. 64. B. I.*) we find the Motion of those Points *in antecedentia*, after the same Manner as that of the Terrestrial Æquinoctial Points. If the Ring, like a Secondary Planet, performs its Motions, not depending upon Saturn's Globe, the Regress of the foresaid Line will be plain, from *Prop. 14. Book IV.*

Moreover, the Angle, which the Plane of the Ring contains with the Plane of Saturn's Orbit, will be greater, by *Prop. 15.* when Saturn is in their common Interfection, than when it is any where else; and least of all, when Saturn is most distant from it: But these Changes are so small as to be scarce observable from the Earth.

PROPOSITION LXIX.

TO describe and define the Phases of Saturn's Ring, seen from the Sun. [Fig. LX.]

We begin from that Phænomena happening when the Spectator's Eye coincides, or is at the illuminating Body, as being the most simple and leading to the others. First, it is plain, that when Saturn is seen from the Sun, as at *A* or *I*, (in the Figure where we have represented Saturn,

turn, together with its Ring carried parallel to it self, with the Figure of the Disc's Appearance drawn respectively behind each Representation of Saturn) we can see neither Face of the Ring, and that upon a double Account: For first, since in that Case the Plane of the Ring produc'd goes thro' the Sun, neither Face of it will be enlighten'd, the Sun's Rays only sliding by on each Side of it, therefore it can't be seen, as being obscure; then, since the Eye is also plac'd in the Sun, that is, in the Planes to be seen produc'd, those Surfaces, tho' enlighten'd, cou'd not be seen. If the Edge of the Ring shou'd, like the Planets, every way reflect the Sun's Rays falling upon it, two *bright Arms* wou'd stand out in a Right Line inclin'd to the Ecliptic, in an Angle of $23\frac{1}{2}$ Degrees; but since this Edge does not reflect the Rays (as has been shewn in the foregoing Proposition) the Arms will not appear extended beyond Saturn's Disc; but because the Ring crosses Saturn's enlighten'd Disc, it will look like a dark Line inclin'd in the Manner before-mention'd, drawn thro' the Center of the Disc: But this dark Line also will appear in other Positions, sometimes above the Center of the Disc, and sometimes bent below it, as the Situation of the dark Edge requires.

As Saturn goes from *A*, thro' *B*, *C*, &c. towards *E*, or from *I*, thro' *K*, *L*, &c. towards *N*, the Sun is by degrees rais'd above the Face of the Ring, either the one or the other, till Saturn being come to *E* or *N*, a Plane drawn thro' the Sun and Saturn, perpendicular to the Plane of the Ecliptic, is also perpendicular to the Plane of the Ring, and the Sun is rais'd about $23\frac{1}{2}$ Degrees above the Plane of the Ring,

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in which Case that Face of the Ring, which the Sun enlightens, is most illuminated of any, by reason of the Incidence of the Sun's Rays, which are then very direct: And the circular Limb of the Ring, the Eye being also plac'd in the Sun, will, from the Laws of Opticks, appear an Ellipse, whose greater Axis is three times the Length of the lesser, as the Radius is almost three times the Sine of $23\frac{1}{2}$ Degrees. And then as Saturn goes from *E*, thro' *F*, *G*, &c. towards *I*, or from *N*, thro' *O*, *P*, &c. towards *A*, the Sun's Elevation, above the enlighten'd Face, is by degrees diminish'd, till at last it enlightens neither Face, but throws its Rays along or parallel to both of them. In any intermediate Situation the enlighten'd Face will be more or less so, according to the greater or less Distance of Saturn from the Situation *A* or *I*, and the circular Ring will appear under the Shape of a more contracted or more dilated Ellipse; namely, its greater Axis will be every where to its lesser, as the Radius to the Sine of the Angle, in which a Right Line, drawn from the Eye to its Center, is inclin'd to the Plane of the Ring. The foresaid Angle in which the Sun (or an Eye in the Sun) is rais'd above the Plane of the Ring, in any Situation of Saturn (namely *C*) is equal to the Angle in which the Sun is rais'd above the Plane of the Terrestrial *Æ*quator, (that is, to the Sun's Declination from the *Æ*quator) when the Earth is distant from the Point where it is *Æ*quinox to the Inhabitants of the Earth, by an Angle equal to *ASC*, and therefore it may easily be found by the Table of Declination of the Points of the Ecliptic: For (by the foregoing *Prop.*) the greatest Elevation of the Sun, above the Plane of the Terrestrial

stial *Æquator*, is equal to its greatest Elevation above the Plane of Saturn's Ring.

When Saturn sticks near the Point *A*, the Ring will least of all be perceiv'd, not only because the lesser Axis of the Ellipse is very little in such a Case, but chiefly because it will be but very little enlighten'd, by reason of the Obliquity with which the Sun's Rays fall upon it, and therefore will be scarce perceivable to the Eye. But it is plain by Experience, that to have the Face of the Ring enlighten'd, so that it may be visible, the Sun must be at least three Degrees above its Plane; in order to which the Heliocentric Place of Saturn must be seven or eight Degrees distant from *A* or *I*, and therefore all the Time that Saturn, seen from the Sun, is on this Side or beyond the Point *A* or *I*, (already often described) it sticks between seven or eight Degrees, that is, when, in the opposite Part of its Orbit, it runs thro' those fifteen Degrees, which lasts about as many Months, its Ring and Arms or Handles, which it makes, will not appear; but Saturn will appear round and alone without any Ring. But these Limits will be different according to the different Goodness of the Observer's Eye, and Length and Goodness of his Telescope. These are the chief Phenomena of the Ring that encompasses Saturn, seen from the Sun; and this is the Manner of determining them.

SCHOLIUM.

For the same Reason, the Ways of the Satellites revolving about any Primary Planet, seen from the Sun, appear sometimes streight, sometimes elliptical: Streight, when the Line of the Nodes of the Satellites produc'd goes thro' the Sun; but when the Satellite, seen from the Sun,

Book IV. of ASTRONOMY. 685

is very much distant from its Primary Planet, it most of all declines from the Way of the Primary Planet seen from the Sun, in which Case the Eclipse of that Satellite, running into the Shadow of the Primary Planet, will be central, and therefore (*cæteris paribus*) very lasting, and the Shadow of the Satellite, running thro' the lower Part of its Orbit, falling upon the Primary Planet, runs over the Center of the Disc seen from the Sun. Whence the Phænomena of the Sun, and of a Satellite seen from its Primary Planet, will be defin'd according to the foregoing Rule. Out of this Situation, the Way of a Satellite, seen from the Sun, will appear to be an Ellipse, (that is, a Circle seen obliquely) whose greater Axis is not yet in the apparent Way of the Primary Planet. This Ellipse will have its shorter Axis, the less by how much nearer the Primary Planet is to those Points where the former Phasis happens, and the greater by how much it is more remov'd from them; and therefore the longest of all (that is, this Ellipse, which the Circular Orbit of the Satellite appears to be, will be broadest) when the Primary Planet is in the Places intermediate between those mentioned; namely, when a Right Line, joining the Sun and a Primary Planet, is very little oblique to the Plane of the Orbit of a Satellite; in which Case the Satellite, in its greatest Elongation from its Primary Planet, will appear to be in the Way of the Primary Planet seen from the Sun; but in its Conjunction with the Primary Planet, it declines very much from the said Way, on the one or the other Side, as it is in the inferior or superior Semicircle of the Orbit. Whence it happens, that a Satellite, in the upper Part of its Orbit, will be very far distant

686 *The* ELEMENTS Book IV.

stant from the Axis of the Shadow of the Primary Planet, and in the lower Part of it, the Shadow of the Satellite will also be very distant from the Center of the Disc of its Primary Planet seen from the Sun; therefore, according to the Inclination of the Plane of the Orbit of the Primary Planet, and according to the Magnitude of the Satellite's Orbit, the Eclipses arising hence will be (*cæteris paribus*) either very short, or perhaps none at all. In the intermediate Place, the Satellite's Way about the Primary Planet, seen from the Sun, will be an Eclipse of less Breadth, and all the foresaid Things will be mix'd. And all these Phases will happen twice in one Revolution of the Primary Planet about the Sun, if the Line of the Nodes always remain parallel to itself; and if it moves, all Variety of Phases will happen, from the Time that that Line of Nodes produc'd falls into the Sun, till it be again directed towards it. And conversly, from these Observations of these longer or shorter Eclipses, the Position of the Line of the Nodes of a Satellite, and the Inclination of the Plane of the Orbit of a Satellite, to the Plane of the Orbit of a Primary Planet, will be defin'd.

PROPOSITION LXX.

TO describe and define the Phases of Saturn's Ring, seen from the Earth.

Tho' the Phases of Saturn's Ring, seen from the Sun, are nearly the same with the Phases of it seen from the Earth, (because Saturn's Geocentric Place, at most, scarce differs six Degrees from its Heliocentrick Place) and therefore are to be defin'd by the foregoing *Prop.* Yet since they differ in some Things, especially when
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Book IV. of ASTRONOMY. 687

Saturn appears round, we shall in a few Words shew wherein they differ.

Let S , [Fig. LXI.] be the Sun, $ABIN$ the Orbit of Saturn, DCT the annual Orbit of the Earth; (for we don't here consider its Inclination to the Orbit of Saturn as being very small) and let A and I be the Places of Saturn's Orbit, in which Saturn is, when the Plane of its Ring, produc'd, goes thro' the Sun, and it is $\text{\AA}quinox$ in Saturn, if the Ring be in the Plane of the $\text{\AA}quator$, and Saturn revolves about its own Axis. It is plain that when Saturn is in A or I the Ring will not be seen by an Eye in the Earth, whatever Part of its Orbit the Earth is in: For since in that Case neither Face of the Ring is enlighten'd by the Sun, neither Face will be seen by the Eye, tho' rais'd high enough above its Plane. The Edge only of the Ring is enlighten'd, which yet is not seen, because it does not reflect the Rays, at least so as they may reach the Earth: Yet since it crosses the bright Disc of Saturn, it appears obscure, or like a dark Line, for the Reasons explain'd in the foregoing Propositions.

Moreover, if when Saturn is got to B , the Earth be in such a Point C of its Orbit, that BC be parallel to AI , Saturn will also appear round: For tho' one Face of the Ring be in that Case enlighten'd by the Sun, an Eye on the Earth is also found in the Plane of the Ring produc'd, and therefore it does not see it, but has only the dark Edge turn'd towards it: This happens when Saturn, seen from the Earth, has behind it the same Fix'd Stars, which it had when the Eye was suppos'd in the Sun, and the Sun in the Plane of the Ring produc'd; that is, when the Geocentric Plane of Saturn is the same with
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the Heliocentric Place, (which happens when it is in the Plane of the Ring produc'd) as may easily be known from Astronomical Tables.

Again, let Saturn be supposed in the Situation H , and the Earth in E , so that the Plane of the Ring, or rather its common Section with the Plane of Saturn's Orbit, which is the Right Line HM , parallel to IA , being produc'd, falls between the Sun S , and the Earth E , which will happen when Saturn's Geocentric Place, and its Heliocentric Place, are at different Parts of the Place, describ'd above in *Prop.* 68, where the Plane of the Ring, produc'd, falls on the Sun or Earth, which is readily known by Astronomical Tables, especially Ephemeris's; for the Right Line SH , shewing the Heliocentric Place of Saturn, falls on one Side of the Right Line HM , and EH , shewing its Geocentric Place, falls on the other Side of the same Right Line: I say, that in such a Case an Observer on the Earth will not see the Ring, (only a dark Line will appear to cross Saturn's Disc) because he can't see that Face of the Ring which is expos'd to the Sun's Rays, as being turn'd from him. For in that Case, that Face, which is as a Shadow, is turn'd towards the Earth.

On the Contrary, when the Right Lines TN , SN , drawn to Saturn from the Sun and the Earth, fall towards the same Parts of the said Line NR , (the common Section of the Planes of the Ring, and of the Orb of Saturn) as it happens when Saturn is in N ; that is, when Saturn's Geocentric Place, and its Heliocentric Place are on the same Side, in respect of the Place above describ'd, where the Plane of the Ring, produc'd, falls into the Sun; then the same Face of the Ring is turn'd to the Sun and to the Earth,

Book IV. of ASTRONOMY. 689

Earth, and therefore (if the Reasons given in the foregoing *Prop.* don't interfere) the enlightened Face seems to give Saturn Arms or Handles; *viz.* if the Sun does enough enlighten the Ring, that its Light may affect the Eye; that is, if it be rais'd three or more Degrees above the Plane of the Ring, or if Saturn be distant from the Point *A* or *I*, eight Degrees or more, and the Earth also be rais'd one Degree, or more, above the Face of it, (for some sensible Elevation is necessary for the Eye to have above the Plane of the Ring, lest the Eclipse, which the Ring represents, shou'd appear like a very small Line, and so not reflect Light enough to affect the Eye;) that is, if the Geocentric Place of Saturn be about three Degrees, or more, from the Place where the Plane of its Ring, produc'd, falls upon the Earth, the Ring will be visible, as has been shewn in the foregoing *Prop.*

To define what Kind of Ellipse the Ring, at any Time, represents; we must observe, that when its Plane is inclin'd to the Plane of the Orbit of the Earth, about 23 Degrees, where the Circle of the Ring appears the most direct of all, a Line, joining the Earth and the said Circle will be inclin'd to the Plane of the Circle, in the said Angle; and therefore in that Case the Circle will appear to be an Ellipse, whose greater Axis is three times longer than the lesser Axis. Moreover, in any other Inclination of a Right Line joining the Earth and Saturn to the Plane of the Circle, (we must find the Distance of the Geocentric Place of Saturn from that Geocentric Place of it, where the Plane of the Ring produc'd goes thro' the Earth, by Help of the Table of Declination

clination of the Points of the Ecliptic, as above) its greater Axis will be to its lesser, as the Radius to the Right Sine of the Angle lately found, in which the Eye is rais'd above the Plane of the Ring. And these are the chief Phases of the Ring seen from the Earth, and the Manner of defining them. Q. E. F.

The Ellipse also, which represents the inner Circle of the Ring is of the same Kind, as being in the same Plane with the outer one: But this Ellipse is not the same with the perforated Figure, upon Account of the Thickness of the Ring. The hithermost Half of this Figure is the same with Part of the foresaid Ellipse; but the furthestmost Part of it is equal and similar to a Part of the Ellipse, brought a little nearer to the Axis; as is plain from *Fig. 59.*

SCHOLIUM.

After the same Manner, from the Way or Track of a Satellite about its Primary Planet, seen from the Sun, and determin'd by the foregoing Proposition, may be determin'd the Figure of it, seen from the Earth at any Time propos'd. For it is plain, that a Satellite, to an Inhabitant of the Earth, will appear to have describ'd a Right Line, when the Earth is found in the Plane of the Orbit of a Satellite, tho' the Sun at that Time is not in it; and that the Ellipse (which, in all other Cases, it represents) comes nearest to a Circle, when a Right Line, joining the Earth and a Primary Planet, is the least oblique to the Plane of the Orbit of a Satellite, tho' at that Time a Right Line, drawn from the Sun to the Primary Planet, is not the least oblique that it can be to the said Plane. And, on the other hand, from the Observation of the
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Book IV. of ASTRONOMY. 691

said Phases, may be defin'd the Situation of the Line of Nodes, and the Inclination of the Plane of the Orbit of a Satellite, to the Plane of the Orbit, which its Primary Planet describes about the Sun.

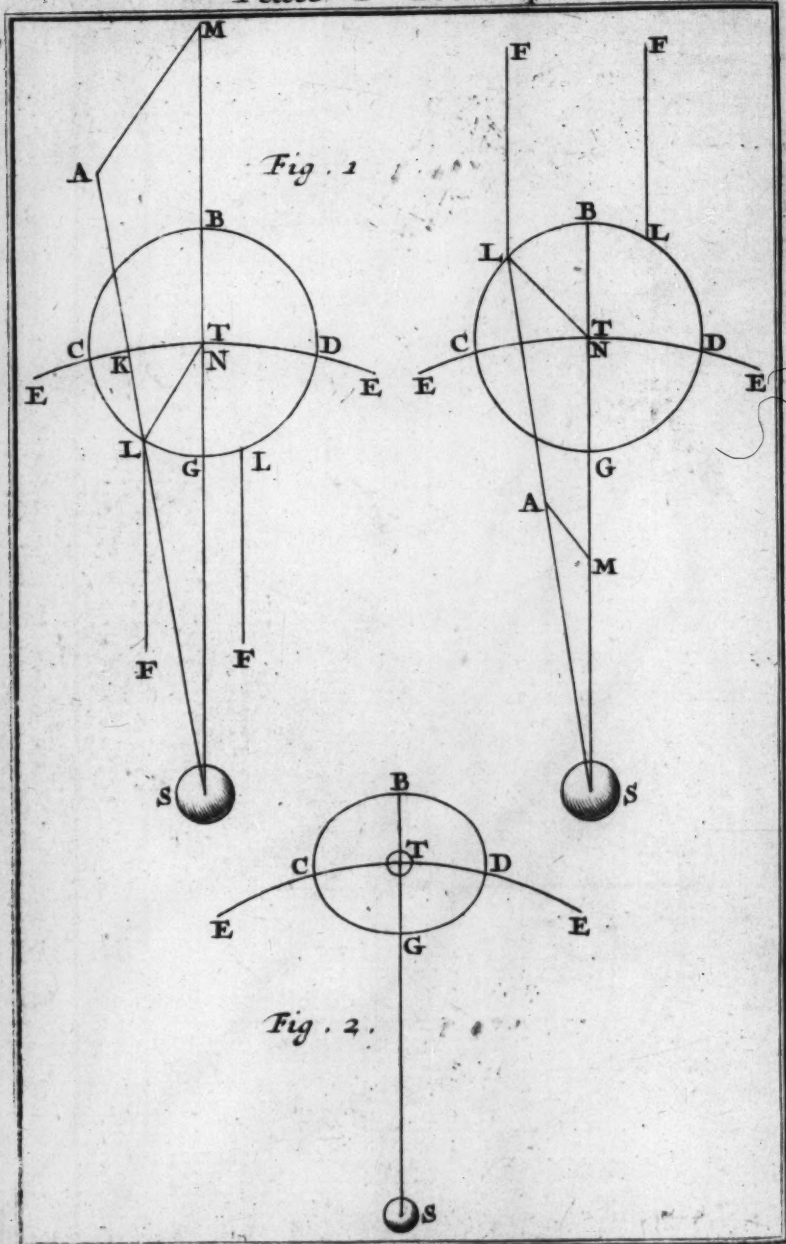
Moreover, the same Things are true of the apparent Way of a Spot, or any other visible Mark in the Surface of a Primary Planet, or of the Sun itself, in its Rotation about its Axis, or of a Circular Order or Series of such Marks: For, such a Circle, by the Laws of Opticks, will appear sometimes a Circle, sometimes an Ellipse, as the Earth is in its Plane produc'd, or out of it.

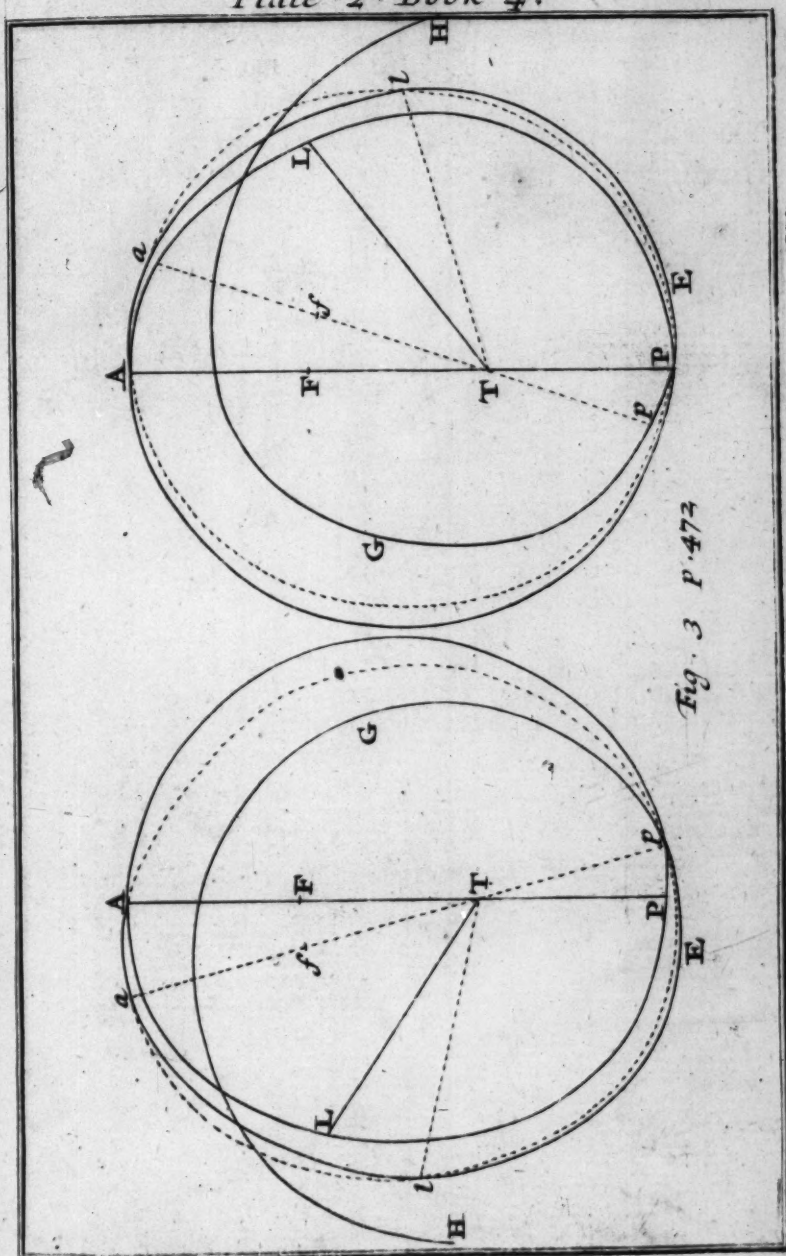


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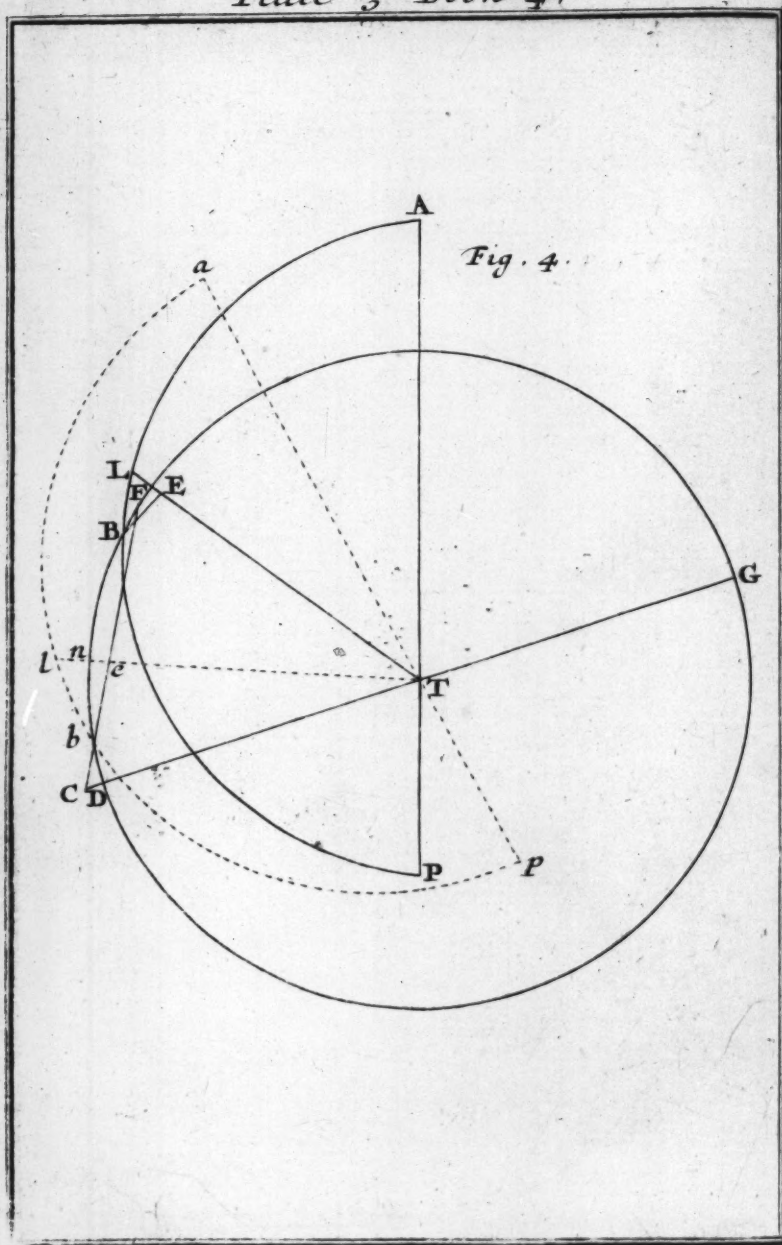


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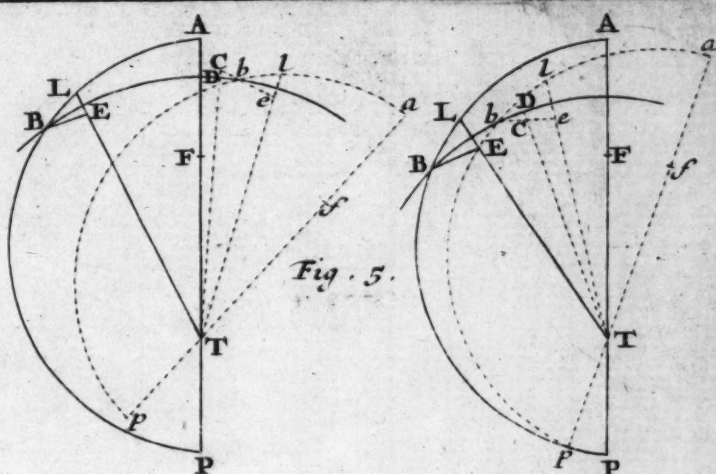


Fig. 5.

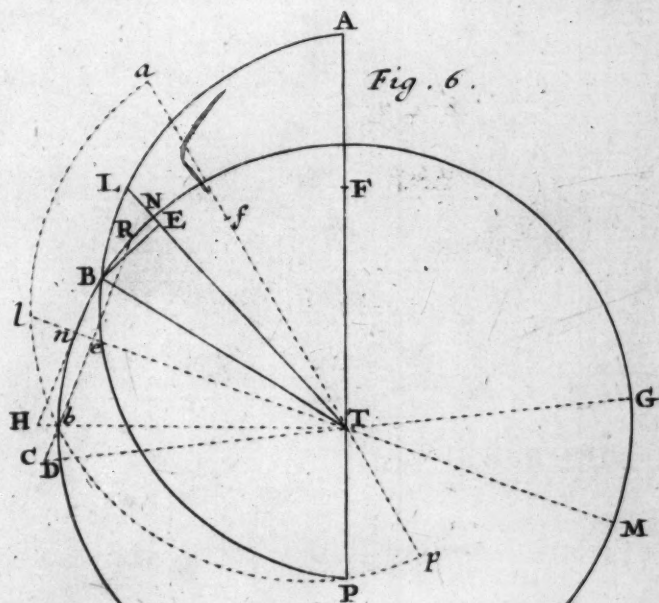


Fig. 6.

Fig. 7.
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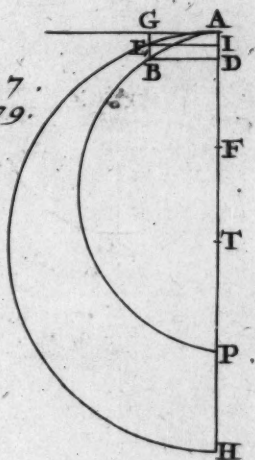


Fig. 8.
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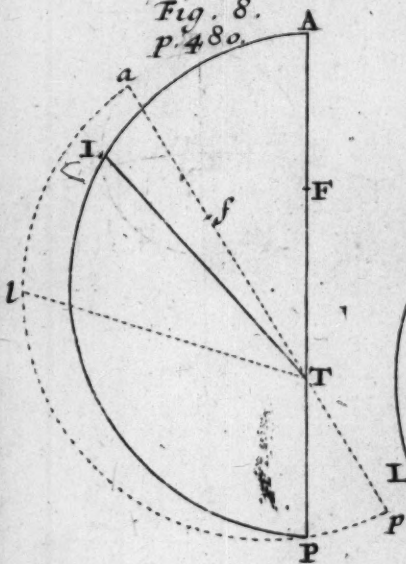


Fig. 9.
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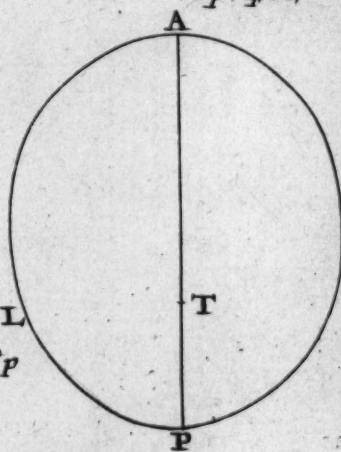


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Fig. 10.
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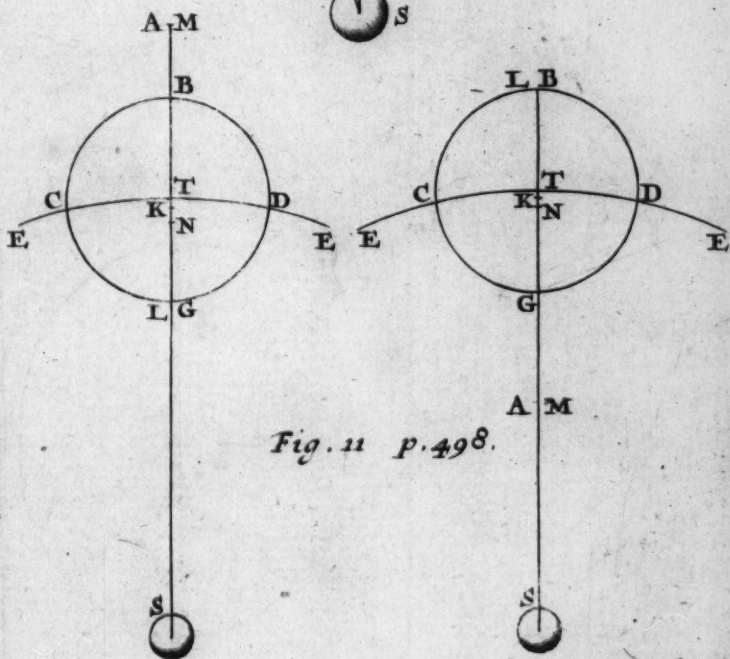
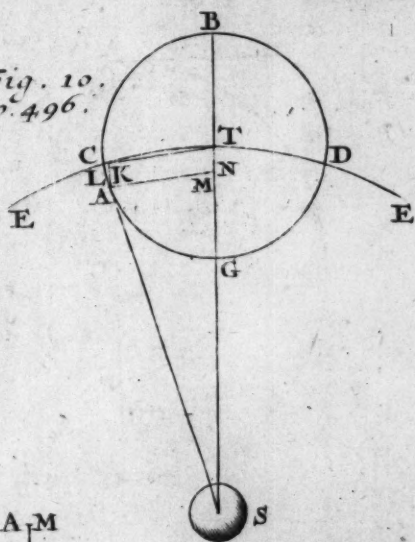


Fig. 11 p. 498.

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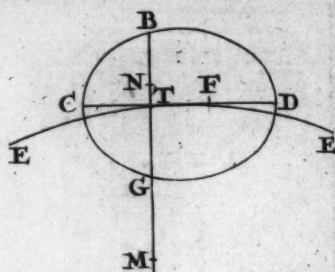
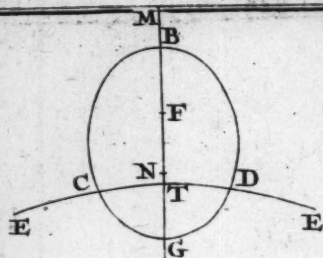


Fig. 12 p. 500.

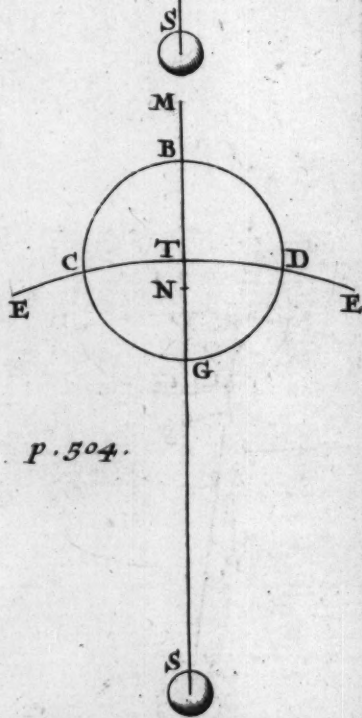
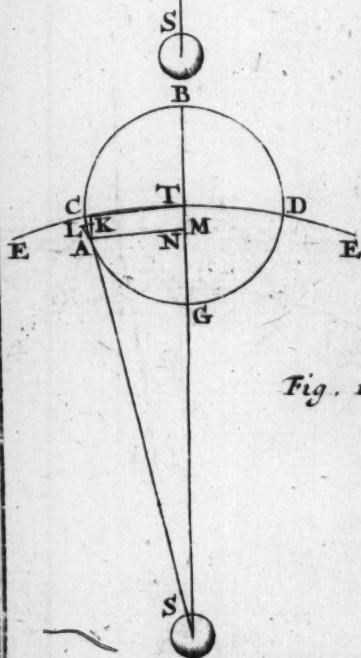


Fig. 13 p. 504.

Plate 8 Book 4.

Fig. 14.
P. 510.

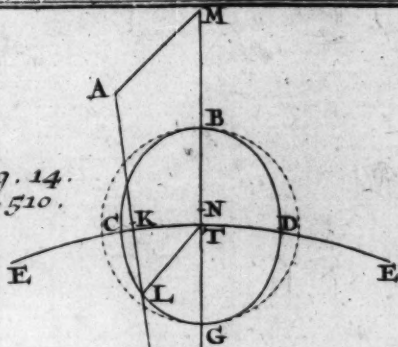
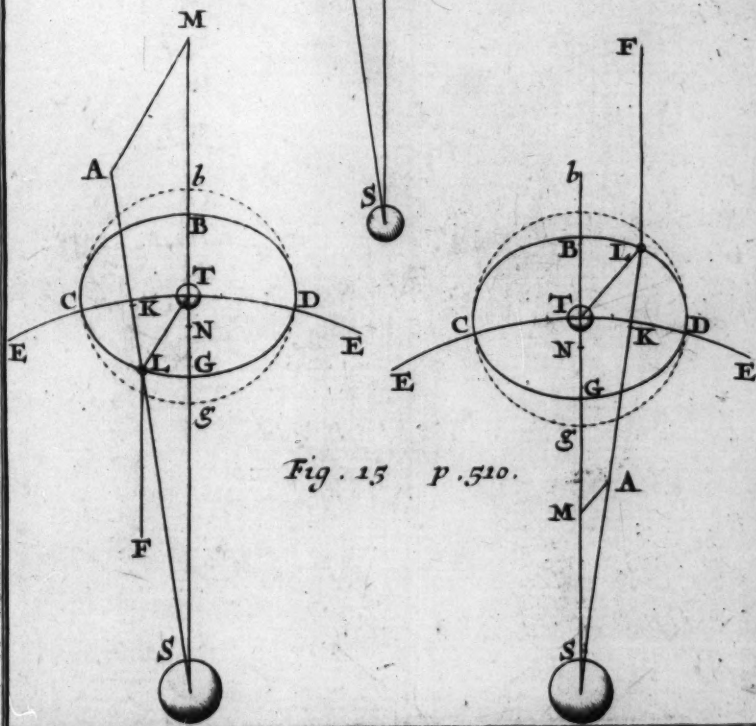


Fig. 15 P. 510.



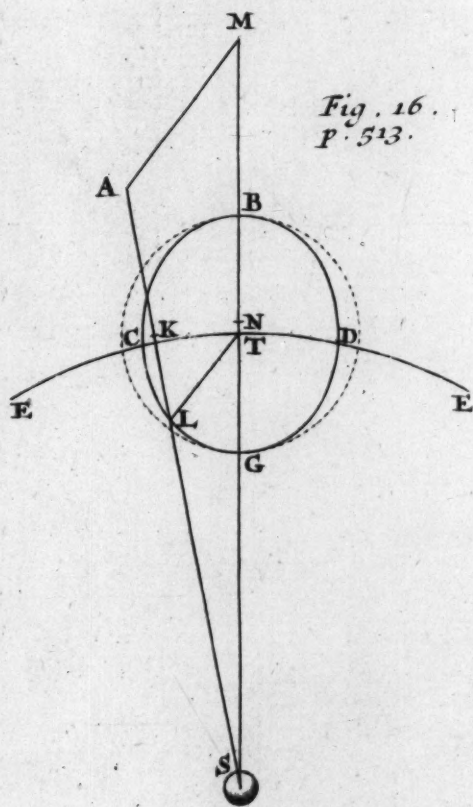


Plate 10 Book 4.

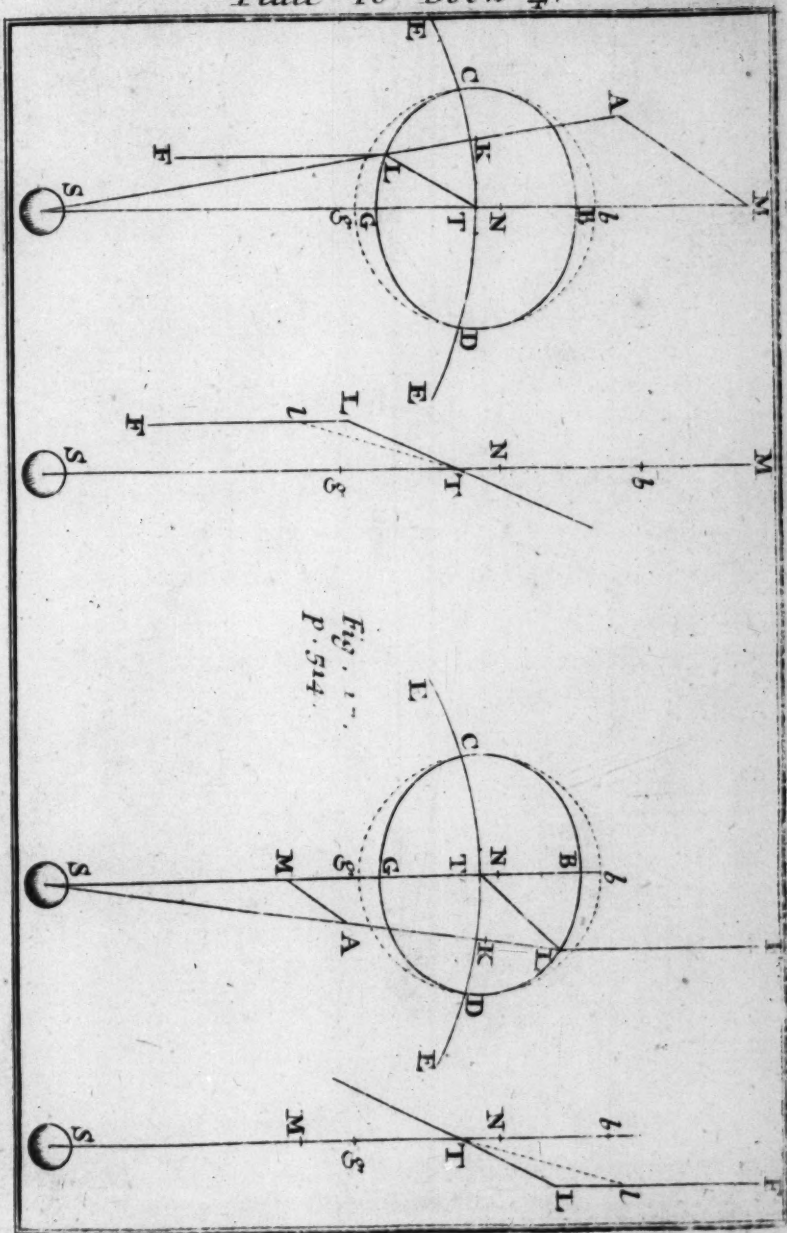


Plate 11 Book 4.

Fig. 18.
P. 515.

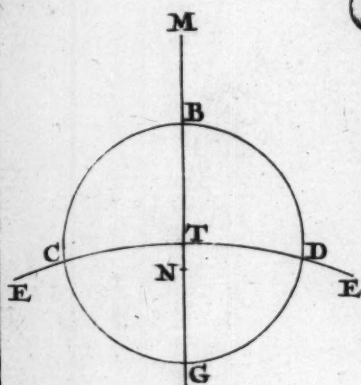


Fig. 19.
P. 518.

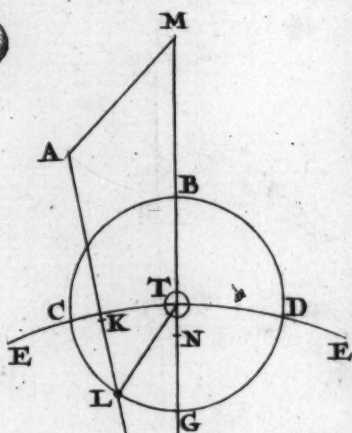


Fig. 20.
P. 521.



Plate 12 Book 4.

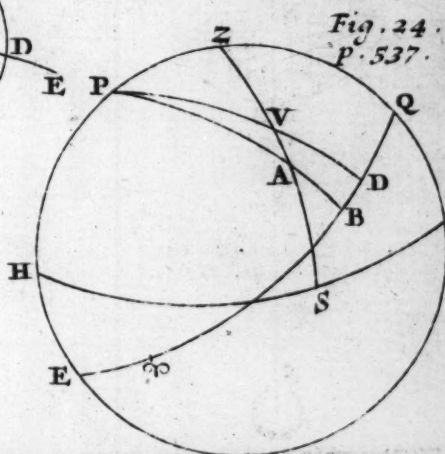
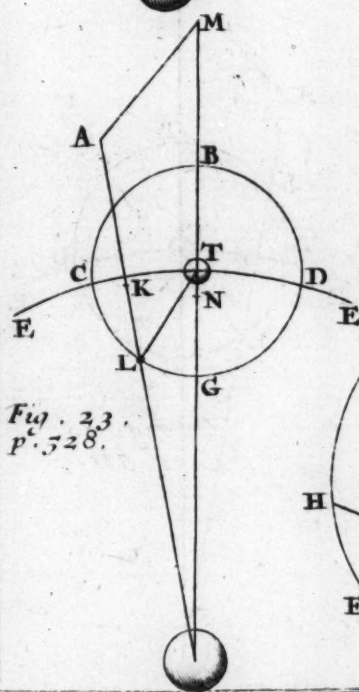
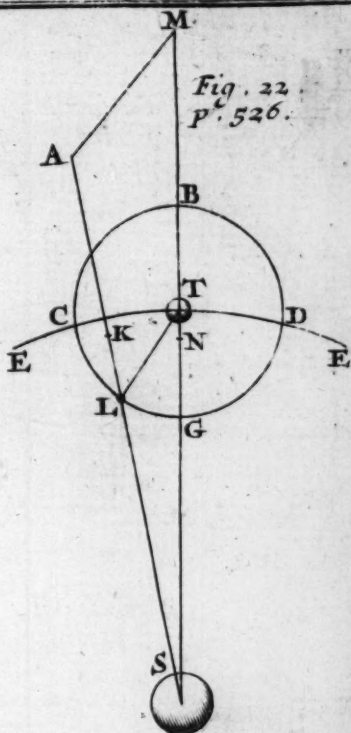
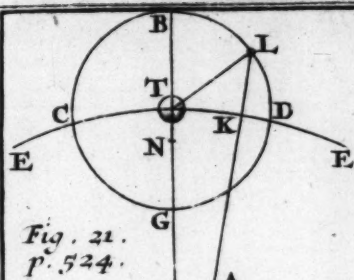


Plate 13 Book 4.

Fig. 25.
P. 538.

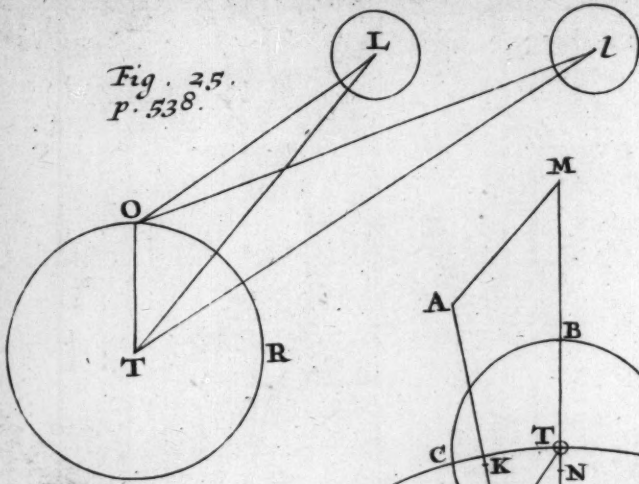


Fig. 26.
P. 540.

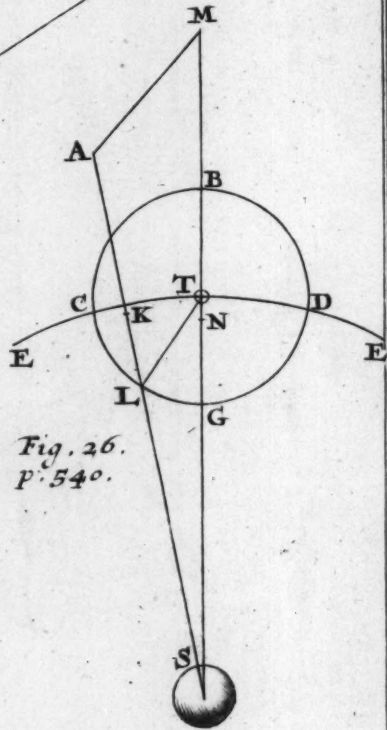


Fig. 27.
P. 566.

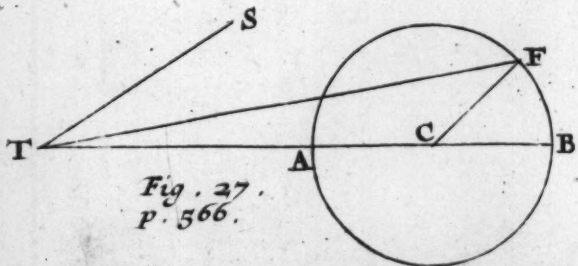
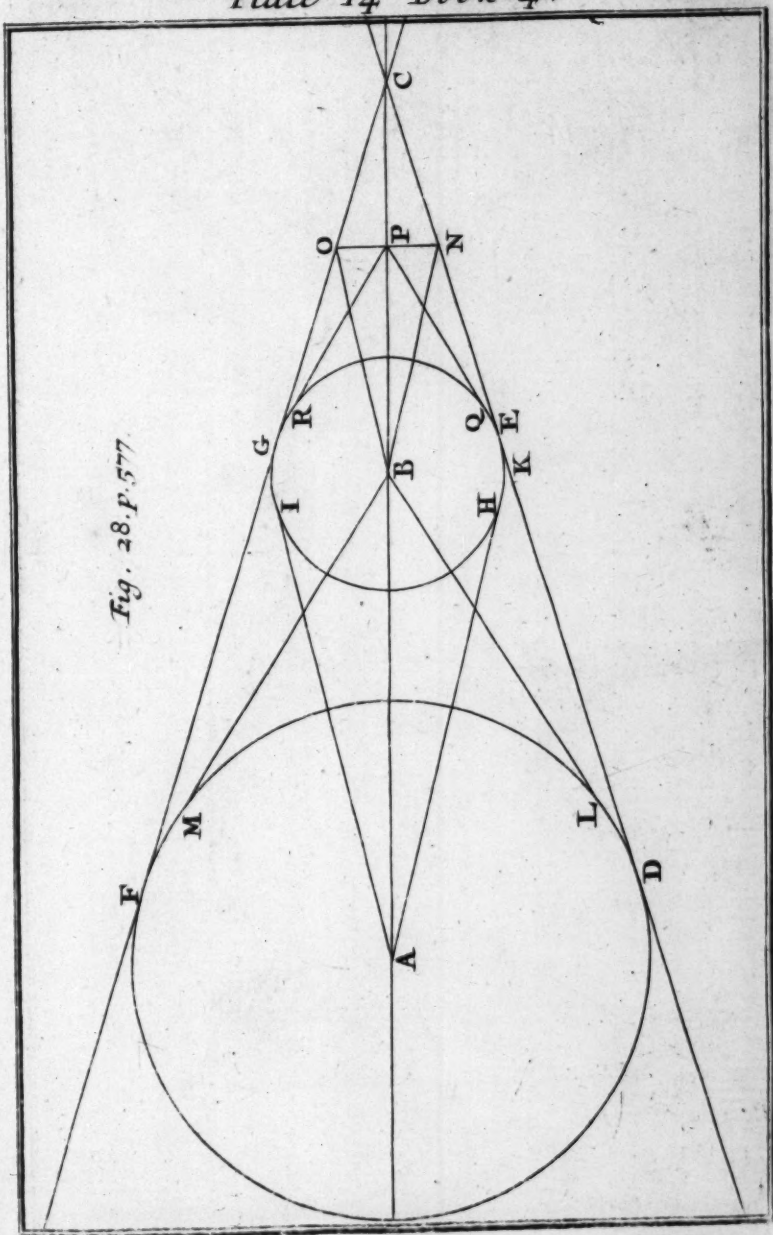


Fig. 28. p. 577.



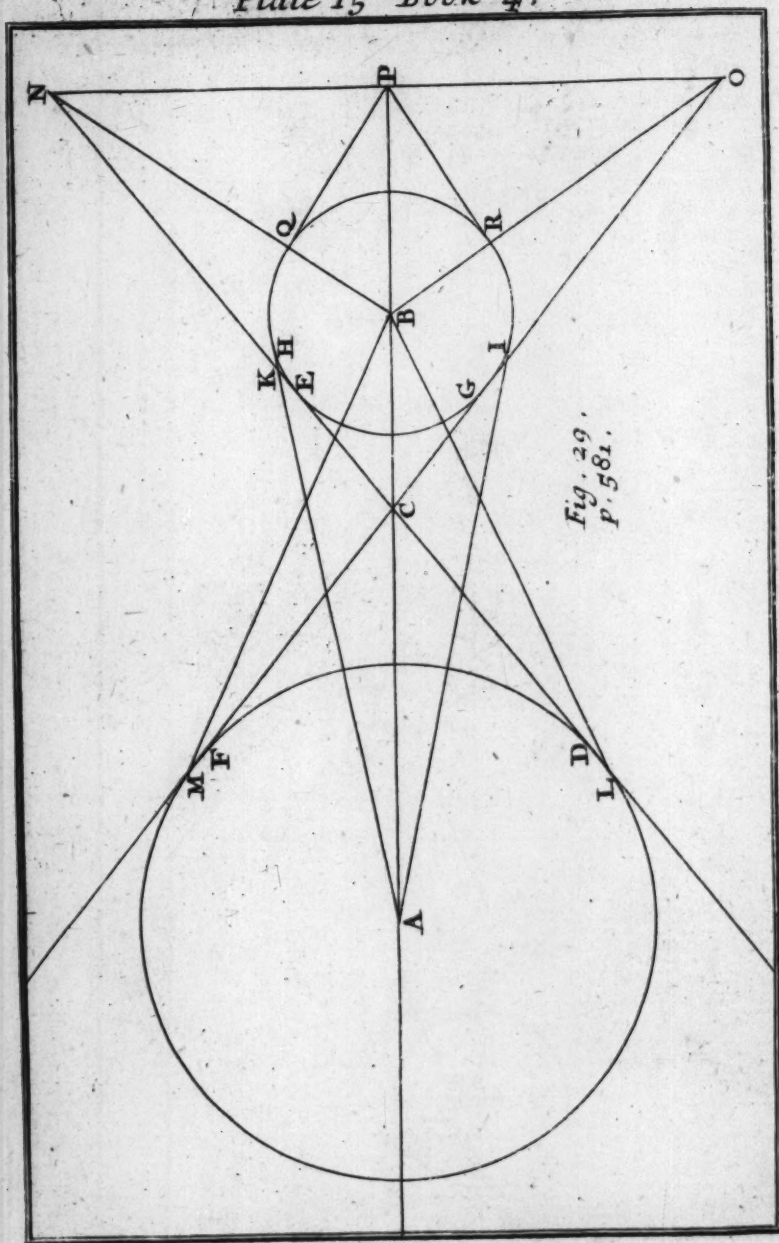


Fig. 29.
P. 581.

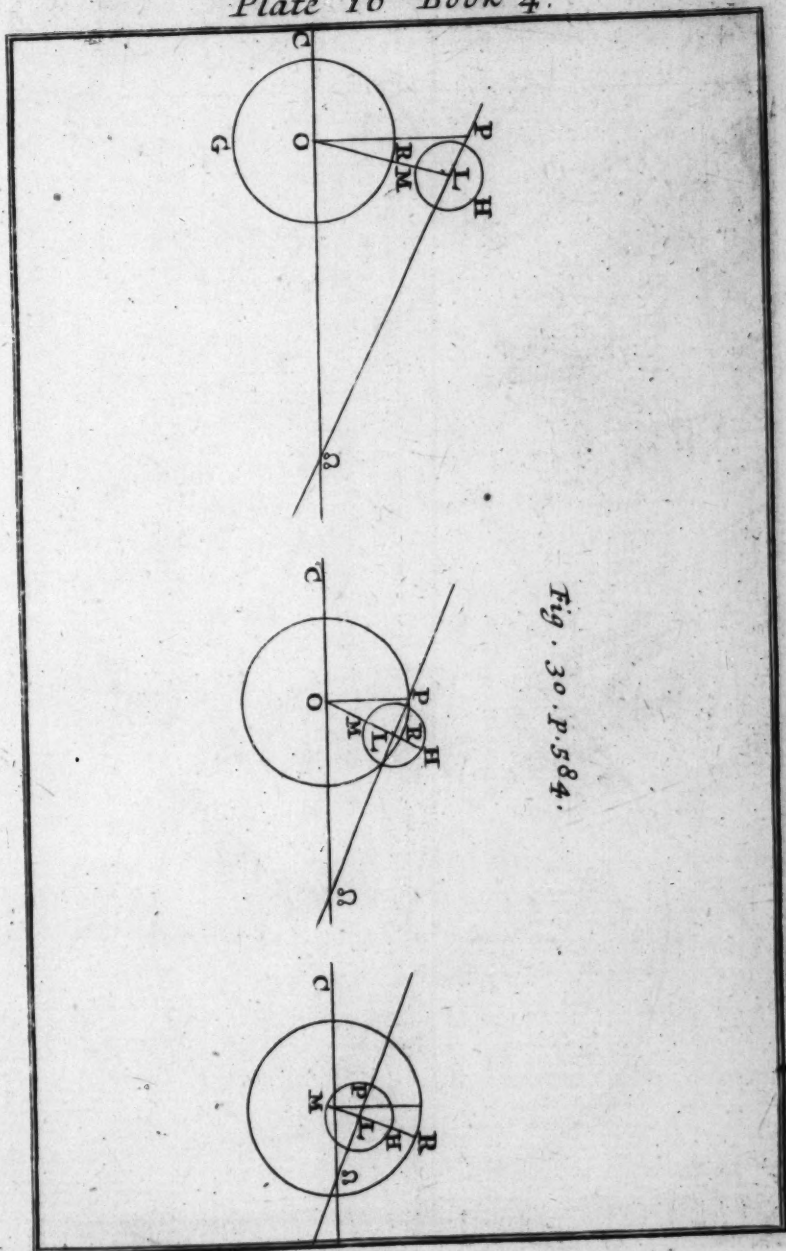
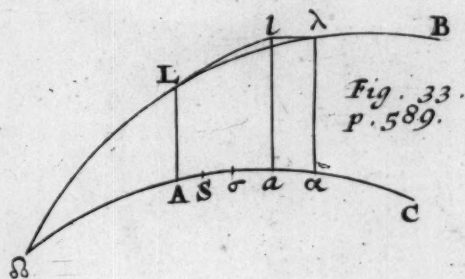
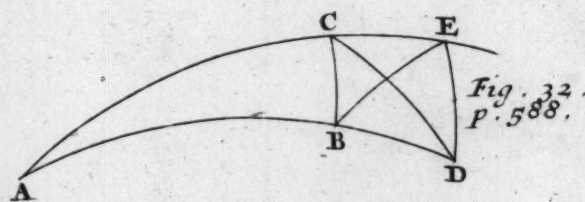
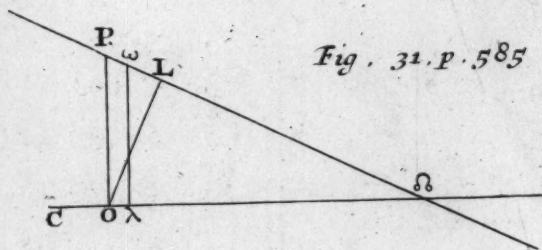


Fig. 30. P. 584.



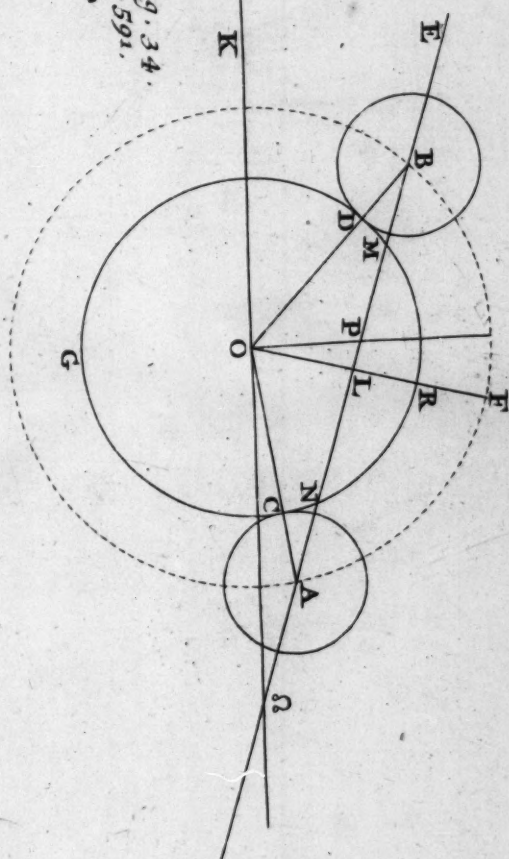


Fig. 34.
p. 591.

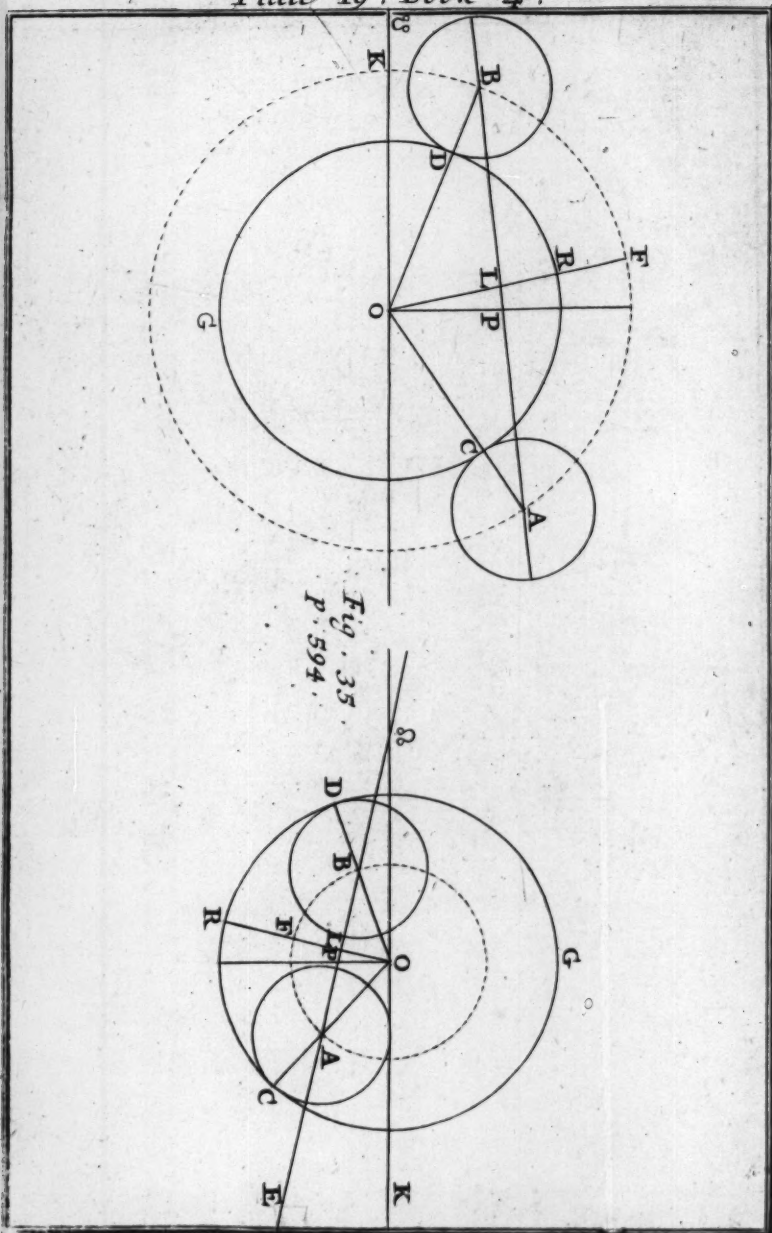


Fig. 36.
P. 596.

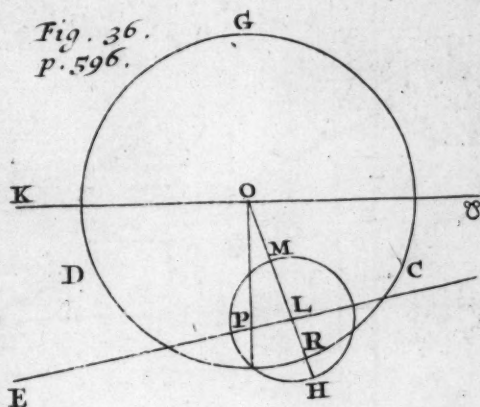


Fig. 38.
P. 601.

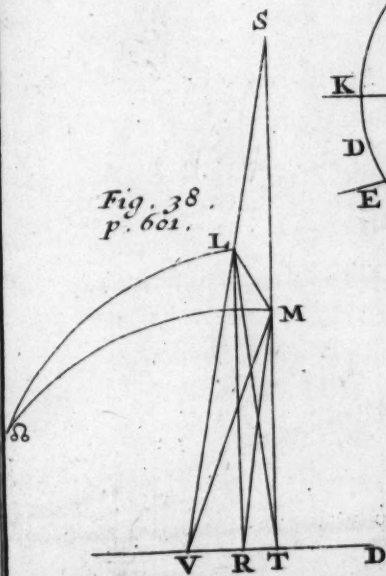
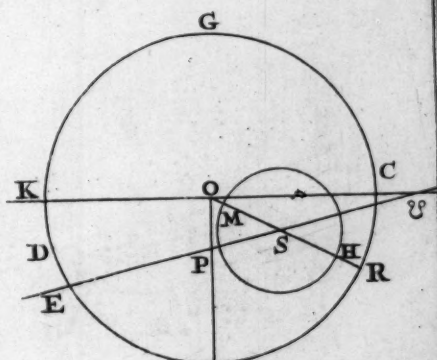


Fig. 37.
P. 598.



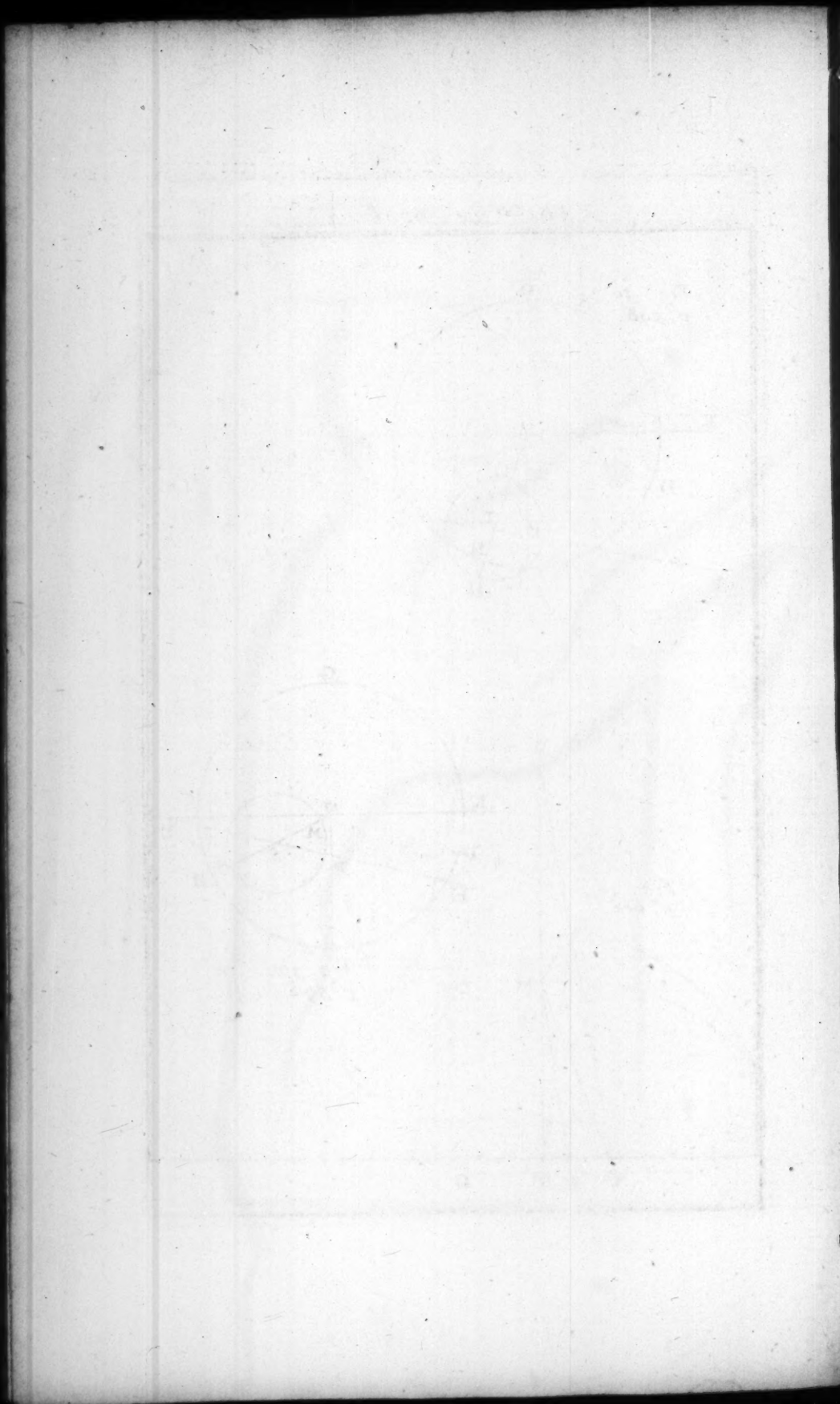


Plate 21. Book 4.

Fig. 39. p. 603.

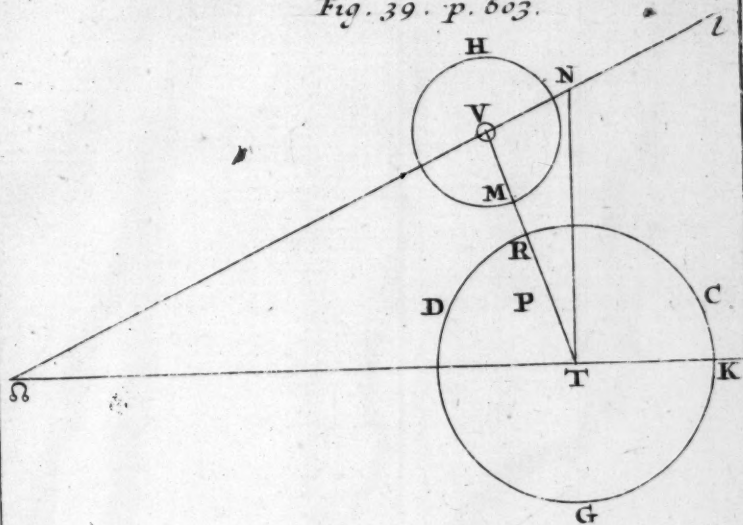
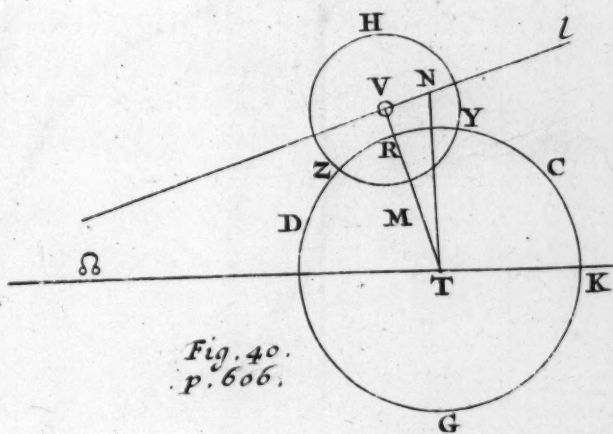


Fig. 40.
p. 606.



Book 4: Plate 22.

Fig. 41.
p. 608.

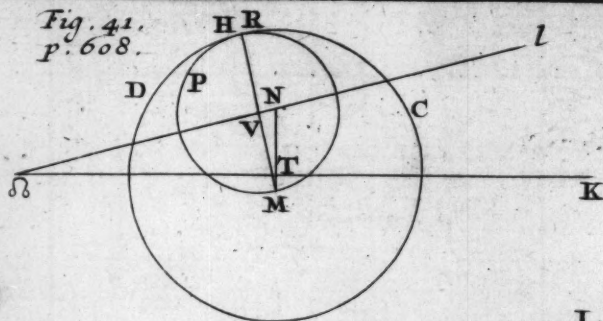


Fig. 42.
p. 609.

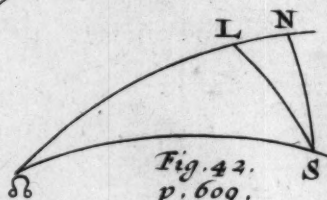


Fig. 43.
p. 611.

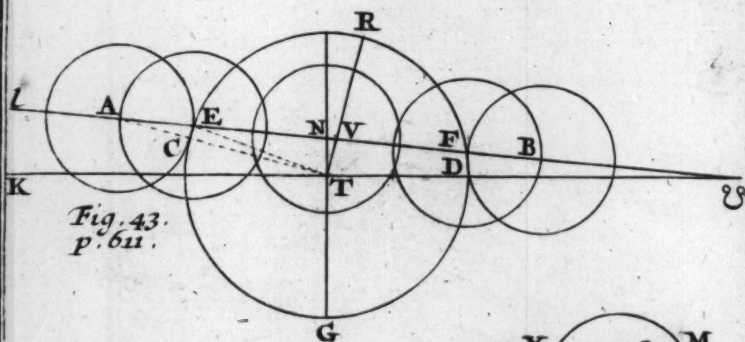


Fig. 44.
p. 615.

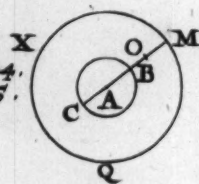
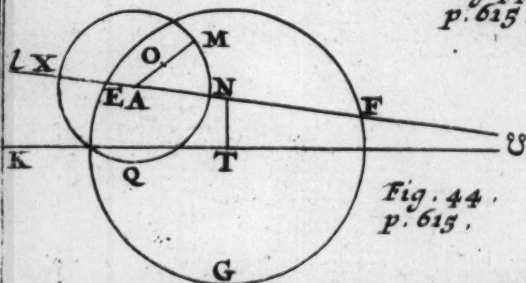


Fig. 44.
p. 615.



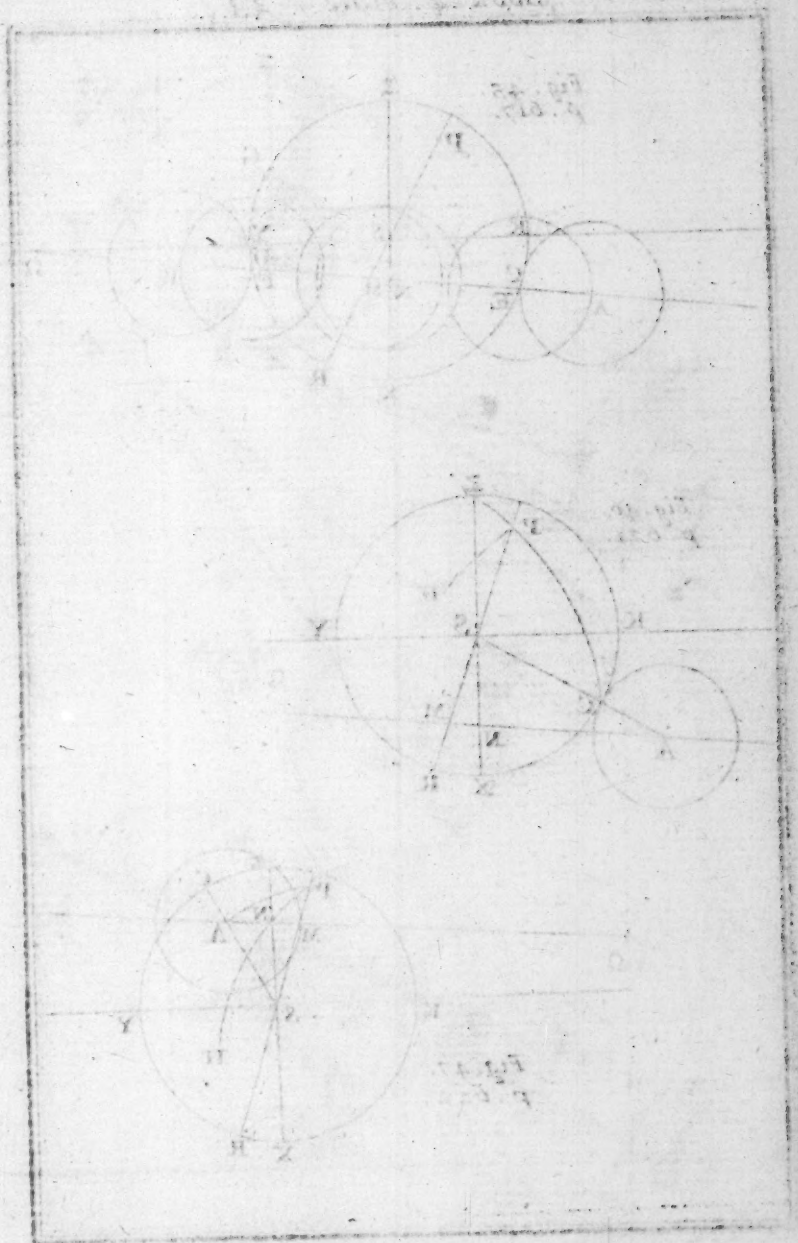


Fig. 45...
p. 617.

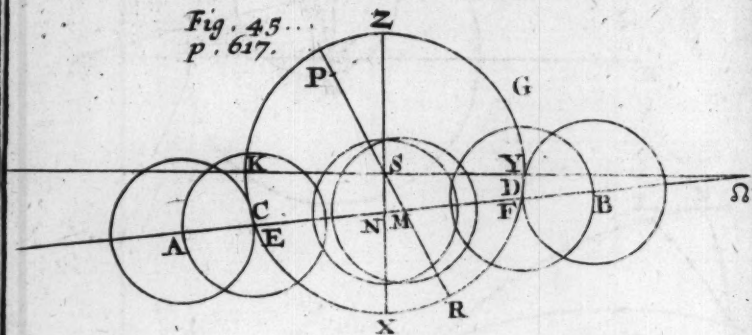


Fig. 46.
p. 621.

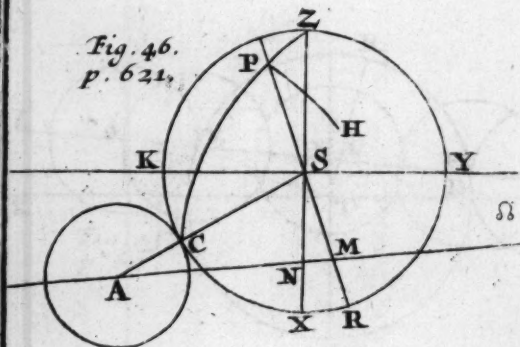


Fig. 47.
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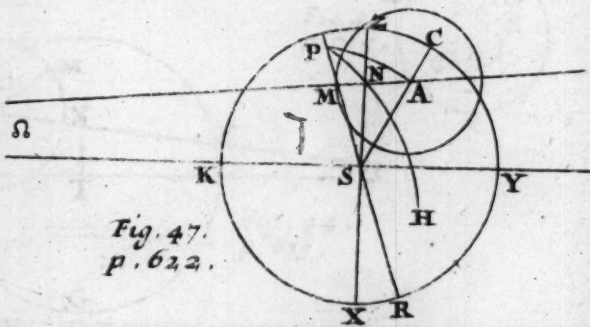


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p. 625.

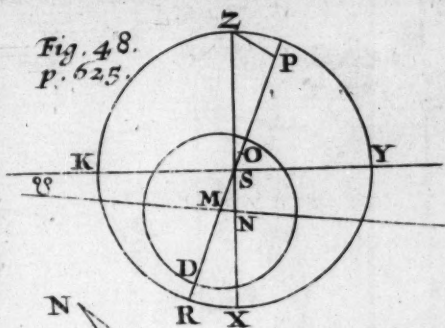


Fig. 49.
p. 627.

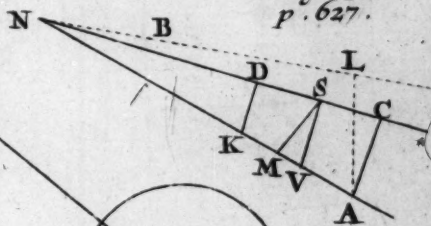


Fig. 50.
p. 630.

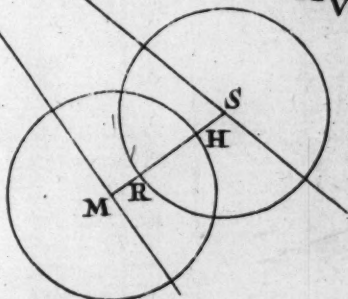
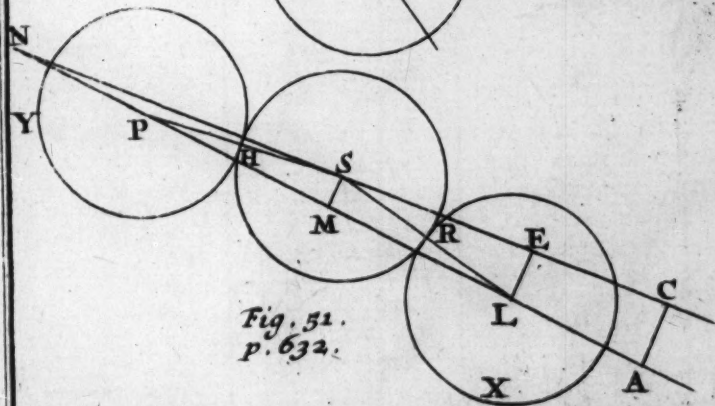


Fig. 51.
p. 632.



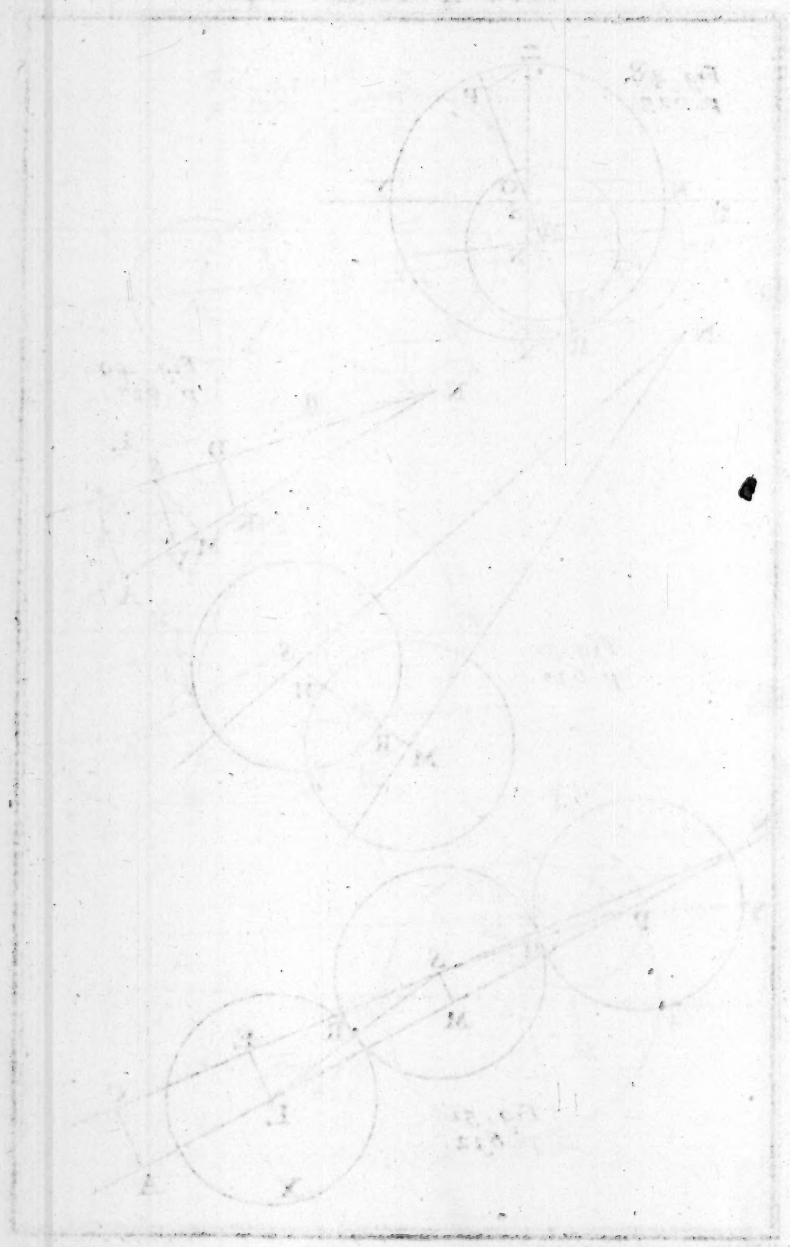


Plate 25. Book 4.



Fig. 52.
p. 647.

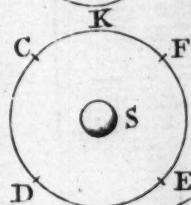


Fig. 53.
p. 650.

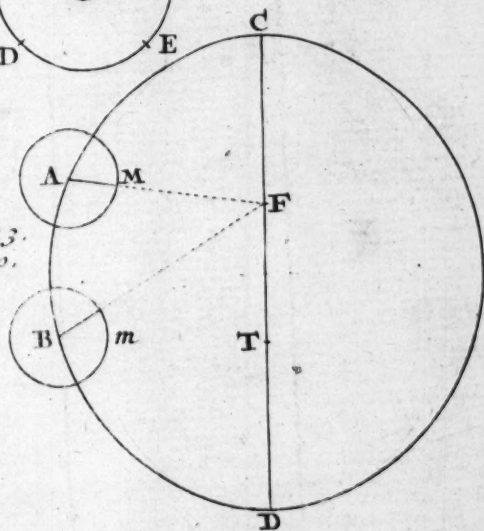


Fig. 54.
p. 657.

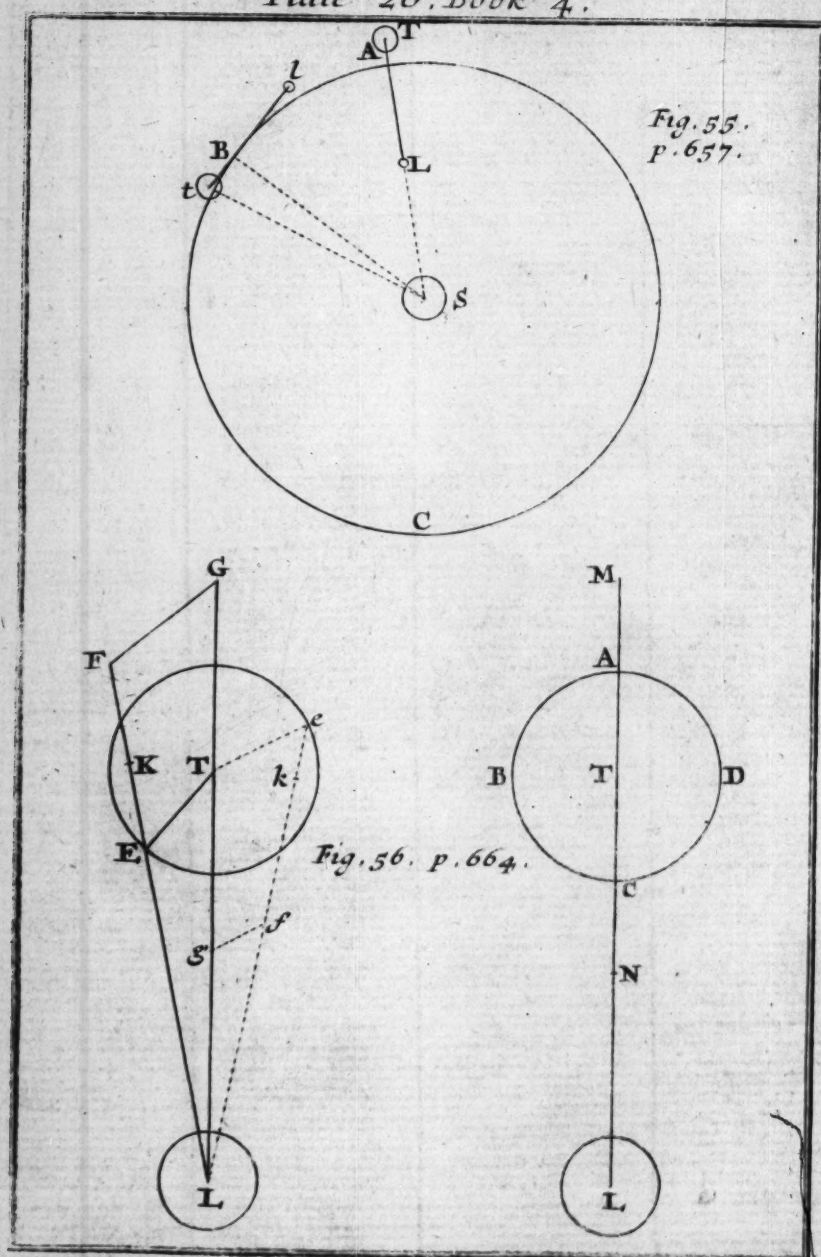
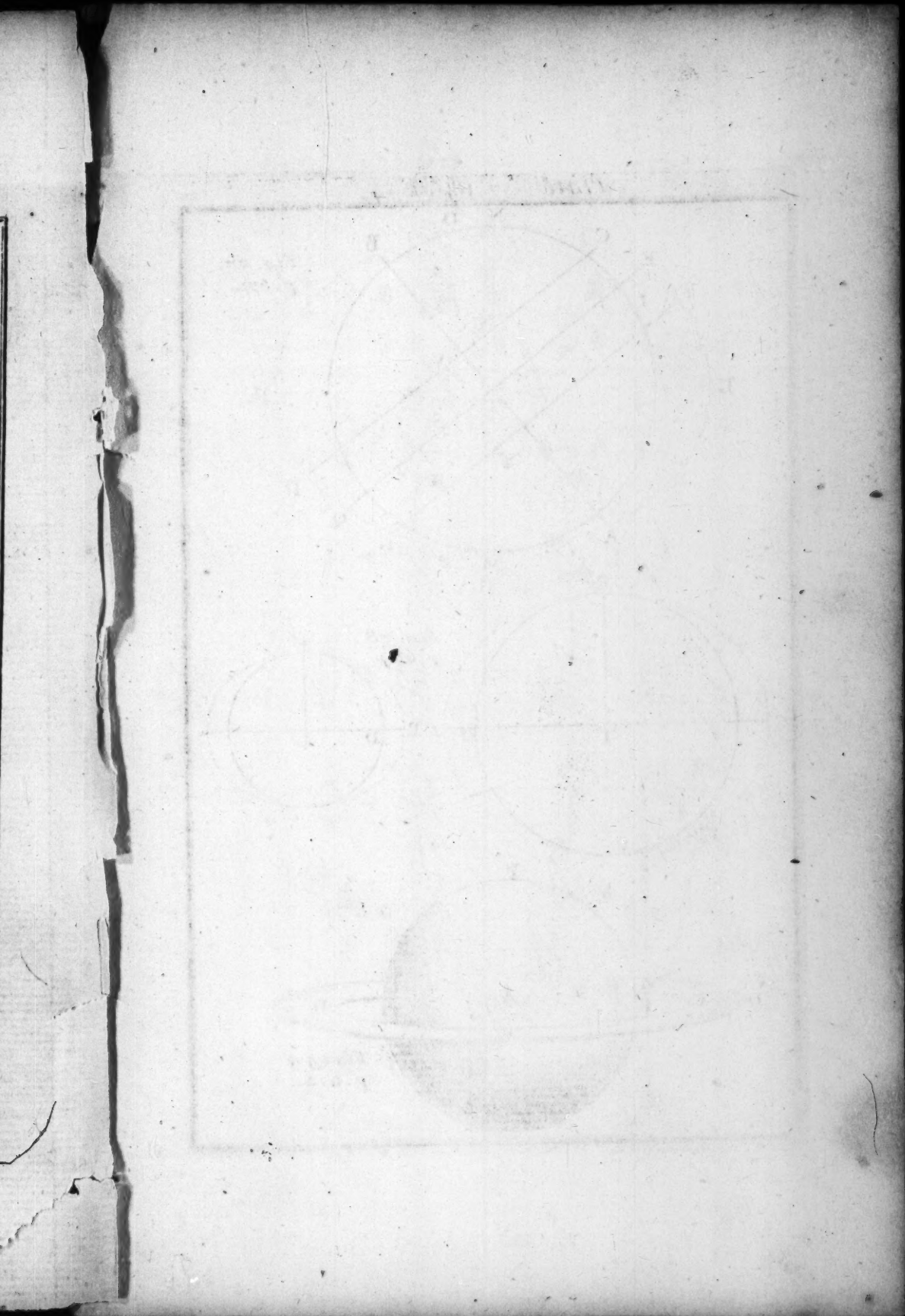


Fig. 55.
P. 657.

Fig. 56. p. 664.



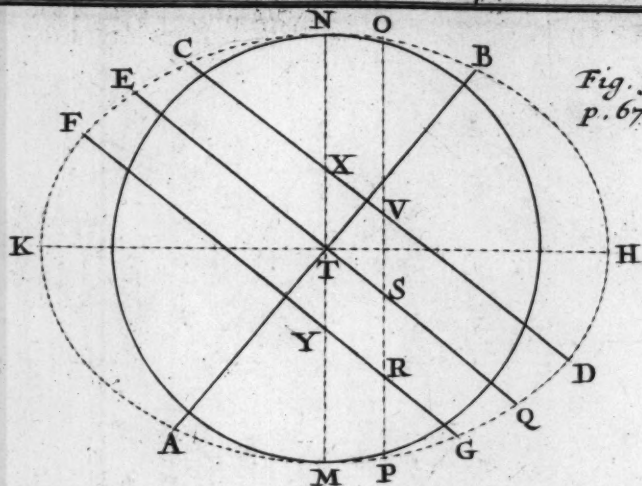


Fig. 57.
p. 671.

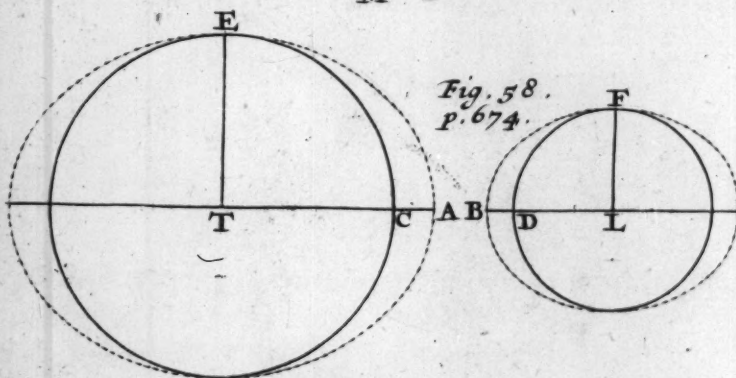


Fig. 58.
p. 674.

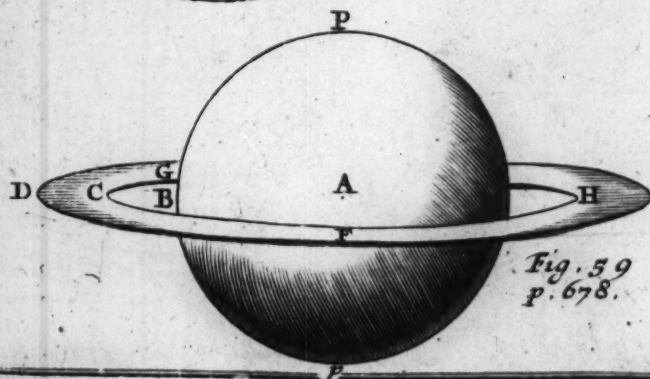
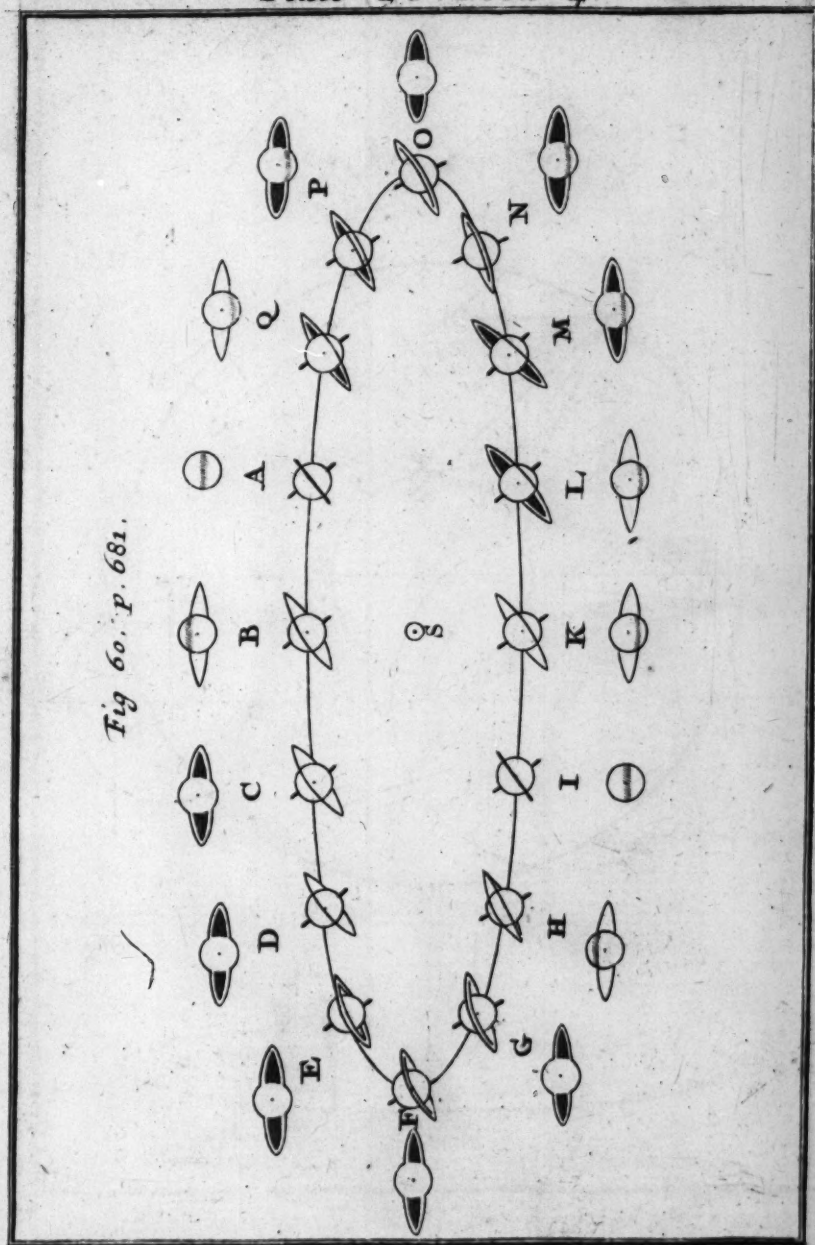


Fig. 59
p. 678.

Fig 60. p. 681.



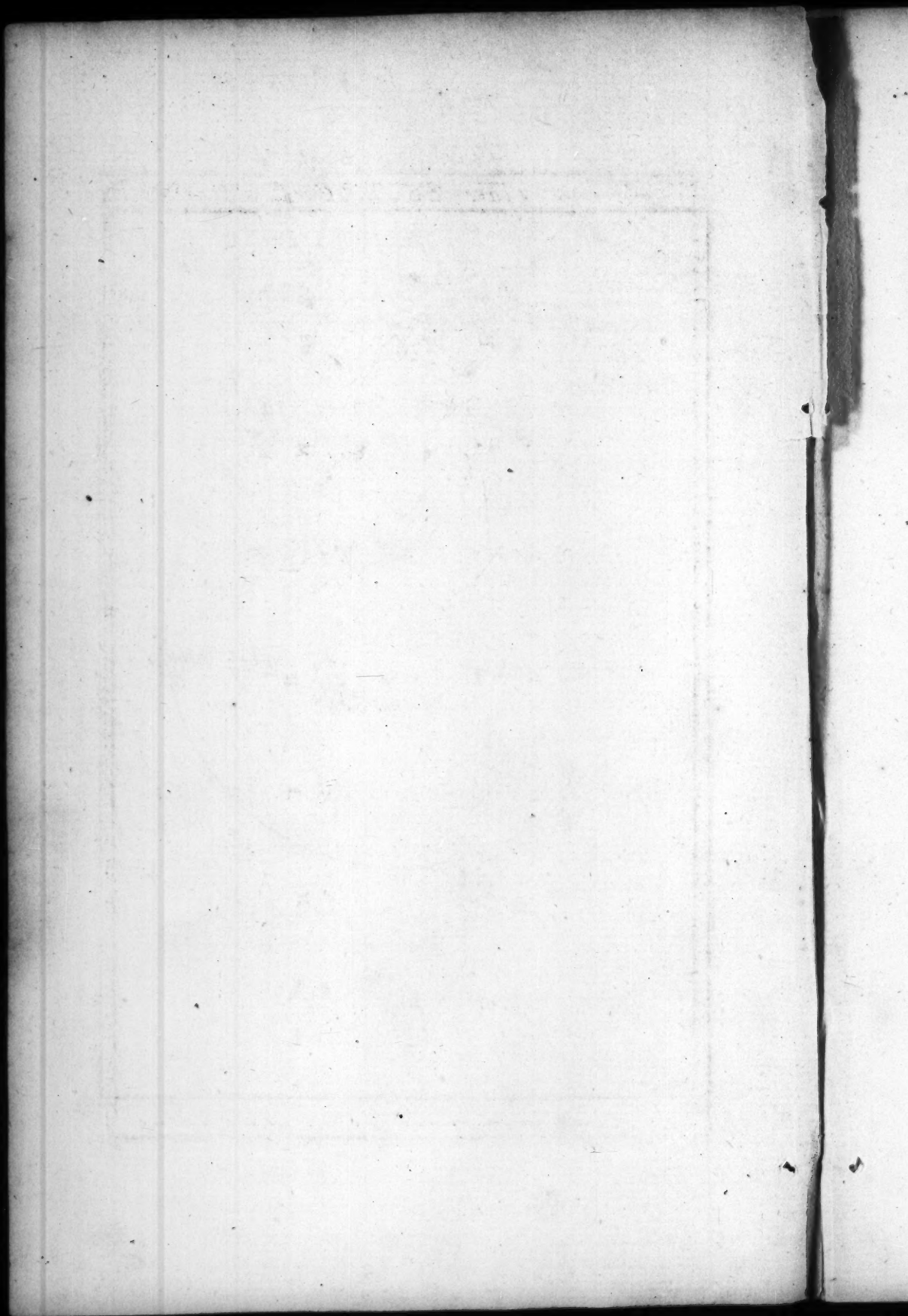
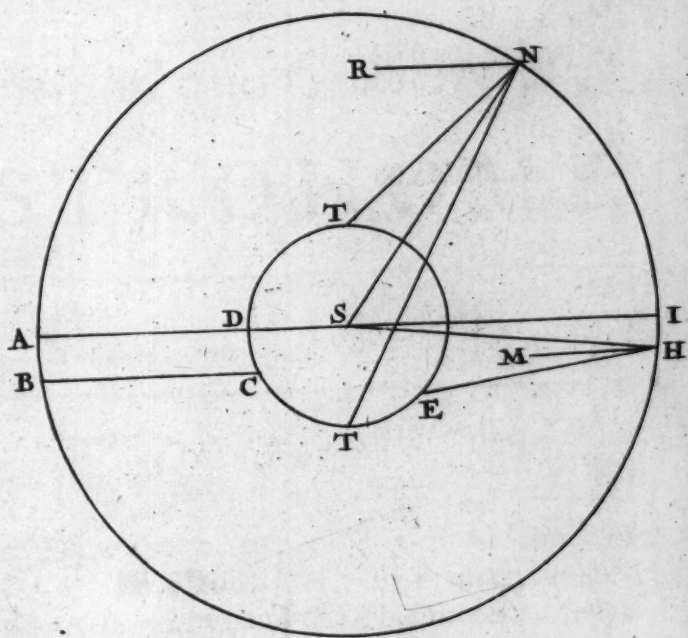


Fig. 61. p. 687.





THE
ELEMENTS
OF
Physical and Geometrical
ASTRONOMY.

The FIFTH BOOK.

Of Comets.



HAVING explain'd the Motions of the Planets, as well Primary as Secondary, and the Phænomena thence arising, we proceed to explain the Motions of another Sort of Bodies, namely, of Comets; a great many Kinds of which are mentioned by Authors: For they distinguish them according to the various Colours which they appear to shine with, and the Planets which they look like, and the Figures of their Tails. We that treat of Astronomy, and not Astrology, own only one Kind of Comets, which have those different Appearances,

694 *The* ELEMENTS Book V.

according to the Difference of the Vapours which make their Tails, and the different Situation of the Comet itself in respect to the Sun.

In treating of them we shall first take Notice of some Physical Matters, that we may afterwards go on to Geometrical Propositions, namely, to determine their Orbits. In the Physical Things which we shall take Notice of, it will first be necessary to mention, and afterwards reject the Opinions of some Philosophers, that we may the better understand those Opinions which we must embrace. The Reader is desir'd to pardon our taking no Notice of these Things in the foregoing Books, but reserving them for this; because the Geometricians have not hitherto had that Regard to the Theory of the Comets, which they have had to the Theory of the Planets and Fix'd Stars.

SECTION I.

General Propositions concerning Comets.

PROPOSITION I.

TO explain in a few Words the Opinions of Philosophers concerning the Duration and Places of the Comets.

The most antient Philosophers and Astronomers suppos'd Comets to be eternal or constant Bodies of the World, which, like Planets, perform'd their Revolutions in stated Times: For, *Aristotle*, speaking of Comets, in his *First Book of Meteors*, Chap. 6. says, Τῶν δὲ Ἰταλικῶν πρὸς καλεμῶν Πυθαγορείων, ἵνα λέγουσι αὐτὸν εἶναι πλανήτων ἀστέρων, ἀλλὰ διὰ πολλῶν τε χεῖν τὴν φασασίαν αὐτῷ εἶναι καὶ τῆν

Book V. of ASTRONOMY. 695

καὶ τὴν ὑπερβολὴν ἐπὶ μικρὸν, ὅπως συμβαίνει καὶ τοῖς τῷ Ἑρμῇ
 ἀστέρες. But some of the Italians, call'd Pythagoreans, say, That a Comet is one of the Planets or
 Wandring Stars, but that they don't appear un-
 less after a long time, and are seen but a small
 time, which happens also to Mercury. And Plu-
 tarch, in his Third Book of Philosophical Opinions,
 Chap. 2. says, Τὸν ἀπὸ Πυθαγόρου πρὸς μὲν ἀστέρες φασὶν
 εἶναι τὸν κομήτην ὅτι ἐκ αἰὶ φαινομένην, διὰ τοῦτο δὲ ὀρίσμεν
 χρόνους περιόδου αὐτοῦ τελλόντων. Some of the Pytha-
 goreans say, that a Comet is a Star of the Num-
 ber of those which do not constantly appear in
 the Heavens, but in stated Times, having gone
 thro' a determinate Circuit, arise or appear a-
 gain; And Seneca, in his Seventh Book of Nat.
 Questions, Chap. 3. says thus, Democritus, the
 most subtile of all the Antients, said that he sus-
 pected that there were several Running Stars,
 (that is, Comets) but he neither determin'd
 their Number nor Names, not having yet right-
 ly understood the Courses of the Five Stars, that
 is, of the Planets, except the Sun and Moon.
 And again, Seneca, in the same Place, says,
 that Apollonius, the Myndian, who was very
 skilful in Natural Sciences, affirm'd, That the
 Comets were, by the Chaldeans, reckon'd among
 the Planets, and had their Periods, or Courses,
 like them. This Apollonius (as the same Seneca
 says in the 17th Chap.) us'd to say, That a
 Comet was a Star or Cælestial Body as the Sun
 or Moon. But that he did not know its Course;
 because it cuts thro' higher Parts of the World,
 and then at last appears, when it comes to the
 Bottom of its Course; and that is the very same
 thing which was shewn in Prop. 10. B. I. Of
 which Opinion Seneca himself professes himself
 to be, in the 22^d Chapter, in these Words, I

don't think a Comet to be a sudden Fire, but one of the eternal Works of Nature. In the 2^d Chapter, he gives the sole and only Method by which this Question may be solv'd, saying, that there ought to be a Collection of former Rises or Appearances of Comets; because, by Reason of their seldom appearing, their Courses cannot yet be understood, neither can it be discover'd whether they return or no; or, that they appear in Order, at a settled Time. And at last he thus prophesies, in the 25th Chapter, *A Time will come, when those things which are now hid will at last be brought to Light, by Length of Time and the Diligence of Posterity. One Age is not sufficient to make such great Discoveries. A Time will come, when those that come after us will wonder that we were ignorant of Things so plain.* And in the 26th Chapter, *Some Body will demonstrate, which Way Comets wander, why they go so far from the rest of the Cælestial Bodies, how big, and what Sort of Bodies they are.*

Afterwards the whole School of the Peripateticks, to keep up their Doctrine concerning the Heavens, which they suppos'd not capable of being generated or corrupted, did not look upon the Comets as eternal and permanent Bodies of the World, but as Bodies newly produc'd, and in a short Time to perish again, and affirm'd them to be Sublunary, and made of Exhalations in the Terrestrial Regions; which Opinion was the more believ'd after solid Orbs were introduc'd, because Comets cou'd not pass thro' them.

But at last *Tycho Brahe* and *Kepler*, finding by Observation that Comets had no diurnal Parallax, restor'd them to Places above the Moon. The easiest and readiest Way to find the diurnal Parallax of a Comet, if it has any, is the following;

Book V. of ASTRONOMY. 697

lowing: By the Observations of many Days, made before and after, let the Track be defin'd among the Fix'd Stars, which a Comet wou'd have, if it had no diurnal Parallax. If therefore, from its Rising till it gets to the Meridian, or from thence to its Setting, it has the same Place among the Fix'd Stars respectively, which it wou'd have according to the Track above defin'd, it is a Sign that it has no diurnal Parallax: For, when it is near the Horizon, if it had a diurnal Parallax, it wou'd appear lower among the neighbouring Fix'd Stars, which have no Parallax; therefore, as the Comet is towards its Rising it wou'd appear nearer to the East, and at Setting nearer to the West, than it is according to the Place agreeable to that Moment of Time, according to former Observations. And this Method is no Way affected by the Uncertainty of the Refraction by which Cœlestial Bodies are seen; for the Refraction of the neighbouring Stars is nearly the same with that of a Comet. But the readiest Way that this Observation is made, is to find the true Place of a Comet, in a Right Line drawn between two Fix'd Stars, so that those Fix'd Stars, at that Time at a small Height above the Horizon, be in, or nearly in the same Almucantar, which will be easily done among such a Multitude of Fix'd Stars, at least if we take in such as the Eye perceives with the Help of a Telescope: For, if a Comet has a diurnal Parallax, it will appear out of that Right Line, and nearer to the Horizon; if not, it will even then appear in a Right Line among the Fix'd Stars.

That Comets are above the Moon is farther confirm'd, from the Observation which Mr. *Cassini* made, of that which appear'd in the
Year

698 *The ELEMENTS* Book V.

Year 1680, which he observ'd to be but 22 or 23 Degrees distant from the Sun, and yet to shine at that Time with its full Orb; which he had also observ'd of the Comet which appear'd in *April*, 1665. whence one may certainly conclude, either that those Comets at that Time were not only above the Moon, but also above the Sun, (since Mercury and Venus, at that apparent Distance from the Sun, do not shine with a full Orb, unless they be above the Sun;) or that they are lucid Bodies, like the Sun or Fix'd Stars, or at least pellucid and transmitting the Sun's Rays, which can hardly be suppos'd of such great Bodies, since the Sun's Spots, however rare their Texture is, are opaque enough. But that Comets are opaque Bodies, will be shewn below, from the Shadows which they cast.

PROPOSITION II.

TO give a short Account of the Opinions of Philosophers concerning the Rise and Orbits of Comets.

Philosophers are not yet agreed about the Orbits of Comets: *Kepler*, and after him *Cartesius*, *Hevelius*, and others, say, That the Trajectories of Comets are Right Lines. For, *Kepler* did not like *Circles*, or other recurring Curves, for the Motion of Comets, as thinking it not likely that the Motion of Comets, which never return, shou'd be Circular. But *Kepler*, in his *Physiolog. of Com.* believ'd, That the vast Depth of the Liquid *Æther* was of such a Nature, that it produc'd Comets of itself, lest that Region shou'd be empty, and that that was perform'd by a Faculty like the Animal or Vital one, for the refining and purging of the *Æthereal Aura*, which in some Places grows thick. And therefore, that this gross fat Part of the Air, being gathered together like some Excrement into

Book V. of ASTRONOMY. 699

an Imposthume; it happens, from the Nature of the Place, to be enlightened, (because the Sun's Light is every where) and to get a Motion like some Star: An exact Example of this, how this fat and Cælestial Matter of Comets has Light and Motion, may be seen in Aerial Trajectories (or Star-shooting.) But Hevelius suppos'd Comets to be generated from the most subtile Exhalations of Planets joining together by Chance in the Æther, and retaining the Form of a Disc, one Side of which is turn'd to the Sun, after the Comet, moving in a Spiral Line, is gone out of the Atmosphere of the Planet, from which it had its first Rise.

Cartesius suppos'd the Body of every Comet permanent, like that of a Planet, and made (like a Planet) of some Fix'd Star, or Sun cover'd with Spots, and carried away by the next Vortex, as it encreas'd. But he wou'd have a Comet be constantly carried in a Right Line from one Vortex to another.

But those that believe a Comet to be a permanent Body, and to return again in the same Track, as Planets do, will have its Orbit, tho' very Excentric, such as either to contain the Earth in its Circle (as Mr. Cassini says of the Comet which appear'd in the Year 1577, which he suspects to be the same Comet that appear'd again in the Years 1680, and 1681) and that the Comet runs equably thro' the Periphery of such a Circle, contrary to what the Planets do; or not to contain it, so that the Convexity of the Orbit be turn'd towards the Earth, as the same Cassini seems inclinable to believe, in his Theory of the Motion of the Comet of the Year 1664. But it is true, that Mr. Cassini only makes Use of such Orbits chiefly for calculating the Motion

Motion of a Comet, being too cautious a Natural Philosopher to affirm or define any thing concerning that Matter.

From the Observation of several Comets, Mr. *Cassini* has settled the Paths of Comets among the Fix'd Stars, within whose Limits most of the Comets (tho' not all of them) are contain'd, as the Planets are within the Limits of the Zodiac. He finds that this Cometary Zone, which is as broad as the *Æquator*, and like it mark'd with the Figures of Animals, goes thro' *Antinous*, *Pegasus*, *Andromeda*, the *Bull*, *Orion*, the *lesser Dog*, the *Hydra*, the *Centaur*, the *Scorpion* and the *Bow of Sagitarius*; and he calls it the *Zodiac of Comets*.

Something like this is the *System of Comets*, publish'd by the ingenious Mr. *James Bernoulli*, in the Year 1682, in which he supposes that a certain Primary Planet revolves about the Sun at a Distance from the Sun, equal to 2583 Semidiameters of the *Magnus Orbis*, in the Space of four Years and 157 Days, tho' *Saturn*, who is two hundred and fifty eight times nearer, does not make its Revolution in less than 30 Years. About this Primary Planet, (which we can never see, as well by Reason of its Smallness as its immense Distance) at different Distances, several Comets revolve, tho' none of them come down quite to the Orbit of *Saturn*; therefore the Comets, according to Mr. *Bernoulli*, are nothing else but the *Satellites* of the said invisible Primary Planet, which only begin to be visible when they descend towards us in their *Perigæum*, and so are nearest to us.

The Opinion concerning the Orbits of Comets, which seems most probable to us, has been already explain'd in the 35th Prop. of B. I. which, besides

Book V. of ASTRONOMY. 701

besides its being most agreeable to the Simplicity of Nature, may put on the Appearance of almost every Opinion mention'd before. For, if S , which represents the Sun in the first Figure of the first Scheme, be the common Focus of the Orbit of the Earth $TGHK$, which is nearly circular, and the very excentric Orbit of a Comet be $ABDIEC$; it is plain that the Phænomena of the Comet going thro' the Part EC , or BD , of its Orbit, will be well enough explain'd by an equable Motion in a Right Line, because BD , or CE , is but little different from a Right Line; and besides, a Comet will move equably in it, because equal Triangles standing upon it, and having S for their common Vertex, will also have their Bases nearly equal. But yet it will, at first Sight, hardly appear to be the same Comet, which first describes EC , and afterwards BD , there being a considerable Time between those two Apparitions; namely, that in which it describes the Arc CAB , and is then hid under the Sun's Rays. Now, when the Earth is at T , a curve Convex towards T will better agree with the Phænomena of a Comet describing that Part of the Orbit which is at A ; for it is truly such in the 1st Fig. of Scheme 1. But a curve Convex towards the Earth, when the Comet is in Opposition with the Sun, will never agree with the present Theory; but if it is concave, it will very well agree with it, if (as in the 2^d Fig. of Scheme 1.) SA , the Perihelion of the Comet, be greater than ST , the Semidiameter of the *Magnus Orbis*: For, a curve Concave, towards T , will very well answer the Phænomena of a Comet passing at A ; because such a Curve is its real Trajectory.

PRO-

PROPOSITION III. LEMMA.

IF the Air should expand it self according to the following Law, which is confirm'd by Experiments, viz. That the Spaces, in which it is compress'd, are reciprocally proportionable to the Weights compressing it; a Globe of our Air of an Inch Diameter, if it shou'd become as rare as it wou'd be at the Height of the Semidiameter of the Earth, wou'd fill all the Planetary Regions as far as the Sphere of Saturn, and far beyond. [Fig. II.]

That the Law of the Expansion of the Air is the same as was laid down here; namely, that the Spaces, in which a given Quantity of Air is compress'd, are reciprocally proportional to the Weights compressing it, is plain from the Experiments mention'd in the *Philosophical Transactions* of July 1671. N^o 73. This Law then being laid down, let $ABZZ$ represent an upright infinite Tube, full of Air, whose Sides AZ , BZ , are parallel to one another, and perpendicular to the Horizon, and the Bottom AB standing upon the Earth's Surface: It is plain the that Air at AB is press'd by the Weight of the whole incumbent Air $ABZZ$, and that the Air at ED (a Parallel to AB drawn at any Distance from it) is press'd by the Weight of the incumbent Air $EDZZ$. Let us conceive the Air expanded along the whole Right Line ED to be compress'd into Part of the said Line, as CD , so that it may have the same Density as that of the Air AB , and that this is also done to any other Right Line, similarly drawn as $e d$, that is, that all the Air, which before was expanded all over the Tube $ABZZ$, is compress'd into the Space $ABZX$, so that it may be every where of the same Density. The Right Line BZ will

Book V. of ASTRONOMY. 703

will be the Asymptotes of the Curve $ACcX$, terminating the Space $ACcXZdDBA$ on one Side, because at any Height above the Earth there is some Air, which therefore being compress'd, till it be equally dense with the Air AB , there still remains some.

Since then the same Quantity of Air compress'd as much as AB was compress'd, (that is, by the incumbent Air $AZZB$) will take up the Space CD , but, compress'd as ED , will take up the Space ED ; it is plain, by the foremention'd Law of the Compression of Air, that CD is to ED , that is, to AB , as the Weight of the incumbent Air $EDZZ$, to the Weight of the incumbent Air $ABZZ$; that is, (by reason of Gravity, which we suppos'd the same at all Distances from the Center) as the incumbent Air $CDZX$, to the incumbent Air $ABZX$, or as the untermimated Figure $CDZX$, to the untermimated Figure $ABZX$: But this is the Property of the Logarithmic Curve very well known to Geometricians; therefore the Curve ACX is a Logarithmic Curve, whose Asymptote is the Right Line BZ . Whence by *Prob. 3. Append. 3. of Barrow's Twelfth Geometrical Lecture*, (if CG , at cg be drawn parallel to BZ and meets with AB , and Gg ,) BG , or CD is in the same Order between BA and Bg or dc a Geometrical Mean proportional, as BD is an Arithmetical Mean between Bd and Nothing; therefore the o (or Nothing) BD and Bd are as the Logarithms of the Right Lines BA , DC , and dc respectively. Therefore BD , Bd , BA , and DC being given, it will be easy to find cd , which, as has been already shewn, represents the Quantity of Air compress'd as much as it is at the Surface of the Earth, which in the
Height

Height Bd above the Earth will expand itself into a Space equal to BA ; that is, making DB to dB as the Differences of Logarithms of AB and CD to another, this will be the Difference of the Logarithms of AB and cd . And therefore the Logarithm of AB being given, the the Logarithm of cd , and therefore cd itself will be known.

Let AB be 33 Foot long, that is, of the Height of a Column of Water whose Weight is equal to a Column of Air of the same Thickness, extended from the Surface of the Earth up to the Top of the Atmosphere: And as the Air near the Surface of the Earth takes up a Space nearly 850 Times greater than Water of the same Weight, as innumerable Experiments shew; a Column of Air, 850 Foot high, is of the same Weight with a Column of Water of equal Thickness and one Foot high. If therefore we were to rise 850 Feet above the Surface of the Earth, the Column of Air, pressing upon any Place at that Height, is equal in Weight to a Column of Water of the same Thickness and 32 Foot high: If therefore BD be taken 850 Feet, DC will be 32 Feet: For AB , CD , cd , &c. will shew the Heights of the Baroscope (with whatsoever Liquor it is fill'd) in the Places B, D and d ; for they are respectively proportional to the Weights of the incumbent Air. Therefore to find out dc from the Premises, supposing Bd to be equal to the Semidiameter of the Earth, that is, in a round Number equal to 20000000 Feet, (for according to some late Experiments made in *France*, its Measure is 19615800,) let the Operation be perform'd by the Rule of Proportion, finding a fourth proportional to DB 850, dB 20000000, and the Difference of the Logarithms of AB ,
 CD taken

Book V. of ASTRONOMY. 705

CD, taken out of the Tables, that is, 0,0133639, which will be 314, 00,00000, namely, the Difference of the Logarithms of *AB* and *cd*, which being taken from 1,5185139, the Logarithm of *AB* first taken from the Tables, and there will remain the Logarithm of *CD*, which is —312, 0000000, whose Characteristic (as they call it) is —312. Moreover, from the Nature of Logarithms, the Number of the Tables, whose first Place is Unity, has Nothing for the Characteristic of its Logarithm; but the Number whose first Place is 2, or the second of Integers, has One for the Characteristic of its Logarithm; that, whose First Place is the Third of the Integers, has Two for its Characteristic, and so on ascending; and likewise the Characteristic of a Number whose first Place is the next below One in a decimal Progression, or the first Place of Decimals is —1, and so on in descending: Whence, conversly, from the Characteristic of the Logarithm, you may know the Place of the first Place of the Number, or of how many Places the Number is; and hence the Number, the Characteristic of whose Logarithm is —312, is the first Place in the 312th Place of Decimals; therefore the Right Line *cd* is denoted by a Number whose first Place is the 312th of Decimals.

Then, taking away the Calculation, it is plain that a Portion of our Air, denoted by a Number whose first Place is the 312th of Decimals, at the Height of the Semidiameter of the Earth expands, it self to a Magnitude which is denoted by 33; that is, the Space, taken up by our Air, is to the Space that it wou'd fill up, if it was as rare as it wou'd be at the Distance of the Semidiameter of the Earth, as the Number one to a Number express'd by the two Ternaries

Q q q

with

with 312 Cyphers annex'd. But this Ratio is vastly less than that which (according to *Aristarchus*) a small Grain of Sand has to the Sphere of the Fix'd Stars, as *Archimedes* has demonstrated it in his *Arenarium*; namely, greater than the Ratio which One has to a Number whose Notation is Unity with 63 Cyphers annex'd: But we shall presently shew, that the said Ratio is much less than that, which is between a Globe of an Inch Diameter and the Orb of Saturn, much more nearly computed, than the Sphere of the Fix'd Stars was by *Aristarchus*.

The Semidiameter of the Earth is, as was said before, in a round Number 20000000 Feet: Now if the Sun's Parallax be 10" (which is the least that Astronomers have hitherto attributed to it) whose Right Sine is 485 Parts, supposing the Radius to be 10000000; the Semidiameter of the *Magnus Orbis* will be 500000000000 Feet, that is, to the Semidiameter of the Earth, as the Radius to the Sine of the Sun's Parallax; and therefore the Semidiameter of the Orb of Saturn (by *Prop. 41. B. III.* scarce ten times as great) will be 5000000000000 Feet, and its Diameter 10000000000000, or 12000000000000 Inches. Wherefore the Sphere of Saturn is to a Globe of an Inch Diameter, as the Cube of the foregoing Number, or 1728 with 39 Cyphers to Unity, or in a round Number, and, at most, as a Number express'd by 2 and 42 Cyphers is to One, or Unity. Therefore, since a Globe of an Inch Diameter has a much greater Ratio to the Sphere of Saturn, than to the Sphere, which Air, like our Air, wou'd fill, if it shou'd become as rare as the Air at the Height of a Semidiameter of the Earth above the Earth, it is plain that that Globe of Air, whose Diameter is one Inch, when

Book V. of ASTRONOMY. 707

when it comes to be as rare as it wou'd be at the Distance of a Semidiameter of the Earth from it, wou'd fill all the Planetary Regions as far as the Sphere of Saturn, and much farther. Q. E. D.

If the Gravity towards the Earth be suppos'd to diminish as you recede from the Center, as it really does; it is plain that such a Globe of our Air of an Inch Diameter, as we have mention'd, at the Height of a Semidiameter of the Earth above the Earth, wou'd yet take up a greater Space.

PROPOSITION IV.

TO explain the Opinions of Philosophers concerning the Tails of Comets, and confirm that which is the true Opinion.

That a Comet's Tail depends upon the Sun is plain from hence; that it is always turn'd from the Sun, which was first of all discover'd by *Peter Apian*, in the Comets which appear'd between the Years 1530 and 1540, tho' *Regiomontanus*, 60 Years before, became almost certain of it. As for the Opinion, about the Manner of the Tail's being caus'd by the Sun, it is threefold.

First, *Apian*, *Cardan*, *Tycho*, *Snell*, and others, believe, that the Sun's Rays being propagated thro' the transparent Head, or Body, of the Comet, are refracted as in a Lens of Glass, and look like a Beam behind the Comet, or in those Parts which are turn'd from the Sun. But this is contrary to the Laws of Dioptrics: For, besides the Figure of it not answering to this, such a Brightness, propagated thro' the pure Æther, will not be perceiv'd by an Eye posited sideways; it is requisite that there shou'd be some reflecting Substance in those Places, where the Tail is to make that Brightness sensible, after

the same Manner that there must be Particles of Dust, or Smoke, flying about in a dark Room, that such a Brightness may be seen by an Eye that looks upon a Sun-beam let into the Room, the said Eye not being in the Way of the Beam; and by how much more gross and dense the Particles are that fly about in the Medium, by so much will the Beam be brighter, and so on the contrary. From this there ought to be a more gross Matter in the Region of the Tail, than any where else in the Æther about the Tail, otherwise the whole Sky round about wou'd be enlighten'd, and shine as much as the Tail; and therefore the Tail of a Comet does not proceed from the Sun's Rays propagated thro' the transparent Body of the Comet. And therefore this Argument will be of as much Force, whether the Rays be suppos'd refracted in the Head of a Comet and the Atmosphere about it, or in the Atmosphere only, the opaque Head hindering the Transmission of the Rays, as *Hevelius* observ'd in the Comet of 1665, which had a Shadow; and the same Author says, in his *Cometography*, B. VIII. that several other Comets have been observ'd. to cast a Shadow. But the famous Dr. *Hook* denies such a Shadow, and affirms a Comet to shine brighter in that Place where the Shadow, if there was any, ought to appear.

The second Opinion, is of *Cartesius* and his Followers, who wou'd have the Tail of a Comet to be owing to the Refraction of the Light from its Head to the Eye of the Spectator. If this was true, the Rays of the Fix'd Stars, and of the Planets, which are propagated thro' the same Mediums, wou'd not reach our Eyes so free from Refraction, as they appear by Observation; neither wou'd the Tails of Comets be so
free

Book V. of ASTRONOMY. 709

free from the Colour of the Rain-bow, which always accompany refracted Light. Neither does *Cartesius* take off those Difficulties, when he says in the 139th *Paragraph* of the third *Book* of his *Principles*, that the twinkling of the Fix'd Stars is owing to this, that their Tails wou'd be longer if the Light was greater; for when we make use of Telescopes, which encrease their Light, their twinkling ceases. Besides, several of the Planets, as Saturn, Jupiter and Venus, whose apparent Diameters are equal to, or even surpass the Diameters of the Comets that send out pretty long Tails, and which have more Light than those Comets that send out no Tails, have no Tails; and the Light of the Heads of those Comets, that have the greatest Tails, is often very small and dull. If either the Fix'd Stars (or even the Planets) as *Aristotle* says in his Discourse on Meteors, have sometimes appear'd Hairy to the *Egyptians*, this does not happen to them as it does to Comets, but is rather to be ascrib'd to an accidental Refraction of a Cloud, such an one as that to which *Cartesius* ascrib'd the Hairiness observ'd by *Aristotle* himself about the Fix'd Star in the Dog's Thigh. Lastly, If the Tail of a Comet did arise from the Refraction of the Cœlestial Medium, it would in the same Regions of the Heavens always go towards the same Parts; which yet does not happen, but is according to the Sun's Situation thrown towards different Parts, as has been said.

There are also other Opinions concerning the Tails of Comets to be met with in several Authors, which we shall not here take Time to confute; among which is this of Mr. *James Bernouilli*, in the before quoted *System of Comets*. For he conceives the Sun to be, in the Middle of

its Vortex, as a great Fire kindled in its Hearth, by whose Heat the Planets round about are warmed, whence it happens, that a great Quantity of very subtile Exhalations comes out of the Planets every Way, and even from the Sun itself, and from the very rapid Motion of the Solar Vortex, they are carried upwards towards the Circumference; to which Place, when these Effluvia are come, they are stopp'd and repell'd by the Resistance of the neighbouring Vortices, so that they can go no farther. Hence, because they have a near Addition of Matter, they are by degrees condens'd, and stick to the Secondary Planets of that invisible Primary one which is above Saturn (of which we spoke in Prop. 2.) when they come near the Circumference of the Solar Vortex; just as when a Pot of Flesh hangs over a Culinary Fire, the Smoak and Steam, rising up the Chimney from the Wood and Flesh, sticks to the Beams, and whatever solid Substance it meets in its Way, so as to make Soot: Thus, he says, the viscous Exhalations stick to the Planets, and encompass them to a considerable Thickness; whence it happens that the Extremity of the Nucleus of a Comet, or its Limb, is commonly very irregularly terminated, and rough, as if it had Soot upon it. Then in the Progress of the Satellite, as it revolves about its Primary Planet, from Time to Time, new Exhalations, (but thinner than those which immediately encompass it, which can't be so strongly condens'd for Want of a solid Foundation) stick to it, and extending themselves every Way round about it, not in the Manner of a Globe, but like a broad Disc, one of whose Planes remains turn'd to the Sun, and the Rays of the Sun make Right-angles with the Disc, whence (as he says) the Reason is plain, why the Tail always points towards the

Parts

Book V. of ASTRONOMY. 711

Parts opposite to the Sun; for tho' the whole Disc at once be enlighten'd by the Sun, yet no Part of the Disc but that which in respect of us is on this Side of the Nucleus, and in the same Plane with us and the Sun, reflects Rays to our Eyes, the others being reflected the contrary Way, and on the other Side of a Comet.

The most obvious Opinion to any one that looks at a Comet is, that the Tail has its Origin from the Head. And this is the Opinion of the most ancient Philosophers, and of *Aristotle* himself, who said that the Tail was a very thin Vapour arising from the Head, but he suppos'd it to be on Fire. The most penetrating *Dr. Hook*, from the Observation of Comets, contriv'd the following Manner of explaining the Nature of a Comet and its Tail, upon Account of the Comet which appear'd at *London* in 1678. He supposes a Comet to arise from a Planet, whose Gravity towards the Center, about which it revolv'd before, is destroy'd by a certain intestine Motion of the Parts (by which the Body is set on Fire and appears in a Flame) and which therefore is gone out of its Place, moving in a Tangent to that Point of its Orbit in which it was, when its said Change was perfected. Whence carried thro' the *Æther* in a Right Line (except so much as that Line is bent upon Account of the Matter of the Comet towards its Center) which not being yet fully chang'd is a little incurvated towards the Sun; the Planet, thus chang'd, has its Parts dissolv'd by the *Æther* and chang'd into it, as Salt is dissolv'd by Water, or any thing else by a proper Menstruum. Moreover, the outer Parts, so chang'd, have not only lost their Gravity, but are become light in respect of the Sun, that is, fly from it. Whence as they are absorb'd

in Æther and carried upwards, and at the same time fly from the Sun, these igneous Particles look like innumerable Projectiles going every Way directly from the Nucleus of the Comet, and tending towards a distant Point opposite to the Sun; therefore (by what has been demonstrated by *Torricelli* in the 30th *Prop. B. II.* of the *Motion of Projectiles*) they have the Figure of a Parabolic Conoid, whose Axis is in a Right Line joining the Sun and a Comet, having the Center of the Nucleus for the Focus of the generating Parabola: Therefore the Tail of the Comet will put on such a Shape. But since the heavy Bodies that we can come at do not intend or remit, that is, increase or diminish their Gravity, but their Gravity is proportionable to their Quantity of Matter, whatever intestine Motion the said Matter has, we must not admit a Loss of Gravity or Acquisition of Levity in other Bodies.

They that will not allow the Tail to be on Fire, and think that to produce the Phænomenon, it is enough for the Vapour that goes from the Comet to be enlighten'd by the Sun, and by reflecting the Rays of the Sun from its Particles, to make that Part of the Æther which they take up more enlighten'd than the rest of the Æther, which, tho' it receives the Light, cannot reflect them, as wanting a reflecting Substance, are oblig'd to find out the Reason, why the Vapour or reflecting Exhalation goes only on that Side which is turn'd from the Sun. *Kepler* explains the thing thus: 'The fat and gross Matter of the Comet being gather'd into an Orb, from the Nature of all Things which are united, and the Sun striking with direct Rays and penetrating this Globe, *he imagines* that some of the inmost Matter of the Comet follows, and goes out

Book V. of ASTRONOMY. 713

out the same Way as the Sun's Rays go, and therefore by that Means the Body of the Comet has Parts of it rubb'd, wash'd, or strain'd off, and thus there is a Quantity of Matter gather'd together on that Side of the Comet which is turn'd from the Sun, that reflects the Sun's Rays, and has the Appearance of a Tail. James Gregory in some physical Things subjoin'd to his *Univ. Parts of Mathemat.* will have a Comet to be a Kind of humid Body emitting its proper Vapours, always wandring about the *Aether*, and retaining almost the same Situation in respect to the Sun; which is also the Opinion of *Hevelius* in his *Cometography*, pag. 430. who thinks the Tail produc'd much after the same Manner: Namely, *That the thinnest Parts of the Atmosphere of a Comet are rarified by the Force of the Heat, and from the fore Part and each Side of the Comet are driven towards the Parts turn'd from the Sun.* And then, according to James Gregory's Opinion, the Half of the Comet which is turn'd from the Sun, and never warm'd or enlighten'd by it, has very gross and very opaque Vapours, which being continually supplied or fed with the vapour-like Matter of the Comet, and no way dissolv'd (by reason of the weak and oblique Light of the Sun) grow to an immense Height, and every Way reflect the Sun's Rays, which are not strong enough to expel the Exhalations.

The incomparable Sir Isaac Newton believes also, *That the Tail of a Comet is nothing else but a very thin Vapour, which the Head, or Nucleus of the Comet, sends out, by reason of its Heat.*

When a Comet descends to its Perihelium, the great Quantity of Vapours which encompass'd it when it was in those Regions which were the most

most remote from the Sun, and therefore the coldest, being rarified by the Sun's Heat ascend; that is, go towards those Parts which are turn'd from the Sun: For all the Particles in a Medium extended far beyond Saturn, (namely the *Æthereal* Aura) gravitate towards the Sun. This Aura heated by the warm Particles of the Vapour, which mixes with it, becomes rarer, and therefore lighter, than the rest of the ambient Aura, and for that Reason, being driven by it, rises upwards, and takes with it the reflecting Particles of which the Tail is made up; much after the Manner that Smoak rises up a Chimney by the Impulse of the Air, which, being rarified by Heat, carries up the Particles of the Smoak which are entangled with it. But because beyond the Atmosphere of the Comet the *æthereal* Aura is very rare and next to nothing, or a Vacuum, therefore I shall attribute something to the Action of the Rays of the Sun, carrying along with it the Particles of the Atmosphere of a Comet, tho' *Kepler* is not of this Opinion; and altho' I believe, that (besides the Impetus acquir'd in the first Ascent of the Vapour) the chief and most effectual Cause of the Ascent of the Vapour, which makes the Tail, is the Rarity above describ'd of the *æthereal* Aura, by reason of the Particles which arise duly from the Comet in the Neighbourhood of the Sun, where the Aura about the Tail is denser and heavier. And when the Tail, lifted upwards, does, like the Nucleus, gravitate towards the Sun after the Manner of another Projectile in a Conick Section, and accompanies the Comet in distant Regions till it is dissipated by degrees.

Tho' the Tails of some Comets, being immensely extended, seem to require a great Quantity

Book V. of ASTRONOMY. 715

tity of Matter to produce them, Experience shews us into what a prodigious Heap of Smoak a small Piece of Wood or Pit-coal is converted, and from the foregoing Lemma, it is plain how small a Portion of our Air may be diffused into immense Spaces by its elastic Force. And tho' the Tails are not of so rare a Texture, yet that they are extremely rare, in respect of Bodies about us, is plain from hence, *viz.* that the least Stars shine thro' those immensely deep Tails notwithstanding they are enlighten'd by the Sun's Rays, when at the same time our Terrestrial Atmosphere, whose Depth is vastly less, makes even the brightest Stars insensible when it is enlighten'd: Neither are lucid Things seen more easily thro' a dense Body, when it is farthest remov'd from the Eye, than when it is nearest to it, but rather with more Difficulty, as daily Experience shews us.

COROLLARY I.

Since the Tail of a Comet is rais'd from its Head and the Atmosphere about it, by the Heat of the Sun, it follows that it increases as the Comet comes to its Perihelium, and again, that it is diminish'd in its Recess from the Sun; therefore it will be the greatest and brightest immediately after the Comet has sustain'd a great Degree of Heat in its Perihelium, and is just got off from the Neighbourhood of the Sun. But also the Atmosphere of the Comet (which supplies the Tail with most of its Matter) is diminished as the Tail encreases, and becomes narrower towards the Sun; and therefore *cæteris paribus*, (together with the Comet itself, the greater Part of which it appears to the naked Eye to make up) it appears least immediately after the Comet has pass'd the Perihelium, where sometimes it appears cover'd with a thick and black Smoak, the Attendant of a vehement Heat.

COROL-

COROLLARY 2.

Hence also it follows, that if the Tail of a Comet shou'd touch the Atmosphere of our Earth, (or if a Part of this Matter scatter'd and diffus'd about the Heavens shou'd fall into it) the Exhalations of it mix'd with our Atmosphere (one Fluid with another) may cause very sensible Changes in our Air, especially in Animals and Vegetables: For Vapours, as they call 'em, brought from strange and distant Regions, and excited by a very intense Heat, may be prejudicial to the Inhabitants or Products of the Earth; wherefore those things which have been observ'd by all Nations, and in all Ages, to follow the Apparition of Comets, may happen; and it is a thing unworthy a Philosopher to look upon them as false and ridiculous.

COROLLARY 3.

The Tail rising like a Vapour directly upwards from a Comet and its Atmosphere, that is towards those Parts turn'd from the Sun, will remain in the Plane of the Orbit of a Comet, (that is, a Plane drawn thro' the Sun and the Track of a Comet) because there is no Reason, why in the quiet *Æther* it shou'd be so disturb'd, as to be carried more on one Side than another.

Moreover, since the Vapour going from the Head joins two Motions, and afterwards continues to go on with that Motion, compounded of the Motion of a streight Ascent, not far from the Sun, and of the Motion of the Progression of the Head; the Tail won't be directly turn'd from the Sun, but a little declining from it towards those Parts left by the Comet; nay, the Tail will not be streight, but bent, being convex to-
wards

Book V. of ASTRONOMY. 717

wards the fore Part, and concave towards the back Part. And tho' the rising Vapour partakes of the Motion of the Head, yet by reason of some Resistance of the Æther, it will not go on so fast as the Head. And by how much higher the Vapour rises which makes up the Tail, that is, by how much longer the Tail is, by so much more will it be resisted by the Æther; whence the fore Side of the Tail, or the Side towards the Parts where the Comet is to go, will be convex, but the following one concave, and therefore the Tail of the Comet itself will not be precisely turn'd from the Sun, and even not streight, but bent. And this Deviation from the Sun's Opposition is so much the less, as the Right Line joining the Sun and a Comet is more oblique to the Track of a Comet: For if a Comet shou'd go directly towards, or from the Sun, the Tail wou'd be directly turn'd from the Sun and streight; whence it happens, that in the very Perihelium the Tail very much deflects from the Sun's Opposition, and is also very curve, because a Right Line joining the Sun and the Comet will then be perpendicular to its Orbit. But by reason of the said Resistance of the Æther, however small, the convex Part of the Tail, (that is, the Part that runs into the Æther) will be something thicker, and consequently will appear brighter and better terminated than the concave one, by which the Parts reflecting the Light, being as it were left, are diffus'd about the Æther. Moreover, this Deviation of the Tail, and Incurvation from the Sun's Opposition, will, *ceteris paribus*, be greater, for both the afore-said Causes, when the Comet's Orbit is most curve: For that the Tail might be constantly turn'd from the Sun in such a Case, its Extremity

mity shou'd describe a Circumference greater than, and exterior to, that describ'd by the Head, which it cou'd not do by the Motion which it receives from the Head, tho' there shou'd be no Resistance of the *Æther* to diminish it. Therefore the Extremity of the Tail which is the most remote from the Comet, because it does not describe that greater Circumference in so short a Time, as the Head does the lesser, (and that for the two Reasons that we mention'd) will in this Case be more curve. But as this happens to a Comet at its Perihelium, where the Tail, which before was lifted up, almost vanishes, and the Vapour, which now rises directly up, chiefly makes up the Tail; for that Reason the Curvity will not be so sensible.

COROLLARY 4.

The Figure of the Tail, which we have describ'd in the foregoing Corollary, is that into which the Tail is really extended from the Head of the Comet; notwithstanding which, if the Earth, and the Spectator together, be in the Plane of the Comet's Orbit, the Curvity of the Tail will not be perceiv'd at all, because it is all in that Plane: And either the whole Tail will disappear when the Head is between the Eye and the Tail, or if the Head be less and the Tail broader towards the other End, the Comet will appear Hairy, that is, encompass'd round with the Appearance of Hairs, (and either of these may happen when the Sun and the Comet are in Opposition) or lastly, the Tail will precisely appear to be in Opposition with the Sun. But if the Spectator be out of the Plane of the Comet's Orbit, the Curvity of the Tail will be perceivable; and will be so much

Book V. of ASTRONOMY. 719

much the more sensible, as a Right Line, drawn from the Earth to the Tail of the Comet, becomes more perpendicular, or nearer to a Perpendicular to the Orbit of the Comet (and consequently to the Orbit of the Tail) and all the Symptoms or Signs of the Curvature of the Tail, describ'd in the following Corollary, will become more sensible. And these are the settled and constant Changes of the Tail of a Comet: For, as to the uncertain and sudden ones which Authors mention, they seem to have had their Rise from the Vapours of our Atmosphere, which, by an accidental Refraction, seemingly alter'd the Figure of the Tail; or from the Motion of the Clouds darkening some Parts of the Tail; or from some neighbouring Stars, or a Heap of them; or the milky Way, sometimes taken for Part of the Tail.

PROPOSITION V.

MORE Comets are seen in the Hemisphere towards the Sun, than in that which is opposite to it. [Fig. III.]

Let S denote the Sun, $\mathcal{T}$ the Earth; and the Circle $DEFH$ describ'd about the Center $\mathcal{T}$, the Sphere of the Fix'd Stars, or rather that which bounds the Sight. Because Comets shine by the reflected Sight of the Sun, they will not be seen before they are so enlighten'd by the Sun, as to be able to affect our Eyes by that Light: And besides, as Comets are very conspicuous, upon Account of their Tails, and do not emit them till they are somewhat heated by the Sun; it is plain, that to have either the Comet, or its Tail, visible, it is necessary for the Comet to come within a defin'd Distance of the Sun. Let $ABCG$ be a Sphere concentric to the Sun,
de-

describ'd at the said Distance from it ; namely, such an one, that no Comet, for Want of being enlighten'd, shall appear, before it comes within the Surface of that Sphere ; having join'd S and T , produce the Line ST every Way, till it meets with the said Surface in A and C : Thro' T draw the Plane HE , to which AC is perpendicular, then it will divide the Sphere $DEFH$ into two Hemispheres, one of which, HFE , is towards the Sun, and HDE opposite to the Sun. All the Comets which are in the Segment of the Sphere BCG , will appear in the Hemisphere towards the Sun ; but all those which are in the Segment BAG will appear in the Hemisphere opposite to the Sun. But it is plain that the Segment BCG is greater than BAG , because the Center of the Sphere is in it ; therefore more Comets will be seen in the Hemisphere towards the Sun, than in that which is opposite to it. *Q. E. D.*

And more of them will be first seen, or discover'd in the Hemisphere towards the Sun, than in the other, because the Part BCG of the Surface of the Sphere is greater than the remaining Part BAG ; but they are suppos'd to be discover'd as soon as they touch that Surface.

SCHOLIUM.

By how much less the Sphere $ABCG$ is suppos'd, that is, by how much nearer a Comet must be to the Sun before it becomes visible, by so much does the Number of the Comets, seen in the Hemisphere towards the Sun, exceed the Number of those which appear in the opposite Hemisphere ; because the Segment BCG does so much exceed the Segment BAG . And for that Reason the Number of those which are first discover'd

Book V. of ASTRONOMY. 721

cover'd, is greater in the former than in the latter.

If $ABCG$ be suppos'd to be a little less than the Sphere of Jupiter, or if the Comets are not visible before they come nearer to the Sun than Jupiter; that is, if SA be suppos'd five times longer than ST (for the Distance of Jupiter from the Sun is more than five times the Distance of the Earth from it,) the Segment BCG will be always twice the Segment BAG , and therefore almost twice as many Comets will be seen in the Hemisphere towards the Sun, than in the opposite Hemisphere: But the Surface BCG is only once and a half the Surface BAG , and therefore half as many more will be first discover'd in that than in this. If the Sphere $ABCG$ be less, as for Example, if SA be once and a half the Distance of Mars from the Sun, four times more Comets (and a greater Number) will be seen in the Hemisphere towards the Sun, than in the opposite one; but the Number of those, which are first discover'd in the former, will be three times greater than the Number of those discover'd in the latter. We are to take Notice, that from this Number we must take those which cannot be seen, by reason of their too great Nearness to the Sun; and on the other Hand, those, which are not too near Sun, are better seen in the Hemisphere towards the Sun, than those which are in the opposite Hemisphere, because they are more strongly enlighten'd.

PROPOSITION VI.

TO make an Estimate of a Comet's Distance from the Sun, by comparing the Light of a Comet with the Light of a Planet.

Since a Comet has no diurnal Parallax, and

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no Observations depending upon it can make its Distance known, we may make an Estimate of it, by comparing the Light of a Comet with that of a Planet. And because (by *Prop. 57. B. III.*) the enlightening of the Comet (or the Strength of its Light) and different Distances from the Sun, are in a duplicate reciprocal Ratio of the said Distances, and therefore (for Reasons alledg'd in the said *Prop.*) by how much more remov'd the lucid Body is, by so much fewer Rays, coming from it, enter the Pupil of the Eye, and that in a duplicate Ratio of the encreas'd Distance, or in a duplicate Ratio of its diminish'd apparent Diameter; that is, the Brightness of equal lucid Bodies are in a duplicate Ratio of their apparent Diameters; therefore (in this less accurate Way of making an Estimate, by abstracting from the different real Magnitude of the Planet and Comet) the Light of a Comet is to the Light of a Planet in a Ratio made up of the duplicate Ratio of the apparent Diameter of a Comet, to the apparent Diameter of a Planet, and the duplicate Ratio of the Distance of a Planet from the Sun, to the Distance of a Comet from it: And therefore the Square of the Distance of a Comet from the Sun, is to the Square of the Distance of a Planet from it, in a Ratio compounded of the duplicate Ratio of the apparent Diameter of the Comet, to the apparent Diameter of the Planet, and a Ratio of the Light or Brightness of the Planet, to the Light of the Comet. Whence the Distance of a Comet from the Sun, is to the Distance of a Planet from it, in a Ratio compounded of the Ratio of the apparent Diameter of a Comet, to the apparent Diameter of the Planet, and a subduplicate Ratio of the Light of the Planet to the

Book V. of ASTRONOMY. 723

the Light of the Comet. But each compounding Ratio is given by Observation ; therefore the compound Ratio is given, namely, that between the Distances of a Comet and a given Planet from the Sun. And from *Prop. 45. B. III.* we have the Distance of a Planet from the Sun; and therefore we shall also have the Distance of a Comet from the Sun.

By reason of the thick Smoak and the Atmosphere itself of the Comet, which darkens its Nucleus, the Nucleus (or solid Body of the Comet which we mean here) does not appear so bright ; and therefore its Distance from the Sun, found after the above-mention'd Manner, must be diminish'd to become true.

SECTION II.

To determine, very nearly, the Way and apparent Place of a Comet.

HAVING premis'd some physical Considerations concerning Comets and their Tails, we shall go on to determine their Orbits by Observation, which is the chief Design of this 5th Book. And first we shall shew how, (by the Method of that penetrating Astronomer, Mr. *Cassini*, from some Observations made at the Beginning of the Appearance of any Comet) the Way which it shall afterwards go among the Fix'd Stars, and the Places which it will be in every Day, may nearly be determin'd: Not that we would have those be look'd upon as accurate, but that from thence it may be foretold in what Place a Comet must be look'd for at a given

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Time ;

Time; whether or no it will in a little Time be hid under the Sun's Rays, when it will be nearest to the Earth, and move swiftest, and other things referring to this.

PROPOSITION VII.

Knowing, by Observation, Two Places of a Comet seen from the Earth: To determine its Way among the Fix'd Stars; that is, its Node and Inclination to the Ecliptic. [Fig. IV.]

Let CE represent the Ecliptic, and let B and D be two Places of a Comet, found by Observation. Let a great Circle be suppos'd to be drawn thro' B and D , cutting the Ecliptic at A , which, by what has been shewn in *Prop. 10. B. I.* will be the Way of a Comet, whose Situation, in respect of the Ecliptic, is thus determin'd: Thro' B and D draw the Circles of Latitude, BC and DE , and also the great Circle DC . In the Spherical Triangle DCE , rectangular at E , you have given the Sides GE and ED , this being the Comet's Latitude when at D , and that the Difference of Longitude of the Places of the Comet B and D ; therefore the Side DC will be found. Again, in the Triangle DBC , you have all the Sides given; namely, DC lately found, and BC the Latitude of B , the Place observ'd, and BD the observ'd Distance of the Places; and therefore the Angle CBD will be known, and therefore its Complement CBA . Lastly, in the Triangle BAC you have, besides the Right Angle at C , the Side BC , and the Angle ABC ; therefore the Side CA , and the Angle at A will be known, the Angle being the Inclination of the Comet's Way to the Ecliptic, and the Side, the Interval of the Node, and the known Point C in the Ecliptic.

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PROPOSITION VIII. LEMMA.

Three Right Lines TA, TB, TD , being given in Position, drawn from the same Point and lying in the same Plane, from the Point A given in any one of them, (as in TA) to draw the Right Line AC , whose Parts AE, EC , intercepted between the said Right Lines, shall have a given Ratio, namely, that of R to S . [Fig. V.]

Let the Right Line AB be drawn any how from A , to meet with TB at the Point B ; in this Line (produc'd if there be Occasion) on the Part of it towards TD , take (by Prop. 12. El. 6.) BF , so that it may be to AB , as S to R ; thro' F draw FC parallel to BT meeting with TD at C : and you will have AC requir'd. For (by Prop. 2. El. 6.) AE will be to EC , as AB to BF , that is, (by Construction) as R to S .

SCHOLIUM.

The Position and Magnitude of the Right Line AC are easily defin'd, by Trigonometrical Calculation: For in the Triangle ATB , you have given the Side AT , the Angle ATB , and the Angle TAB , known by drawing the Line AB ; whence the Angle ABT , and the Side AB will be known; therefore BF , having a Ratio to the given Line AB is given, and for that Reason its parallel, and (consequently) equal CM . Again, in the Triangle TCM , knowing the Angle CTM , (given at first) and CMT equal to FBM , (that is, equal to the Complement of the before found ABT to two Right Angles) and the Side CM , you will also have CT . Lastly, in the Triangle ACT , having the Sides TA, CA , and the Angle between them, ATC , you will find the third Side CA , and the Triangle TAC and $TC A$; that

is, the Magnitude and Position of the Right Line *AC*.

PROPOSITION IX.

From three Observations made at the Beginning of the Appearance of a Comet, to determine the Place of the Perigæum, at what Time a Comet will be in it, and to find the Place of a Comet at a given Time. [Fig. VI.]

Let *T* represent the Earth, which here, where we go not to the greatest Accuracy, we shall consider as at Rest, not considering its Motion) *ABDG* a Circle among the Fix'd Stars, in which the Comet seems to go, determin'd by *Prop. 7.* in which the three observ'd Places of a Comet are, *A*, *B* and *D*. Let *R* be put for the Time between the Observations at *A* and *B*, and *S* for the Time between the Observations at *B* and *D*: Let the Right Lines *TA*, *TB* and *TD*, be suppos'd to be drawn, and, by the foregoing Lemma, let the Right Line *AO* be drawn thro' *A*, whose Parts *AE*, *EC*, intercepted between the two Right Lines *TA*, *TB*, and the two *TB*, *TD*, are as *R* to *S*: This will be nearly the Trajectory of the Comet, especially in some remarkable Distance of the Comet from the Perihelium. For in that Case the Way of the Comet is nearly strait, and the Comet goes equably thro' it; therefore the Spaces gone thro' are proportionable to the Times: But *AE* is to *EC*, as the Time in which the Comet is seen to move from *A* to *B* among the Fix'd Stars, to the Time in which it is gone from *B* to *D*; and that agrees to no Right Line drawn from *A* to the same Parts of *T*, except *AO*: This is therefore nearly the Comet's Trajectory.

You

Book V. of ASTRONOMY. 727

You have (by the foregoing Lem.) the Situation of the Right Line AO , and the Magnitude of AC in respect to AT ; that is, in the Triangle ACT , you have all the Angles and all the Sides given. Let TP be drawn from T , perpendicular to AO , the Right-lin'd Trajectory, and let it be produc'd till it meets with the Circle ABD at G . It is plain that the Comet, moving in the Trajectory AO , is in its Perigæum at P ; for the Right Line TP is the shortest that can be drawn from T to AO : And G will be the Place among the Fix'd Stars, which a Comet in its Perigæum will appear to be in. To find out which, in the Triangle APT , rectangular at P , you have the Side AT , and the Angle PAT ; from which you will know the Sides AP , TP , and the Angle ATP , the Complement of PAT to a Right Angle, and its Measure the Arc AG , and therefore the Place of the Perigæum in a great Circle between the Fix'd Stars, which the Comet describes, when the Point A is known by Observation. This Place G , of the Perigæum, is thus very easily reduc'd to the Ecliptic NKL : Thro' G draw the Circle of Latitude GL , meeting with the Ecliptic at L : And in the Triangle NLG , besides the Right Angle at L , you have the Angle N , (found by *Prop. 7.*) and NG , the Sum (or Difference) of the known Arcs NA , AG ; whence will be known NL , the Longitude of the Perigæum from the Node, and GL the Latitude of the Comet in the Perigæum. Moreover, knowing the Ratio of the Right Lines AC and AP , to the same AT , you will also know the Ratio between AC and AP : But because a Comet moves equably, the Ratio between the Times, in which the Comet moves thro' AC , AP , is the same with the Ratio of

AC to AP ; and the Time in which it has moved from A to C is known by Observation, being the same which is spent between the first Observation in A and the third in D ; and therefore the Time will be known in which a Comet does move really from A to P , and apparently from A to G ; that is, between a known Point of Time, and the Time in which the Comet will come to the Perigæum. Therefore the Time will be known in which a Comet will be in its Perigæum.

Besides, let the Interval between the given Time, for which the Place is sought, and the Time of any Observation (as for Example, of the first in A) be requir'd. Let the Line AQ be taken in the Line AO , so that it may be to AC , as the Interval of Time lately found, to the known Interval of the Observations of the Comet at A and D : Draw TQ and produce it till it meets with the Circle NAD in the Point V , thro' which we must suppose the Circle of Latitude VX to be drawn. It is plain from what has been said before, that the Comet is in the Point Q , of the Trajectory AO , at the given Time. In the Triangle QAT , from the given Sides AT , AQ , and the intercepted Angle TAQ , you will find the Angle ATQ , and therefore its Measure (namely, the Arc AV) will be known: But the Point A is known; therefore also the Point V , or the requir'd Place of the Comet in the apparent Way at the Time given. But the Situation of this Place, in respect to the Ecliptic, is found, by having already, in the Triangle NXV , rectangular at X , the Angle VNX , and Side NU , in finding NX , the Longitude of the Comet from the Node,

V. Book V. of ASTRONOMY. 729

Node, and *XU* the Latitude agreeable to a given Time.

Instead of the first and second Observations, any two Observations may be taken in this Practice, or any one of them with the Place and Time of the Perigæum above found. And this Way *Ephemeris's* for Comets may be made.

SCHOLIUM.

In a Matter of so little Accuracy, it will perhaps be sufficient to describe the Way of a Comet, at its first Appearance on the Surface of the Cœlestial Globe, and to mark its Places for every Midnight of an Observer's Habitation, by Protraction. For if, at the Beginning of the Appearance of a Comet, its Situation among the Fix'd Stars be observ'd, namely, by looking at it with a stretch'd Thread, and so knowing when it lies in a Right Line with two known Fix'd Stars, and at the same Time finding two other Fix'd Stars, with which also it lies in a Right Line, which may easily be done among such a Multitude of Fix'd Stars; a great Circle going through two (or more) Places observ'd, mark'd in the Globe (which the farther they are from one another, the more exact the Practice will be) will be the future Way of a Comet. But a great Circle going thro' them will easily be found, by raising or depressing one of the Poles, and at the same Time turning the Globe, till the Places mark'd are at the same Time found in the Horizon; for then the Circle, which the Horizon at that Time denotes on the Surface of the Globe, will be the Circle requir'd. Therefore its Intersections with the Ecliptic will be the Nodes of the Orbit of the Comet, and the Angle which the Ecliptic makes

makes with the Horizon (measur'd by the Altitude of the Nonagesime Degree) will be the Inclination of the Orbit of a Comet to the Ecliptic. Let a Circle be describ'd with its Diameter equal to that of the Globe, as for Example ABD , whose Center is T , in whose Circumference, take the Point A , representing the Place among the Fix'd Stars, in which the Comet was first observ'd: Let the Arcs AB , AD , be taken equal to the Distances of the Places of the Comet, mark'd on the Surface of the Globe, from the Place first observ'd, and let TB , TD be drawn: And (by *Prop. 8.*) let the Right Line AO be drawn in such a Manner thro' A , that AE may be to EC , as R to S , that is, (by Construction) as the Time between the first Observation and the second, to the Time between the second and the third. From T letting fall TP perpendicular to AO , and producing it (if Need requires) as far as the Circumference at G ; an Arc equal to AG being transferr'd from the Place first observ'd, to the Way of the Comet, describ'd above, on the Surface of the Globe, the Point G will shew the Place among the Fix'd Stars which the Comet will be in, in its Perigæum.

Moreover, if the Right Line AC be divided (either actually, or by Help of *Galilæus's* Compasses of Proportion) into so many equal Parts, as there are Hours (or Minutes, if the Scheme be large enough) between the first Observation in A , and the third in C ; each of those Parts will be the Way of a Comet, moving equably in AO , for the Space of an Hour, and twenty four of them will be the Way of it in the Space of a natural Day, and so on. Therefore the Place of a Comet among the Fix'd Stars, for

Book V. of ASTRONOMY. 731

for a given Time, is determin'd upon the Globe, if so many of the foresaid Parts be taken from A towards O , in the Right Line AO , as Hours are pass'd between the Time of the first Observation and the given Time; as for Example, AQ , and let the Right Line TQ , joining the Center T , and the End Q , of the assumed Parts, be produced to V , and let an Arc be taken equal to AV , from a Point answering to A in the Comet's Way; and one Extremity of it will shew the Place of the Comet among the Fix'd Stars, agreeable to the said Time. The Place of a Comet, for every Midnight, may after the same Manner be protracted upon the Surface of the Globe: Whence the Longitude, Latitude, and other Things belonging to a Comet, will (by the Operations taught in *B. II.*) be easily defin'd. These things, which are done on a Globe, may likewise be done in any Planisphere, in which the Fix'd Stars are drawn.

The Way of a Comet being first drawn, and the Places agreeable to different Times, will be corrected by the following Observations: For this Operation and Calculation is not to be expected to be without any Error.

PROPOSITION X.

TO define in what Time a Vapour, going out of a Comet, rises from the Head to the Extremity of the Tail. [Fig. VII.]

Let S denote the Sun; AB Part of the Trajectory of a Comet, in which let N be the Nucleus of a Comet moving from A towards B ; and C the Extremity of the Tail. Now since (by *Corol. 3. Prop. 4.*) the Tail NC is not exactly turn'd from the Sun, that is, NC does not lie in *directum* with SN , but turning towards A , the
Parts

732 *The* ELEMENTS Book V.

Parts which the Comet has left, and whose Inclination is known by Observation; let SE be drawn parallel to NC , cutting the Trajectory at the Point D behind the Comet. The Vapour, which now makes the Extremity C of the Tail, began to rise from the Nucleus, when the Comet was in the Point D ; because the Motion of a Vapour is compounded of a Motion directly from the Sun, and of the progressive Motion of the Comet impress'd on it from the Head, before its Separation. But by the foregoing *Prop.* will be defin'd (exactly enough for such an Enquiry as this, which is not very accurate) the Time that a Comet was at the Point D , before the Time propos'd of the Observation, as great as is requisite for a Comet to go thro' the Right Line DN , given in Magnitude. Therefore the Time is found, in which the Vapour rises from the Head to the End of the Tail.

PROPOSITION XI. LEMMA.

FOUR Right Lines given in Position, lying in the same Plane, to draw a fifth which shall be cut by those four, in three Parts, having a given Ratio and a given Order among themselves. [Fig. VIII.]

Let there be four Right Lines given in Position, namely, AE , AC , BC , and BD produc'd *in infinitum*, and three Quantities given, R , S , and V ; a Right Line is to be drawn, cutting the four first given Lines at M , O , N and K , so that MO may be to ON , as R to S , and ON to NK as S to V .

Let CG be taken, so that it may be to CE as S and V together to R ; take also FC to CD , as R and S together to V : Draw the Right Lines AG , BF , and produce them till they meet; thro' their Concurrence H , draw HK , parallel to DC ,
and

Book V. of ASTRONOMY. 733

and HM parallel to BC , with respective Right Lines, concurring as in the Scheme: Draw MK , and it will be the Line requir'd.

For by reason of the Parallels HM and TN , KN is to NM , as KT to TH . But because HK is parallel to FD , KT is to TH as DC to CF , that is, by Construction, as V to R and S together; and therefore KN is to NM , as V to R and S together. Again, by reason of the Parallels HK and OX , MO is to OK as MX to XH : But because HM is parallel to EG , MX is to XH as EC to CG , that is, by Construction, as R , to S and V together; and therefore MO is to OK as R to S and V together. But it has already been shewn, that NK is to MN , as V to R and S together. Therefore, by Division, the three Right Lines MO , ON and NK have the same Ratio to one another, as the three Quantities R , S and V . Q. E. F.

We do not say any thing here of the Limitations and different Cases of this Problem, as being manifest enough to any one that considers this Matter.

This Construction is the Invention of the famous Geometrician Sir *Christopher Wren*.

SCHOLIUM.

The Position of the Right Line MK and of the Points M , O , N and K , of the Distance from A , B , C , D or E , are thus easily determin'd by Calculation. All the Intersections of the four Right Lines first given, AE , AC , BC , and BD , as the Points A , B , C , D and E , are given (by 25 *Dat.* and the Angles at them, by the Inverse of 29 *Dat.* wherefore the Right Lines AC , BC , CB , CE and BD (by 26 *Dat.*) are given in Magnitude. But (by 2 *Dat.*)

CF

734 *The* ELEMENTS Book V.

CF and CG are also given, having a given Ratio to CD and CE respectively. In the Triangle BCF the Sides CB , CF , and the Angle between them, BCF being given, you have the Side BF and the Angle BFC , and consequently the Angle on the other Side of it AFH . Likewise, in the Triangle ACG , from the given Sides AC and CG , with the Angle ACG , you will have the Angle CAG . Therefore in the Triangle AFH two Angles, AFH and FAH , which were before found, being given, with the Side between them AF , (the Sum or Difference of the known Right Lines AC and CF) the Side AH will be known. Lastly, in the Triangle AHM , having the Angle HAM , (the Sum or Difference of the known Angles CAE , HAC) and the Angle AMH , equal to the given one AEC , as also the Side AH , you will know the Sides HM and AM , the Distance of the Point M from the known Point A . You may, by the same Steps, find the Sides BK and HK in the Triangle BKH . Therefore in the Triangle MHK , knowing the Sides HM , HK , and the Angle between them MHK (equal to ECA before known) you will know the Angles HMK , HKM and the Base MK : But the Angles HMA , HMK being given, you will have their Sum or Difference AMK ; that is, the Position of the Right Line MK , AM being given in Position. After the same Manner AO , or BN , or BK , and the Angles which MK makes with them, are found by Trigonometry.

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PROPOSITION XII.

Four observ'd Places of a Comet being given, to determine its Trajectory, if rectilineal. [Fig. IX.]

Let S represent the Sun; $ABCD$ the *Magnus Orbis*; and let the Points A , B , C and D be known Places of the Earth at the Times of the Observations; AE , BF , CG and DH , four Longitudes of an observ'd Comet. By *Prop. II.* draw the Right Line EH in such Manner that its Part EF , intercepted between AE and BF , may be to FG , its Part intercepted between BF and CG , as the Time between the first and second Observation, to the Time between the second and third; and that the said FG may be to GH , its Part intercepted between CG and DH , as the Time between the second and third Observation, to the Time between the third and fourth; and EH thus describ'd will be the Mark of the Orbit of a Comet, in the Plane of the Ecliptic; that is, if thro' a Right Line, which is the Trajectory of a Comet, a Plane be suppos'd to be drawn perpendicular to the Ecliptic, this Right Line EH will be the common Section of those Planes. For since, according to the present Hypothesis, a Comet moves equably in a Right Line, if from the Points of that Right Line, in which the Comet was at the Time of the four Observations, Perpendiculars be let fall to the Plane of the Ecliptic, they will fall on the Right Lines AE , BF , CG , and DH respectively, because these denote the Places of the Comet in the Ecliptic; and these four Points will be in the same Right Line; namely, the common Section of a Plane thro' the Trajectory perpendicular to the Plane of the Ecliptic, with the said Plane of the Ecliptic; and

and besides (by *Prop. 2. El. 6.*) the Distances between those Points will be as the Distances of the Points in which the Comet was found in its proper Trajectory, and therefore, as the Times between the Observations: But EH is the only Right Line, whose Parts lying between the Right Lines AE , BF , CG and DH are as the Times between the four Observations respectively; and therefore the Right Line EH is the Mark of the Orbit of a Comet in the Plane of the Ecliptic.

But to compare the Distance of a given Point of this Right Line with the mean Distance of the Earth from the Sun, for a Time given, we must work thus: In the Triangle ASB you have (by *Prop. 18. B. III.*) the Sides SA , SB , the Distances of the Earth from the Sun for given Times, and the intercepted Angle ASB , which is the Difference of the Places of the Earth, seen from the Sun; therefore you will have the Angles SAB , SBA , and the Right Line AB the Chord of the Arc gone thro' by the Earth in that Time; that is, the Ratio of that Chord, to the Semidiameter of the *Magnus Orbis*. Again, in the Triangle KAB all the Angles are given, because you have the two KAB , KBA (namely, this last being the Sum or Difference of the known Angles SBA , SBK , and the first likewise the Sum or Difference of the known Angles SAB , SAK ;) wherefore you have the Ratio of the Sides AK and AB : But the Ratio of SA and AB was given before; therefore the Ratio of SA to KA will be known. Just after the same Manner, from two Places of the Earth, A and D , the Ratio between SA and AO will be found; wherefore the Ratio between SA and KO will be known, being the Difference between AO and KO : But it has been shewn in the foregoing

Book V. of ASTRONOMY. 737

going Proposition, how to find the Ratio between KO and KE , wherefore the Ratio between SA and KE will be known, and therefore also the Ratio between SA and AE . Moreover, in the foregoing Proposition, the Angle AEH also is found; therefore AE is found, being the Distance (in a Right Line given in Position) of the Mark of a Comet's Trajectory from the Earth, express'd in Parts of the Semidiameter of the *Magnus Orbis*, and its Inclination to the said Right Line. After the same Manner BF , or CG , or DH , may be compar'd with the Semidiameter of the *Magnus Orbis*, and the Angles at F , G and H , will be found.

To determine this Trajectory, from E to the Plane of the Ecliptic, (which is the same with the Plane of this Scheme) let EM be suppos'd to be erected equal to the Tangent of the Angle of the Comet's Latitude in the first Observation, supposing AE the Radius; and M will be the Place of the Comet in the Mundane Space for the first Observation, because the Position of the Right Line AE is its Longitude, and the Angle MAE its Latitude. Likewise at H let the Right Line HL be rais'd perpendicular to the Plane of the Ecliptic, and equal to the Tangent of the Latitude observ'd the fourth Time to the Radius DH ; and L will be the Place of a Comet in the Mundane Space for the fourth Time: And therefore ML , being drawn, is the requir'd rectilineal Trajectory of a Comet, in which a Perpendicular, let fall from S , will shew the Perihelium. It is plain, that the Distance of the Point M from A , or the Right Line AM , is to the Right Line AE as the Secant of the Latitude in the first Observation to the Radius, and

S f f

that

738 *The* ELEMENTS Book V.

that DL is to DH , as the Secant of the Latitude observ'd the fourth Time to the Radius; but LM will be found in the Triangle $LP M$, LP being drawn thro' L , parallel to HE , meeting with ME at P : For besides the Right Angle at P , the Sides LP , PM , are given, the first of them equal to the before-known Line HE , and the last equal to the Difference of the known ones ME , HL ; wherefore LM will be known, being a Right Line equably gone through by a Comet during a known Time, namely, whilst the Earth moves from A to D .

Moreover, for finding the Node, the Time when the Comet will be in the Node, the Inclination of the Comet's Way to the Ecliptic, and other Things of this Nature, let ML be produc'd till it meets with EH , as for Example at N ; then NH will be to HL , as HE to PM , and therefore known; also LN will be to LH , as LM to MP , and therefore also given: Let a Time be taken, which is to the Time between the first and fourth Observation, as NL to LM , and it will be the Time between the fourth Observation, and the Moment when the Comet will be in the Node; for in an equable Motion (such as we suppose this of a Comet) the Times are as the Spaces gone thro': But the Place of the Comet in the Node will be given: For since the Point N is given, and (by reason of the Time known between the fourth Observation, and the Moment when the Comet is in the Node) the Place also of the Earth is given for that Moment; you will have (by 26 *Dat.*) the Position of a Right Line joining those Points, that is, the Longitude of the Comet, when it is in the Node. Moreover, because you have the Distance of the Node from the Point H (the Place of the Comet
at

Book V. of ASTRONOMY. 739

at the fourth Observation) and the Latitude of the Comet at the said Point H , (namely the two Sides about the Right Angle, in a rectangular spherical Triangle;) you will have the Inclination of the Hypotenuse, or of the Comet's Way to the Ecliptic.

PROPOSITION XIII.

T*Hings being as before, to find the Place of a Comet seen from the Earth for a given Time,*
[Fig. X.]

Let the Mark or Section of the Orbit of a Comet be, by the foregoing *Prop.* determin'd on the Plane of the Ecliptic, namely $EHN R$, and MLN the Orbit itself in the Mundane Space, the Points N, H, E, M and L , denoting the same things as before: Let ER be taken to EH as the Space between the first Observation and the Time given, for which the Comet's Place is requir'd, to the Time between the first and fourth Observation: Moreover, find (by *Prop.* 18. B.III.) the Place of the Earth for the Time propos'd, viz. the Point Q . The Position of the Right Line QR , whose two Extremities Q and R are known, (being given by 26 *Dat.*) gives the requir'd Longitude of the Comet. Moreover, let RV be taken to RN , as ME to EN , and the Latitude of the Comet will be at the given Time an Angle, whose Radius is QR , and the Tangent the Right Line RV .

After this Manner may you make Ephemeris's of the Motion of a Comet, either Graphically, or by Calculation.

SCHOLIUM.

After the same Manner you may find the Distance of a Comet from the Earth, either in its Perigæum, or for a Time given, as well curta-

ted, as in its Trajectory, express'd in Parts of the Diameter of the *Magnus Orbis*; as also the Angle which the Trajectory of a Comet makes with a Right Line joining a Comet and the Earth, and which the Mark or Section of the Orbit of a Comet in the Plane of the Ecliptic contains with a Right Line drawn from the Earth, to the Place of the Comet in the Ecliptic.

COROLLARY.

Hence the true Magnitude, as well of the Nucleus of a Comet, as of the Peruke or Hairiness about it, like an Atmosphere, will be determin'd; for when once you know the Distance of a Comet, and the apparent Diameter of the Nucleus, and of the Atmosphere about the Nucleus, by Observation, their Magnitude will be known, after the same Manner that the Magnitude of the Planets is known from the like *Data* in *Prop.* 46. *B.* III. For by knowing the Places of the Sun and of a Comet, the Angle which the Tail makes with a Right Line, joining the Earth and a Comet, will be known, and the Angle at the Earth, which the Tail of a Comet subtends, is known by Observation; from knowing of which Things, and the given Distance of a Comet from the Earth (namely, the Side adjacent to the said Angles) the Length of the Tail will be known, and its Thickness will be found the same Way as that of a Comet.

SECTION.

SECTION III.

To determine the true Trajectory of a Comet.

IN some of the foregoing Propositions we have shewn how to find such a Trajectory of a Comet, as does not much disagree with the Phenomena, not only because such a Trajectory will suffice for nearly determining the Places of several Comets, and because a pretty many of the chief Philosophers did believe that a Comet was really carried in a Right Line; but also because by that Means we have laid a Foundation for finding the true Parabolical or Ecliptical Trajectory to be describ'd from Observations, which we shall do in the following Pages, after we have premised some Propositions concerning the Motion of a Comet in a Conic Section, for the better understanding of it, and laid down necessary Lemmata.

PROPOSITION XIV. LEMMA.

IF the Right Line GAB touches the Parabola VA, whose Vertex is V and Axis VD, in the Point A; and from F the Focus of the Parabola FC be let fall perpendicularly on the said Line, it will be a Mean Proportional between the Right Lines FA and FV. [Fig. XI.]

Let the Right Line touching the Parabola, and the Axis of the Parabola, be produc'd till they meet together, as for Example at B: Thro' A let AE be drawn parallel, and AD perpendicular to the Axis; and BV (by Prop. 35. B. I. El. Conic.) will be equal to VD, and (by the well-known catoptrical Property of the Parabola) the Angle

Sff 3

GAE

742 *The* ELEMENTS Book V.

GAE is equal to the Angle FAB ; But GAE is equal to ABF ; whence FAB is equal to FBA , and therefore FB is equal to FA , and AC equal to CB ; therefore (by *Prop. 2. El. 6.*) VC , being drawn, is parallel to AD , that is, perpendicular to BF . Whence (by *Prop. 8. El. 6.*) CF is a mean Proportional between BF and VF : But AF has been shewn equal to BF , and therefore CF is a mean Proportional between FA and FV . Q. E. D.

C O R O L L A R Y 1.

Hence it follows, that in the same Parabola, FC is in a subduplicate Ratio of FA , or that this FC is to any other FC in a subduplicate Ratio of this FA to another FA , agreeable to another FC ; or since VF is constant and unchangeable, FC^2 will be as FA itself, and therefore the Right Line FC itself, in a subduplicate Ratio of the Right Line FA .

C O R O L L A R Y 2.

If in FV produc'd be taken VH equal to VF , and the infinite Line HK be drawn parallel to VC , on which from any Point A of the Parabola AK be let fall perpendicular, it will be equal to FA , the Distance of the Point A from the Focus. And conversly if AK , let fall perpendicularly from A , on the Right Line HK , be equal to the Right Line AF ; the Point A will be in the Parabola above describ'd, whose Focus is F , and Vertex V .

The First appears from hence: That BV is equal to VD (by *Prop. 33. B. I. Conic.*) and HV , by Construction, is equal to VF , and therefore BF is equal to HD : But it has been before shewn, that AF is equal to BF , and (by *Prop. 34. El. 1.*) KA is equal to HD ; whence KA and FA are equal.

But

Book V. of ASTRONOMY. 743

But the second from hence: By Supposition AK is equal to AF , wherefore the Point A is in a Right Line perpendicular to FK (F, K , being join'd) rais'd from its middle Point. Now, if a Parabola, whose Vertex is V and Focus F , does not go thro' A , let it go thro' any other Point of the Right Line KA , (or produc'd, if Occasion require) as for *Example* a : Because (by what has been already shewn) Fa is equal to Ka ; whence the Point a is in the foresaid Perpendicular to FK , rais'd from its middle Point. Therefore the said Perpendicular will meet with the two Points A and a of the Right Line KA , which is absurd; and therefore a Parabola, whose Focus is F and Vertex V , will go thro' the Point A , if AK and AF be equal.

PROPOSITION XV.

If a Body moves in a Parabola, and at the same time be urg'd by Forces tending to the Focus of the Parabola, which Forces are reciprocally proportionable to the Square of the Distance from the Focus, its Velocity, in any Point of the Parabola, is reciprocally in a subduplicate Ratio of the Distance from the Focus of the Parabola. [Fig. XII.]

Let DC be the Parabolical Trajectory of a Body, whose Focus is S ; it is plain (from *Corol. 2. Prop. 41. B. I.*) that its Velocity in the Point C is reciprocally as SE , drawn perpendicularly from the Focus S , on the Right Line CE , which touches the Parabola at C , or that the Velocity of the Body in C , is to its Velocity in c , as Se to SE : But (from *Corol. 1. of the foregoing Prop.*) Se is to SE in a subduplicate Ratio of the Right Line Sc to the Right Line SC ; and therefore the Velocity of the Body in C , is to its Velocity in c , in a subduplicate Ratio of the Right Line Sc to

744 *The* ELEMENTS Book V.

the Right Line *SC*; that is, the Velocity of the Body is inversly in a subduplicate Ratio of its Distance from the Focus. *Q. E. D.*

PROPOSITION XVI.

THE same Things being suppos'd as before, the Velocity of a Body in any Point of a Parabola is to the Velocity of a Body revolving in a Circle, about the same Center of Forces describ'd at the same Distance, as the Diameter of a Square to its Side.

For (by *Cor. 1. Prop. 41. B. I.*) the Velocity of a Body moving in a Parabola at the least Distance from the Focus (that is, in the Vertex of the Parabola) is, to the Velocity of a Body moving in a Circle at the same Distance about the same Center of Forces, in a subduplicate Ratio of the principal *Latus rectum* of the Parabola, to twice the Distance of the Vertex from the Focus: But the principal *Latus rectum* of the Parabola is equal to four Times the Distance between the Focus of the Parabola and its Vertex, and therefore the Velocity of a Body moving in a Parabola after the Manner mention'd, whilst it is in its Vertex, is to the Velocity of a Body moving in a Circle about the same Center of Forces at the same Distance, in a subduplicate Ratio of Four to Two, or of Two to One; that is, in the same Ratio which the Diameter of a Square has to its Side. Again, (by the foregoing *Prop.*) the Velocity of a Body in any other Point, taken at Pleasure, is to its Velocity in the Vertex in a subduplicate Ratio of the Distance of the Vertex from the Focus, to the Distance of the said Point taken at Pleasure from it: And (by *Prop. 27. B. I.*) if Bodies, which are urg'd by Forces reciprocally proportional to the Square of the Distance from the Center, describe about a given Center Circles,

Book V. of ASTRONOMY. 745

Circles, at the same Distances as the forementioned Points of the Parabola, the Velocity of a Body, revolving in a Circle passing thro' the said Point, will be to the Velocity of a Body in a Circle passing thro' the Vertex of the Parabola in a subduplicate Ratio of the Distance of the Vertex from the Focus, to the Distance of this other assum'd Point from it; that is, in the same Ratio that the Velocities of a Body, carried in a Parabola at the same Distances, have been shewn to be to each other. And therefore by Permutation the Velocity of a Body moving in a Parabola at a Point taken at Pleasure is to the Velocity of a Body moving in a Circle at the same Distance, as the Velocity of a Body moving in a Parabola at the Vertex, to the Velocity of it moving in a Circle at the Distance of the said Vertex; that is, as was before shewn, in a Ratio of the Diameter of the Square to its Side. Q. E. D.

PROPOSITION XVII.

THE Velocity of any Comet is to the Velocity of any Primary Planet, in a subduplicate Ratio twice the Distance of a Planet from the Sun to the Distance of a Comet from it.

It is plain from the First Book, that tho' the Way of a Comet about the Sun be Elliptical, yet, by Reason of the great Excentricity of such an Ellipsis, it may be look'd upon as a Parabola in whose Focus the Sun is; at least in that Part of it, which, being in our Planetary Region, is subject to Observation, it does not sensibly differ from a Parabola. And it is plain from the Third Book, that the Orbits of the Primary Planets, by reason of their small Excentricity, differ very little from Circles concentric to the Sun.

746 *The* ELEMENTS Book V,

Sun. Whence, from the foregoing *Prop.* it is plain, that the Velocity of a Comet is, to the Velocity of a Primary Planet revolving at the same Distance as the Comet, in a subduplicate Ratio of Two to One. And the Velocity of such a suppos'd Planet is, to the Velocity of any other Primary Planet revolving at any Distance (by *Prop.* 27. B. I.), in a subduplicate Ratio of the Distance of this said last real Primary Planet's Distance from the Sun, to the Distance of the suppos'd one, that is, to the Distance of the Comet itself from it : And therefore the Ratio of the Velocity of a Comet, to the Velocity of any Primary Planet, made up of the Two aforesaid, is equal to a subduplicate Ratio of twice the Distance of the Planet from the Sun to the Distance of the Comet from it. *Q. E. D.*

C O R O L L A R Y.

Hence the Velocity of a Comet may be compar'd with the Velocity of any Primary Planet (as for Example, of the Earth) carried about the Sun, if the Ratio be known between the Distance of a Comet from the Sun, and the mean Distance of the said Planet from it.

P R O P O S I T I O N XVIII.

LET *PAB* be any Parabola whose Focus is *S*, and any Arc of it *ACB*, whose Chord *AC* is bisected in the Point *M*, thro' which the Right Line *MV* is drawn parallel to the Axis of the Parabola meeting the Parabola in *V*, and produc'd to *G*, so that *MV* be the double of *VG*; let *GS* be drawn cutting *AB* in *Q*, and be produc'd to *E*, so that *SE* be the double of *SG*: If a Comet describes the Parabolic Arc *ACB* about the Sun *S* (as in the 35th *Prop.* B. I.) the Right Line *EC*, joining the Comet when it is any where in the said Arc (as at *C*) and the Point *E*, deter-

Book V. of ASTRONOMY. 747

E, determin'd before, will cut the Chord *AB* in *D*, in such Manner that *AD* will be nearly to *DB* as the Time in which a Comet describes the Arc *AC*, to the Time that it describes the Arc *CB*. [Fig. XIII.]

Draw the Right Lines *SB*, *SD*, *SC*, *GA* and *AV*; draw also *GD* meeting with the Parabola at *K*. Because *MV* is parallel to *SX*, it will (by *Prop. 46. B. I. El. Conic.*) cut all the Right Lines, parallel to *AB* terminated by the Parabola into two equal Parts, and therefore it will divide the parabolical Segment comprehended under the Curve *AVB*, and the Right Line *AB*, into two equal Parts, or half Segments *AHVM* and *BCVM*. Thro' *V* draw *VF* parallel to *AB*, (which by *Prop. 17. B. I. El. Conic.*) will touch the Parabola in *V*) meeting the Right Line *GD* at *F*, and let us suppose the Right Lines *SF*, *CF*, to be drawn.

Because, by Construction, *MV* is the double *VG*, and *ES* also the double of *SG*, *MV* will be to *VG* as *ES* to *SG*. But by reason of the Parallels *VF* and *MD*, *DF* is (by *Prop. 2. El. 6.*) to *FG* as *MV* to *VG*; and therefore *DF* is to *FG* as *ES* to *SG*. Therefore the Right Line *SF* will be parallel to the Right Line *EC* (by *Prop. 2. El. 6.* and therefore the Triangles *DSC*, *DFC* (by *Prop. 37. El. 1.*) will be equal: But the Triangle *DFC* is nearly equal to the trilineal Figure *DKC*, by reason of the Nearness of the Points *F* and *K*; therefore the Triangle *DSC* will be nearly equal to the trilineal Figure *DKC*. Moreover, the Triangle *MAG* (by *Prop. 1. El. 6.*) to the Triangle *MAV* is in a sesquialteral Ratio, because they have the same Vertex *A*, and the Base *GM* is to the Base *MV* as one and a half to one. But the Triangle *AVM* (by the 24th *Prop. Archim. of the Quadrature of the Parabola*) is subsesquiter-

tial of the Segment $AHVM$; therefore the Triangle AGM is eight times and a half the Semisegment $AHVM$, namely, in a Ratio compounded of a sesquialteral and subsesquitermial Ratio. Again, the Triangle MGD is nine times its similar one VGF , the Side MG being three times greater than the homologous Side VG ; therefore the Triangle MGD is eight times and a half the Trapezium $MVFD$: Therefore the Triangle AGM is to the Semisegment $AHVM$, as the Triangle MGD to the Trapezium $MVFD$; and again, the half Segment $AHVM$ is to the Trapezium $MVFD$, as the Triangle AGM to the Triangle MGD , that is, (by *Prop. 1. El. 6.*) as the Right Line AM to the Right Line MD . But the Trapezium $MVFD$ is nearly equal to a Portion of the Parabolical Segment $MVKD$ contain'd under the Right Lines KD , DM , MV and the small Part VK of the Parabolic Curve; and therefore the Semisegment $AHVM$ is nearly to the Portion $MVKD$, as the Right Line AM to the Right Line MD . Wherefore the half Segment $AHVM$ encreas'd by $MVKD$ (that is, the parabolical Area $AHVKDA$) is to the Semisegment when the said Part is taken from it (that is, the Part $BCKDB$) as AM and MD together to MB when MD is taken from it, or as AD to DB ; and the Triangle ASD is also to the Triangle BSD (by *Prop. 1. El. 6.*) as AD to DB . Therefore the Space $SAHVKDS$ is to the Space $SBCCKDS$ as AD to DB . Now, if to the first Space you add the trilineal Figure DKC , and from the Sum take away the Triangle DSC , equal to the foresaid trilineal Figure, by what has been already shewn, you will have the Space $SAHKCS$ equal to the said first Space. Likewise, if to the last Space be added the Triangle DSC ,
and

Book V. of ASTRONOMY. 749

and from the Sum be taken the Triangle DKC equal to it, you will have the Space $SBCS$ equal to the said last Space. Therefore the Right Line AD is to the Right Line DB , nearly as the Space $SAHVCS$ to the Space $SCBS$. But the Space $SAHVCS$ is the Area which a Comet, by a Radius drawn to the Sun, describes, whilst it goes thro' the parabolic Arc AC , and the Space $SBCS$ is the Area which the same Comet describes after the same Manner, as it goes thro' the Arc CB ; and these Areas (by *Prop. 11. B.I.*) are as the Times in which a Comet describes the Arcs AC and CB : Therefore the Right Line AD is to DB nearly as the Time in which a Comet describes the parabolic Arc AC to the Time that it describes the Arc BC . But the Point C (the Comet's Place) is taken at Pleasure: Therefore, universally the Right Line EC does so divide the Right Line AB at D , that AD is nearly to DB , as the Time in which a Comet describes the Arc AC , to the Time in which it describes the Arc CB . *Q. E. D.*

SCHOLIUM.

If the Point C coincides with the Vertex V of the parabolic Arc, then AD will be to DB accurately as the Time in which a Comet describes the Arc AC , to the Time in which it describes the Arc CB . For [*Fig. XIV.*] the Triangle ASD is (as it was before) to the Triangle BSD as AD to BD , and also the Semisegment $AHVM$, when the Trapezium $MVFD$ is taken from it, is to the Semisegment BVM , encreas'd by the said Trapezium $MVFD$, as AD to BD : But the Trapezium $MVFD$ is made up of the two Triangles DMV and DFV ; and therefore the Triangle ASD + the Semisegment $AHVM$ —, the Triangle DMV — the Triangle DFV is
to

750 *The ELEMENTS* Book V.

to the Triangle BSD + the Semisegment BV
 M + the Triangle DMV + the Triangle DFV ,
as AD to DB . Instead of the Triangle DFV ,
let us take the Triangle DSC equal to it;
and the Triangle ASD + the Semisegment
 $AHVM$ — the Triangle DMV — the Tri-
angle DSC will be to the Triangle BSD + the
Semisegment BVM + the Triangle DMV +
the Triangle DSC , as AD to DB ; that is, the
Area $ASCHA$ is to the Area $CSBC$, or (by
Prop. 11. B. 1.) the Time in which the Comet
describes the Arc AC , to the Time in which it
describes the Arc CB , as the Right Line AD to
the Right Line DB ; and that accurately: For
the curvilinear Space is not taken nearly equal
to the rectilinear one; as it was done in the
Proposition, when the Point C was different
from the Point V .

COROLLARY i.

Hence it follows, that the nearer the Point C
is to V , the Vertex of the Segment, so much more
accurately is the Chord AB divided by the Right
Line EC into the two Segments AD, BD , having
to one another the Ratio of the Times in which
the Comet goes thro' the Arcs AC, CB : For if
it coincides with V , this will be exactly so; if
not, then it is nearly so. But we must observe,
that the Error will be less, if C is distant from V
towards B , than if it shou'd be as far distant from
it towards A ; because the Portion VA of the
Parabola nearer to the principal Vertex X is more
curve, and declines sooner from the Right Line
 VF , a Tangent to it, than the Portion VB , re-
moter from X . And hence it happens, that if
three Moments of Time be requir'd at which the
Comet is in three Points of the Parabola (such as
are

Book V. of ASTRONOMY. 751

are A, C and B) so dispos'd, that the Right Line EC may cut the Chord AB in D , so that AD be to DB accurately, as the two Times between the three Moments, the said Times must be nearly equal to one another. For in such Parts of the parabolic Trajectory as we can observe, the Point D (where the Right Line joining the Vertex V and the Point E cuts the Chord AB) is not very far distant from M , the middle Point of the Chord. But that Interval, of Time where the Comet mov'd slowest, must be something greater than the other; for it is plain that when the Comet is in V , where the thing requir'd is exactly found, that the Right Line DB is greater than AD ; that is, the Time in which the Comet (then moving slowest by *Prop. 15.*) describes the Arc BC , is greater than the Time in which the said Comet, by a swifter Motion, describes the Arc AC . But it is safer to take Times that are not very unequal, that the Point D may rather decline towards B than A , upon Account of the Reason before alledg'd; namely, that the Error may be less, if C is distant from V towards B , than towards A .

COROLLARY 2. [Fig. XV.]

If V , the Vertex of a Segment, is not far distant from X , the principal Vertex, nor the Point C from V , the Right Line SC , drawn from S , the true Focus of the Parabola, will divide the Chord AB in γ , nearly in a Ratio of the Times in which the Comet describes the Arcs AC and CB . If SV is very great in respect of MV , $M\mathcal{Q}$ will be the triple of $\mathcal{Q}\gamma$: For in that Case the Right Lines $S\mathcal{Q}G$, $S\gamma V$ are to be look'd upon as parallel; therefore (by *Prop. 2. El. 6.*) $M\mathcal{Q}$ is to $\mathcal{Q}\gamma$, as MG to GV ; that is, by Construction, as 3 to 1.

COROL-

COROLLARY 3.

Things remaining as before, QE will be equal to $3 SQ + 3 MV$: For, by Construction, SE is equal to $2SG$, therefore GE is equal to $3SG$, that is, $GE = 3SQ + 3QG$: If therefore from each Side be taken QG , QE will be equal to $3SQ + 2QG$: But by reason of the Parallels $QG, \gamma V$, QG is to γV , as MG to MV , that is, as 3 to 2; wherefore $2QG = 3\gamma V$; and γV is equal to MV , because the Angles γMV and $M\gamma V$ are equal, by reason of SV and MV , which make equal Angles with a Right Line touching the Parabola at V , which (by Prop. 46. B. I. of Conics) is parallel to the Chord AB ; therefore $3MV = 2QG$; and therefore $QE = 3SQ + 3MV$. If the Position of the Right Line AB be a little chang'd, all the foregoing things will notwithstanding be nearly true.

PROPOSITION XIX. LEMMA.

IF, in the Parabola BAP , whose Focus is S , from the Chord drawn AB be cut off the Segment AVB , whose Vertex is V , and Diameter VM , meeting with the Chord in its middle Point M , and from M to AB be drawn perpendicularly MI meeting at I , with the Right Line LV parallel to AB , and the Parallelogram $VIMK$ be compleated; when the Points V and M come together, or when the Segment AVB is nascent, the Diagonal IK , produc'd, falls upon S . [Fig. XVI.]

For (by Prop. 17. B. I. Elem. Conic.) the Right Line LI touches the Parabola at V , whence the Angle SVL is equal to the Angle MVI : But because the Parallelogram $MKVI$ is rectangular, the Angle KIV is equal to the Angle MVI ; and therefore the Angle KIV is equal to the Angle SVL , therefore (by Prop. 28. El. 1.) IK is parallel to VS , and it is always so; therefore when

Book V. of ASTRONOMY. 753

when the Points I and V come together; that is, when M and V come together, the Right Line IK produc'd goes thro' the Focus S . Q. E. D.

COROLLARY [Fig. XVII.]

Hence it follows, that if from any Point C of the Parabola, near the Vertex of the Segment ACB , be drawn the Right Line CI parallel to the Chord AB meeting with the Right Line MI at I , and SI be drawn, which cuts the Chord in K , and the Parallelogram $IMKV$ be compleated; the Point V will be nearly the Vertex of the Segment, if the Distance of the Focus be immensely greater than either of the Right Lines CI , IM , or MK . For it is well known to Geometricians, that the Immensity of the Right Line SK , in respect of the Right Lines CI , IM , MK , brings the Matter to the same as if we make the Segment AB to vanish; in which Case the Proposition is plain from what has been already said.

PROPOSITION XX.

LET PAB be any Parabola whose Focus is S ; AVB any Arc of the Parabola, whose Chord is AB , and Vertex V , and Diameter MV : let the Right Line SV be drawn and produc'd to L and R ; so that VL be the third Part of MV , and SR a third Proportional to SV , SL ; if a Comet, at the same Time that it describes the Parabolic Arc AVB about the Sun S , shou'd go forward equably with the same Velocity which it has at a Distance from the Sun equal to SR , it wou'd describe a Line equal to the Chord AB . [Fig. XVIII.]

Let the Right Line SV meet with the Chord AB at Z : Now since the Angles FVM , GVZ , are equal; therefore also, their alternate Angles

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754 *The ELEMENTS Book V.*

on the two Parallels AB, GF , namely, ZMV , MZV will be equal, and the Right Lines VZ , VM , are equal. Moreover, (by *Corol 5. Prop. 41. B. I.*) if a Comet describing the Parabola PAB , at the same Time that it describes the Arc AVB , shou'd go on with the Velocity that it has in V , equably in the Tangent VF , the Arc, which it wou'd describe by a Radius drawn to the Sun, wou'd be equal to the parabolic Area $AVBSA$: And as SV is inclin'd to the Tangent VF in the same Angle that SZ is inclin'd to AB , the Space contain'd under the Length in the Tangent describ'd (as has been said) and SV wou'd be to the Space contain'd under the Right Line AB and SZ , as the parabolic Area $AVBSA$ to the Triangle ASB ; that is, as the Triangle ASB + the parabolical Segment AVB to the Triangle ASB ; that is, as the Triangle ASB + $\frac{2}{3}$ of the Parallelogram $AGFB$ to the Triangle ASB ; that is, as $AB \times \frac{1}{2} ZS \times AB \times \frac{2}{3} MV$ to $AB + \frac{2}{3} ZS$; that is (dividing by AB) as $\frac{1}{2} ZS + \frac{2}{3} MV$ to $\frac{1}{2} ZS$, or as $ZS + \frac{4}{3} MV$ to ZS . But VL is made equal to $\frac{4}{3} MV$; and it has before been shewn that ZV is equal to MV ; therefore ZL is equal to $\frac{4}{3} MV$. And therefore the Space contain'd under the Length in the Tangent describ'd, and SV wou'd be to the Space contain'd by AB and ZS , as $SZ + ZL$ to SZ ; that is, SL to SZ . And by applying the Antecedents of this Analogy to SV , and the Consequents to SZ , the Length in the Tangent describ'd will be to the Chord AB , as $\frac{SL}{SV}$ to $\frac{SZ}{SZ}$ or Unity; that is as SL to SV . Moreover, (by *Prop. 15.*) the Velocity of a Comet describing a Parabola, in the Height or Distance from the Sun SV , is to the Velocity of it in the Height or Altitude SR ,

Book V. of ASTRONOMY. 755

in a subduplicate Ratio of SR to SV ; that is, as SL to SV : And the Lengths gone thro' in the same Time, by moving Bodies carried equally, are as the Velocities; therefore the Length in the described Tangent, is to the Length described in an equal Time (namely, the Time in which the Comet goes thro' the parabolic Arc AVB) by the Comet equably moving on, with the Velocity that it has in the Altitude SR , as SL to SV . Since therefore the said Length describ'd along the Tangent has the same Ratio, (namely, that of SL to SV) to the Chord AB , as it has to the Length which the Comet wou'd go thro', moving with the Velocity that it has at the Altitude SR , during a Time equal to that in which the same Comet describes the Arc AVB ; these Lengths will be equal to one another, namely, the Chord AB , and the Length gone thro' by a Comet with the same Velocity that it has at the Altitude SR , and in the same Time that the same Comet describes the Arc AVB . Q. E. D.

COROLLARY.

If SV be very great in respect of VL , the three Geometrical Proportionals SV , SL and SR , will be also nearly Arithmetically proportional; that is, LR will be equal to VL , or to the third Part of MV ; therefore VR will be nearly equal to half MV . And therefore a Comet equably mov'd with the Velocity that it has in an Altitude or Distance from the Sun equal to $SV + \frac{1}{2} MV$, and in the same Time that it wou'd describe the parabolic Arc AVB , wou'd go thro' a Length equal to AB , the Chord of the said Arc.

PROPOSITION XXI.

IN the same Figure as before [*viz.* Fig. XVIII.] if a Comet from Rest shou'd be let fall from the Height LS toward the Sun S , and afterward shou'd be urg'd by the centripetal Force which it has at L neither encreas'd nor diminish'd; such a Comet, in half the Time that it wou'd describe the parabolick Arc AVB , would descend along a Right Line equal to half the Length of the Right Line MV .

By the foregoing Proposition in the Time that the Comet describes the parabolic Arc AVB , if it should move equably with the Velocity that it has in the Parabola, at an Altitude or Distance from the Sun equal to SR , it wou'd describe a Length equal to the Chord AB : But the Velocity of a Comet in a Parabola at the Altitude SR (by *Prop.* 16.) is to the Velocity of a Body going round in a Circle at the same Distance from the Sun, in the Circumference of which Circle it is retain'd by the Force of its Gravity, in a subduplicate Ratio of Two to One; therefore the Arc, which a Body, carried in a Circle whose Semidiameter is SR , goes thro' in the Time that a Comet describes the parabolic Arc AVB , is to AB the Chord of a parabolic Arc, as 1 to $\sqrt{2}$, and therefore equal

to $\frac{AB}{\sqrt{2}}$ but (by *Corol.* 1. *Prop.* 25. B. 1.) any Bo-

dy let fall from Rest towards the Sun, the same centripetal Force always acting (as is here suppos'd) in half the Time that it goes thro' the Arc of a Circle describ'd with that Distance from the Sun from whence it had fallen, as a Radius, it falls thro' a Space equal to the Square of half the said circular Arc, (that is to the Square of

Book V. of ASTRONOMY. 757

of half $\frac{AB}{\sqrt{2}}$ applied to the Diameter of the

Circle or $2SR$; that is, to the Square of $\frac{AM}{\sqrt{2}}$

applied to $2SR$, or $\frac{AM^2}{4SR}$. Therefore the

Space thro' which the Body, let fall from Rest from the Height RS , goes thro' in its Fall, in half the Time that a Comet will describe the

parabolic Arc AVB , is equal to $\frac{AM^2}{4SR}$. But (by

Prop. 42. B.I.) the centripetal Force, or the accelerating Gravity towards the Sun at the Distance SL , is to the accelerating Gravity towards it at the Distance SR , as SR^2 to SL^2 ; that is, by reason of the Proportionals SR, SL, SV , as SR to SV : And therefore since a Comet depriv'd of all Motion, and let fall from R , in half the Time that it wou'd describe the parabolic Arc AVB falls towards the Sun, tho' a Space equal to $\frac{AM^2}{4SR}$;

it will in the same Time, being after the same Manner let fall from L towards the Sun, go thro' a Space which is to $\frac{AM^2}{4SR}$ as SR to SV ;

that is, a Space equal to $\frac{AM^2}{4SV}$. Moreover, from

the Nature of a Parabola, $4SV$ is the *Latus rectum* of the Diameter VM ; that is, the Rectangle contain'd under $4SV$ and VM is equal to the

Square of A , therefore $\frac{AM^2}{4SV}$ is equal to the

Right Line VM : Therefore a Comet depriv'd of all Motion, and let fall from the Height LS

758 *The ELEMENTS Book V.*

towards the Sun S , if it be urg'd by an accelerating Force, or uniform Gravity, equal to that by which it was urg'd at the said Altitude SL , in half the Time that it wou'd describe the parabolic Arc AVB , will fall along a Right Line equal to the Right Line VM . Q. E. D.

PROPOSITION XXII. LEMMA.

IE from the Point S , which is the Focus of the Parabola AVB , and the Center of the Circle $T\tau\tau$, be drawn the equal Right Lines SV and St , and there be also drawn the Right Lines AB , $T\tau$, Chords of the Arcs whose Vertices are V and t , cutting the Right Lines SV , St , in the Points D and N , so that VD and tN may be equal, and very small, in respect of the Right Lines SV and St ; the Chord AB will be to the Diameter of the Square, whose Side is the Chord $T\tau$, in a subduplicate Ratio of the Right Line St to the Right Line SD . [Fig. XIX.]

Thro' V draw VM parallel to the Axis of the Parabola, meeting with the Chord at M , and which (by Prop. 46. B. I. Conic.) will be the Diameter of the Segment AVB , whose Parameter is the Quadruple of the Right Line SV ; therefore $AM^2 = 4SV \times VM$. And since (by Prop. 35. El. 3.) $TN^2 = tS + SN \times tN$, equal to $2St \times tN$ because $St = SN$; then $AM^2 : 4SV \times VM :: TN^2 : 2St \times tN$, therefore (by taking the Quadruples of the Antecedents, and by Permutation) the Analogy will be this, $AB^2 : T\tau^2 :: 4SV \times VM : 2St \times tN$. But (by what has been shewn in the Demonstration of Prop. 20.) $VM = VD$, and by Supposition $VD = tN$; therefore, by applying the third Term of the foregoing Analogy to VM , and the fourth to tN , $AB^2 : T\tau^2 :: 4SV : 2St$; that is, (because SV and St are equal by Supposition) $AB^2 : T\tau^2 :: 2St : SV$; that is, as $2ST$ to SV or SD : and

Book V. of ASTRONOMY. 759

and by doubling the Consequents, $AB^2 : 2 \times T\tau^2 :: 2 St : 2 SD$; that is, as DS to St : Therefore AB is to $\sqrt{2 \times T\tau}$ in a subduplicate Ratio of St to SD . Q. E. D.

PROPOSITION XXIII. LEMMA.

IF about the Focus S be drawn any two Parabola's, ACB , EGF , in which the Chords AB and EF being drawn, cut off Segments whose Vertices are C and G , in such Manner that the Right Lines SC and SG cutting the foresaid Chords in D and H , CD is equal to GH ; I say, that AB is to EF in a subduplicate Ratio of the Right Line SC to the Right Line SG . [Fig. XX.]

Thro' C draw CM the Diameter of the Segment ACB , and thro' G the Right Line GK , the Diameter of the Segment EGF , meeting with the respective Chords at M and K . The Parameters of those Diameters are equal to the Quadruples of SC , SG , shewn as before; and (by Prop. 49. B. I. Conic.) AM^2 will be $= 4SC \times CD$ and $EK^2 = 4SG \times GH$: Whence $AM^2 =$ is to $EK^2 = 4SC \times CD$ to $4SG \times GH$, that is, (by reason of the Equability of CD and GH) as SC to SG ; and therefore AM is to EK in a subduplicate Ratio of SC to SG . Whence AB (the double of AM) to EF (the double of EK) is in a subduplicate Ratio of SC to SG .

PROPOSITION XXIV.

TO describe a Parabola about a given Focus, in such Manner that it may pass thro' two given Points. [Fig. XXI.]

Let S be the given Focus, and AB , the two given Points thro' which the Parabola must pass: About the Centers A and B , and with the Distances AS and BS draw two Circles, and (by the Construction of the 118th Prop. of the 7th Book of Pappus's Mathemat. Collect.) draw the Right

Line CD touching each of them, on which from the Point S let fall the Perpendicular SE , which must be bisected at V . With the Vertex V , and the Axis VSX , and a *Latus rectum* equal to the Quadruple of the Right Line VS (by *Prop. 52. B. I. Conic.*) draw a Parabola, and it will be the Parabola requir'd.

That S is the Axis of the describ'd Parabola is plain from hence, that in the Axis of the Parabola, it is distant from the Vertex of it the fourth Part of the *Latus rectum*; which is the well-known Property of the Focus of a Parabola. And that it goes thro' the Points A and B , is prov'd thus: The Axis VX of the Parabola is produc'd beyond the Vertex to E , so that VE is equal to VS , the Distance of the Focus from the Vertex; and thro' E the Right Line CED is drawn perpendicular to the Axis of the Parabola, and the Point A is such, that AS , its Distance from the Focus, is equal to AC , which (by *Prop. 18. El. 3.*) is a Perpendicular drawn from A to the Right Line CD : Whence (by *Corol. 2. Prop. 14.*) the before-describ'd Parabola goes thro' A , and for the same Reasons it goes also thro' B . Q. E. F.

This Problem might have been look'd upon as solv'd by the 26th Proposition of the Third Book, as was there observ'd; but, by reason of its great Use, we have put it in this Place. Two Points given in a Parabola do the same here, as three Points given there in an Ellipsis, because the Ratio given in every Parabola between the *Latus rectum*, and the *Latus transversum*, is equivalent to one Point. It is not necessary to make Use of a Trigonometrical Calculation to determine the Parabola's Vertex, Parameter, &c. for they will be easily found by the latter Part of the 26th Proposition of the 3d Book. PRO-

PROPOSITION XXV.

From some Places of a Comet observ'd, to find its Place for any intermediate Time given; and to determine the Time in which it shall be in any intermediate Place. [Fig. XXII.]

Let the Line $\gamma 123$ be the apparent Way of a Comet, and the observ'd Places 1, 2, 3, 4, 5, 6, 7, in such Manner that an equal Space of Time may be spent between the Moments that a Comet was observ'd in the Points 1 and 2, and that during which it was from 2 to 3, or from 3 to 4, or from 4 to 5, &c. on. Let such a Time be call'd $\mathcal{T}$. The first Differences of those Places from the fix'd Point γ , viz. $\gamma 1$, $\gamma 2$, $\gamma 3$, $\gamma 4$, $\gamma 5$, &c. be taken (as is usual with Astronomers) that is, the Arcs 12, 23, 34, 45, &c. which we shall call A, B, C, D, E, F : Again, take their second Differences, or the Differences of the first Differences, namely, G, H, K, L, M ; and then the Differences of those second Differences, or of the Arcs $\gamma 1$, $\gamma 2$, $\gamma 3$, &c. the third Difference P, Q, R, S ; and so on, till we come to Differences equal to Nothing, namely, the Differences of equal Differences immediately preceding. Every one of the equal Times spent in going thro' the Arcs A, B, C , &c. (namely $\mathcal{T}$) must be suppos'd divided into any Number of equal Parts, (the more of them the more accurate the Thing will be) and let an Arc, gone thro' by a Comet in any Time $\mathcal{T}$, be divided into as many Parts as $\mathcal{T}$ is divided, in such Manner that the Differences of those Parts reckon'd from γ , similar to the Differences of $\gamma 1, \gamma 2, \gamma 3, \gamma 4$, &c. may be equal to Nothing; and let them, being encreas'd or diminish'd, go on in the same Order, towards the same Parts, and

760 *The ELEMENTS* Book V.

Line CD touching each of them, on which from the Point S let fall the Perpendicular SE , which must be bisected at V . With the Vertex V , and the Axis VSX , and a *Latus rectum* equal to the Quadruple of the Right Line VS (by Prop. 52. B. I. Conic.) draw a Parabola, and it will be the Parabola requir'd.

That S is the Axis of the describ'd Parabola is plain from hence, that in the Axis of the Parabola, it is distant from the Vertex of it the fourth Part of the *Latus rectum*; which is the well-known Property of the Focus of a Parabola. And that it goes thro' the Points A and B , is prov'd thus: The Axis VX of the Parabola is produc'd beyond the Vertex to E , so that VE is equal to VS , the Distance of the Focus from the Vertex; and thro' E the Right Line CED is drawn perpendicular to the Axis of the Parabola, and the Point A is such, that AS , its Distance from the Focus, is equal to AC , which (by Prop. 18. El. 3.) is a Perpendicular drawn from A to the Right Line CD : Whence (by Corol. 2. Prop. 14.) the before-describ'd Parabola goes thro' A , and for the same Reasons it goes also thro' B . Q. E. F.

This Problem might have been look'd upon as solv'd by the 26th Proposition of the Third Book, as was there observ'd; but, by reason of its great Use, we have put it in this Place. Two Points given in a Parabola do the same here, as three Points given there in an Ellipsis, because the Ratio given in every Parabola between the *Latus rectum*, and the *Latus transversum*, is equivalent to one Point. It is not necessary to make Use of a Trigonometrical Calculation to determine the Parabola's Vertex, Parameter, &c. for they will be easily found by the latter Part of the 26th Proposition of the 3d Book. PRO-

PROPOSITION XXV.

From some Places of a Comet observ'd, to find its Place for any intermediate Time given; and to determine the Time in which it shall be in any intermediate Place. [Fig. XXII.]

Let the Line $\vee 123$ be the apparent Way of a Comet, and the observ'd Places 1, 2, 3, 4, 5, 6, 7, in such Manner that an equal Space of Time may be spent between the Moments that a Comet was observ'd in the Points 1 and 2, and that during which it moy'd from 2 to 3, or from 3 to 4, or from 4 to 5, and so on. Let such a Time be call'd $\mathcal{T}$. Let the first Differences of those Places from the fix'd Point $\vee$, viz. $\vee 1$, $\vee 2$, $\vee 3$, $\vee 4$, $\vee 5$, &c. be taken (as is usual with Astronomers) that is, the Arcs 12, 23, 34, 45, &c. which we shall call A, B, C, D, E, F : Again, take their second Differences, or the Differences of the first Differences, namely, G, H, K, L, M ; and then the Differences of those second Differences, or of the Arcs $\vee 1$, $\vee 2$, $\vee 3$, &c. the third Difference P, Q, R, S ; and so on, till we come to Differences equal to Nothing, namely, the Differences of equal Differences immediately preceding. Every one of the equal Times spent in going thro' the Arcs A, B, C , &c. (namely $\mathcal{T}$) must be suppos'd divided into any Number of equal Parts, (the more of them the more accurate the Thing will be) and let an Arc, gone thro' by a Comet in any Time $\mathcal{T}$, be divided into as many Parts as $\mathcal{T}$ is divided, in such Manner that the Differences of those Parts reckon'd from $\vee$, similar to the Differences of $\vee 1, \vee 2, \vee 3, \vee 4$, &c. may be equal to Nothing; and let them, being encreas'd or diminish'd, go on in the same Order, towards the same Parts, and

762 *The* ELEMENTS Book V.

and so one of them, being in the Middle, may be contain'd as often in the whole Arc, as the Time, which belongs to it here, is contain'd in the whole Time, which will be easy for any one that understands Astronomical Calculations: As for *Example*, if the third Differences of the given Arcs $\vee 1$, $\vee 2$, $\vee 3$, $\vee 4$, &c. are found to be equal to Nothing; then from $\vee$ to the Divisions of *A*, *B*, *C*, &c. of the computed Arcs, the third Differences are also equal to Nothing; if in the other Case only, the fourth Differences are equal to Nothing; here the fourth Differences will also vanish, and so of the rest; and the Points of Division of the Arcs will be the Places of a Comet, analogous to the Moments of Time: But the intermediate Arcs may safely be reckon'd to be gone thro' equably by a Comet.

For thus it will happen, that every Arc 12, 23, 34, 45, will be gone thro' by the Comet, mov'd after the same Law that it moves thro' the whole Arc 17: For from the Construction every one of the foresaid Arcs, and every Part of them containing as many Parts, as the Arc 17 contains of the Parts 12, 23, 34, &c. is divided after the same Manner, as the Arc itself in the Points 2, 3, 4, 5, 6; and as the Times between the Moments, at which a Comet comes to those Points, are equal, as also those in which the Arcs 12, 23, 34, 45, &c. are describ'd; it is plain that a Comet will come to the found Points, in Times agreeable to the known Times, because a Comet is mov'd after the same Tenour when it describes any of those Arcs, as it was mov'd when it describ'd the preceding, or following Arcs, which plainly happens in every natural Motion. But the Moments of Time being known, that a Comet comes to known Points,

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Book V. of ASTRONOMY. 763

Points, and it being besides suppos'd that a Comet goes equably thro' the little intermediate Arcs, which may safely be allow'd, if these Arcs (namely the Parts of the Arcs 12, 23, 34, 45, &c.) be small enough, that is, if T be suppos'd divided into a Number of Parts great enough; any intermediate Time being given between the Moments that a Comet was at the Points 1 and 7, the Place which the Comet is in will be known: And on the other Hand, the Time will be known when the Comet comes to a given Place between 1 and 7.

SCHOLIUM.

Astronomers commonly make the foresaid Time T a natural Day. But if a Comet cannot be exactly observ'd, after 24 Hours are pass'd, they reduce the Place that it is observed to be in one or two Hours before, or after, to the End of the natural Day, supposing the Motion made in the mean Time to be equable, which in so small a Time can cause no sensible Error. In this Practice they divide the natural Day, or the Time express'd by T in the Proposition, into four Parts, or Parts of six Hours, which they suppose enough, unless the Motion of the Comet be very much accelerated or retarded, and they wou'd proceed very exactly; in which Case they divide that Time, (at least those six Hours in which the given Time is found, for which the Comet's Place is requir'd) into Hours, and at last very safely suppose that the Motion of a Comet, during one Hour, is equable.

It often happens that in the most exact Observations of a Comet, the third Differences of the Places reckon'd from a fix'd Beginning, which are denoted in the said *Fig. 24*, by $P, Q,$
 $R, S,$

764 *The* ELEMENTS Book V.

R, S, are equal to Nothing, or that their second Differences *G, H, K, L, M*, that is, the first Differences of the Arcs *A, B, C, D, E, F*, are equal to each other: Therefore Astronomers divide the Arc, which a Comet goes thro' in 24 Hours, into 24 Parts, such that the Arc, which belongs to the 12th Hour, shall be the 24th Part of the Arc which the Comet is observ'd to go thro' in the Space of a natural Day; or rather (since this Arc is not precisely in the Middle, but together with the Arc of the following Hour, takes up the middle Place) the Division is made in such Manner, that the two Arcs belonging to those two Hours taken together, shall make a twelfth Part of the Arc describ'd in the Space of a natural Day, and so that the first Differences of those Parts shall also be equal to each other, or there shall be no second Differences; and at last the greater Parts shall be on the same Side, in respect of the Lesser, as they are in the Arcs *A, B, C, D, E, F*, and to every Hour they give one of those Parts in the same Order. For it will by that means happen, that a Comet being, after the same Tenour, similarly accelerated or retarded, shall describe any Arc in the Space of one Day, as it describ'd the Arc 15 in the Space of four Days: For there is only Need of the Observations of four Days, near one another, to know whether the Differences of the Arcs, 12, 23, 34, 45, are equal to each other. And if the propos'd Time falls between four other Days, they again investigate what Differences become equal; and so go on as before.

This Method is not only made use of to determine the Place of a Comet in its apparent Way for an intermediate Time, but also (if you divide the Motion of a Comet into that of

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Book V. of ASTRONOMY. 765

Longitude and that of Latitude, Right Ascension and Declination, as is usual, or any Way else) that from some of its Places observ'd, according to any of these Circumstances, its Place may be found, according to the same, for an intermediate Time given: For here it is only suppos'd that the Comet's Motion, consider'd according to any Direction, goes on after the same Tenour for any one single Day, as it does for several Days before and after it.

This Way of proceeding by Differences is very useful also in many other Things, (as for *Example*, for calculating Tables to smaller Divisions) which have been treated of at large by the Reverend Father *Gabriel Mouton* of *Lyons*, who teaches the Manner of working by these Differences in his Book of *Observations of the apparent Diameters of the Sun and Moon*.

PROPOSITION XXVI.

TO determine by Observation the Trajectory of a Comet moving in a Parabola. [Fig. XXIII.]

Let S denote the Sun, $\mathcal{T}t\tau$ the Orbit of the Earth: Choose three Observations, distant from one another almost equal Intervals of Time, yet so that the Time may be greater where the Comet moves the slowest: Let the Points $\mathcal{T}$, t and τ be Places of the Earth for the said Times; that is, $\mathcal{T}$ a Place of the Earth at the first Observation, t a Place at the second Observation, and τ at the third. If such Observations are not given, the Place of the Comet is (by *Prop. 25.*) to be found for One Time in those Circumstances, in respect of two other Times at which the Places agreeable are observ'd. Join $\mathcal{T}\tau$, and from t let tN fall perpendicular to it: Let the Right Lines $\mathcal{T}A$, tC , τB , be three observ'd Longitudes;

gitudes; therefore these three Right Lines are given in Position. In that of them tC which is in the Middle, take any where the Point C for the Trace of a Comet's Way in the Plane of the Ecliptic: From C , to the Plane of the Ecliptic, draw perpendicularly the Right Line $C\gamma$ equal to the Tangent of the known Latitude of the Comet in the second Observation, tC being suppos'd the Radius: Join γS , CS , in the last of which, towards S , take CD in such Manner that it may be to the known Line tN , (which is the vers'd Sine of the known Arc Tt) as a Solid, whose Base is the Square of St , and Altitude a Right Line SC , to the Cube of the Right Line $S\gamma$: Thro' the Point D draw (by *Prop. 8.*) the Right Line ADB meeting with TA , τB at A and B , so that AD may be to DB , as the Time between the first and second Observation is to the Time between the second and third; and A and B will be nearly the Trace of the Comet's Way in the Plane of the Ecliptic in the first and third Observation, if the Point C rightly taken be its Place in the second Observation. Because, by Supposition, C is the Trace of the Comet in the Plane of the Ecliptic in the second Observation, and $C\gamma$ the Tangent of the Latitude observ'd from t , the Radius being tC ; it is plain, that γ is the Comet's Place in the Mundane Space, and γS the Distance of a Comet from the Sun in the second Observation. Thro' D draw $D\delta$ parallel to $C\gamma$, which (by *Prop. 8. El. 11.*) is perpendicular to the Plane of the Ecliptic, and is found in the Plane of the Triangle $SC\gamma$: Let it meet $S\gamma$ at δ . The Ratio of the Right Line $\gamma\delta$ to the Right Line tN , is compounded of the Ratio of $\gamma\delta$, to CD , and the Ratio of CD to tN : But (by *Prop. 2. El. 6.*) $\gamma\delta$ is

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Book V. of ASTRONOMY. 767

is to CD as γS to CS , and CD is to tN as $St^9 \times SC$ to the Cube of the Right Line $S\gamma$; whenc γ^d is to tN in a Ratio compounded of $S\gamma$ to CS and the Ratio of $St^9 \times SC$ to the Cube of the Right Line $S\gamma$: And the Ratio, compounded of these two, is the same with the Ratio of St^9 to $S\gamma^9$; and therefore γ^d is to tN as St^9 to $S\gamma^9$. But (by *Corol. 1. Prop. 25. B. I.*) tN is nearly the Space thro' which the Earth, being let fall from Rest, wou'd, by the Force of its Gravity, fall towards the Sun, during the Time that it describes half the Arc $\mathcal{T}\tau$, if it was every where urg'd with the same accelerating Gravity uniformly continu'd which acted upon it in the Place t , because tN is nearly equal to the Square of the Arc $\mathcal{T}t$ applied to the Diameter of the *Magnus Orbis*: And from *B. I.* the accelerating Gravity towards the Sun in the Place t , is to the accelerating Gravity towards it in the Place γ , as $S\gamma^9$ to St^9 , namely, in a duplicate reciprocal Ratio of the Distances from the Sun, and the Spaces which are gone thro' downwards towards the Sun in the same Time, by the Action of those Forces, are in the same Ratio as these Forces; and therefore the Right Line γ^d is the Space thro' which the Comet let fall from Rest at γ towards the Sun wou'd fall, in half the Time that the Earth describes the Arc $\mathcal{T}\tau$, if it was every Way acted upon by the same accelerating Gravity uniformly continu'd as it is urg'd by, in the Place γ ; then γ^d will be nearly the Space which a Comet, by its Fall towards the Sun, would really describe in half the Time that the Earth describes the Arc $\mathcal{T}\tau$ of its Orbit; that is, in half the Time that a Comet describes an Arc of its Trajectory, intercepted between the two Longitudes $\mathcal{T}A$ and τB ; and therefore the Point d is

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in the Chord of that Arc. And therefore, if as well the Traces of $\alpha \gamma \beta$, the Arc aforefaid of the Trajectory, intercepted between the Ends of the observ'd Lengths τA and τB , as the Traces of the Point δ be conceiv'd in a Plane by letting fall Perpendiculars to the Plane of the Ecliptic, namely ACB and D ; the Point D wou'd be in the Chord of the Arc ACB . But the Chord of the Arc ACB is fuch (by *Corol. 2. Prop. 18.*) as to be divided by SC almost in a Ratio of the Times in which the Comet reduc'd to the Ecliptic describes the Arcs AC , CB ; and, by Construction, the Right Line ADB is divided in the same Ratio, and it is the only Line which can be drawn with these Conditions. Since then the Chord of the Arc, which is the Trace of Part of a Comet's Trajectory intercepted between the Ends of the Lengths τA , τB , must be terminated by the Right Lines τA , τB , and go thro' D , and be divided at D in a Ratio of the two Times between the three Observations; and since the Right Line ADB has all these three Conditions, and is the only Line which has these Properties; it is plain that the Right Line ADB (drawn as before-mention'd) is the Chord of the said Arc, therefore the Points A and B are nearly the Trace of the Way of a Comet in the Plane of the Ecliptic in the first and third Observations, if C be its true Trace in the second Observation.

But, because the Point C is not taken near enough, and tho' it had been the true Point, yet the Points A and B , deduc'd from it, are not accurately enough defin'd; we must from these, however irregularly determin'd, correct all at last. For which End, in *Fig. XXIV.* (where the same Construction is suppos'd as in the foregoing, leaving out only such things as were necessary

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Book V. of ASTRONOMY. 769

to prove the things above, for avoiding Confusion) at M the middle Point of the Right Line AB , raise MI perpendicular to AB , which the Right Line CI will meet with in I , being drawn through C ; parallel to BA ; draw SI cutting AB in K , and compleat the Parallelogram $MIVK$, and take MQ the triple of MK : Draw QS and produce it to E , till QE be equal to $3QS + 3IK$. The Point E being determin'd, let the Signs or Letters A , M , D and B , be suppos'd blotted out; let CE be join'd, and in it, towards E , take the new CD in a duplicate Ratio of the Right Line SC , to a Right Line equal to $SV + \frac{1}{3}IK$, and thro' the Point D lately found (by Prop. 8.) draw a new Line AB , terminated at $T A$ and τB , in such Manner, that AD may be to BD , as the Time between the first and second Observation, to the Time between the second and third Observation; the Points A and B will be more accurately the Trace of the Comet in the Plane of the Ecliptic, at the first and third Observation, than before, if the Point C be so rightly taken as to be its Trace in the second Observation. Let us imagine, from every Point of the Comet's Trajectory Perpendiculars, to be let fall on the Plane of the Ecliptic, or (what is the same) let us imagine that a Cylindric Surface is rais'd perpendicularly upon the Plane of the Ecliptic, and goes thro' the Comet's Trajectory; by the Points mark'd on the Plane of the Ecliptic, by the aforesaid Perpendiculars, will be drawn (or the common Section of the said cylindric Surface with the Plane of the Ecliptic will be) a parabolic Curve ACB , whose Focus is S , or a Curve very little different from a Parabola: The Chord of that Arc ACB comprehended (as we have be-

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fore shewn) between the Right Lines TA , & B , is nearly the Right Line AB , drawn before, whose middle Point is M . And, as in the said Arc, the Point C is taken not far from the Vertex of the Segment ACB , (for, for that Reason the Times of these Observations are taken at almost equal Intervals from one another, yet in such Manner that that Time may be something greater where the Comet moves slower) and a Right Line is drawn parallel to BA , meeting with a Perpendicular drawn from M to AB at I , and SI drawn cutting AB in K , and the Parallelogram $IMKV$ compleated; and since, moreover, upon Account of the immense Distance of the Sun, the Distance of the Points M and V vanishes in Comparison of it, that is, the said Points coincide; (by *Corol. Prop. 19.*) V is the Vertex of the Segment ACB . If VS be drawn cutting the Chord at T (by reason of the immense Distance of the Point S) VT will be nearly parallel to IK , therefore KT , equal to IV , and therefore to MK ; but, by Construction, MQ is the triple of MK , whence it is also triple of KT , and therefore also of the remaining TQ ; for which reason (by *Corol. 2. Prop. 18.*) QS being drawn, is the same as QS , in *Prop. 18.* and in its *Corollaries*, that is, in QS produc'd towards S , you find the Point E , from which any Right Line, being drawn cutting the Chord AB , and the Arc ACB , cuts the Chord into Segments, that have to one another the Ratio of the Times, in which the adjacent Arcs are describ'd by a Comet. Moreover, by Construction, QE is equal to $3QS + 3IK$, and IK equal to MV (they being the Diameters of the same rectangular Parallelogram;) whence QE is equal to $3QS + 3MV$. And therefore (by *Corol. 3. Prop. 18.*)
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Book V. of ASTRONOMY. 771

the Point *E* above determin'd is the same, from which a Right Line, being drawn any how, divides the Chord *AB* into Segments, having the same Ratio as the Times, in which the two Parts of the Arc *AB*, mark'd out by the same Right Line produc'd, are describ'd by a Comet, which before we had falsly (tho' not too erroneously) suppos'd of the Focus *S*. Having therefore, to avoid Confusion, rubbed out the former *CD*, drawn towards *S*, a new Line must be drawn towards *E*, which is to the former as the Square of *SC*, to the Square of $SV + \frac{1}{3} IK$, that is (by reason of the equal Lines *IK* and *MV*) to the Square of $SV + \frac{1}{3} MV$; and *SC* is nearly equal to *SV*: Wherefore the new *CD* is to the former *CD* as the Square of *SV* to the Square of *SL*, supposing *VL* to be the third Part of *MV* or *IK*, as in the Construction of *Prop. 20*, and *Prop. 21*. But (by *Prop. 42. B. I.*) the accelerating Gravity towards the Sun, in the Place *L*, is to the accelerating Gravity towards it in the Place *C* (or *V*) as the Square of *SC* (or *SV*) to the Square of *SL*: And as the accelerating Gravities towards the Sun in different Distances, so are (the said Force remaining) the Spaces run thro' by falling towards the Sun, during the same Time; and therefore the new *CD* is to the former *CD*, as the Space gone thro' by falling towards the Sun (the same accelerating Force acting, which acted at the Place *L*,) during half the Time that the Comet describes the Arc intercepted between the Length *TA* and τB , is to the Space gone thro' in falling towards the Sun, the same accelerating Gravity acting, which acted at *C*. But the Consequents of this Analogy are equal, therefore the Antecedents are also equal; that is, the new drawn *CD* is equal to a Space gone thro' by a heavy Body

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falling towards the Sun, half the Time that a Comet describes the Arc ACB in the Ecliptic, by the Action of the same accelerating Gravity, which obtain'd at the Distance SL from the Sun uniformly continu'd. But (by *Prop.* 21.) the Space, thro' which a Body falls towards the Sun, in half the Time that a Comet describes the Arc AVB , (for now we consider the Motion of a Comet, as made in the Plane of the Ecliptic, looking at the Mark of a Comet's Trajectory in the Ecliptic, instead of the Trajectory itself) when it is urg'd by the same accelerating Gravity towards the Sun, which acted at L , uniformly continu'd, is equal to VT , the Segment of the Right Line VS , intercepted between the Vertex V , and the Chord AB ; therefore nearly equal to the Segment CE of CD intercepted between the Point C (near to V) and the Chord AB . Whence the Point D is in the Chord AB much more accurately than before; that is, the Chord of an Arc, which, in the Plane of the Ecliptic, is the Trace of a Comet's Trajectory, intercepted between the Lengths TA and τB , goes nearly thro' the Point D , determin'd last. Moreover, (by *Prop.* 18.) the foresaid Chord, drawn thro' D , must have such a Situation between TA and τB , that AD may be to DB as the Time in which a Comet describes an Arc of the Ecliptic between the Lengths TA and TC , to the Time in which it describes the Arc intercepted between τC and τB : But AB is (by *Prop.* 8.) drawn in such Manner thro' D , that AD is to DB in that same Ratio, namely, as the Time between the first and second Observation is to the Time between the second and third; therefore AB is rightly drawn, and as it ought to be, namely, going thro' D , and divided at D , as is said before.

Book V. of ASTRONOMY. 773

fore. And therefore, if the Point C is rightly taken for the Trace of the Comet's Place at the second Observation, the Points A and B are nearly the Trace of its Place at the first and third Observations.

Now what remains (*Fig. XXV.*) is to know, whether the Point C in the Middle of the Length be the Trace of a Comet rightly taken; if not, the Error arising from it must be corrected, the other things hitherto done remaining unalter'd. Having rubb'd out the former Parallelogram $IMKV$, made on the former less accurate Chord AB , make another on the last and more accurate Chord AB with the same Construction as before. From A , M and B draw A^α , M^μ and B^β , of which let A^α be the Tangent of the known Latitude in the first Observation for the Radius TA , and B^β the Tangent of the Latitude at the third Observation for the Radius τB : Draw $^\alpha\beta$, with which M^μ meets at μ . If the Trapezium $AB\beta^\alpha$ be erected perpendicularly to the Plane of the Ecliptic, the Right Line AB remaining, the Points $^\alpha$ and $^\beta$ (as was before shewn) are the Places of a Comet in the Mundane Space, if the Point C is its Trace in the Plane of the Ecliptic in the second Observation, and the Plane going thro' the three Points S , $^\alpha$ and $^\beta$, be the Plane of the Comet's Trajectory: Therefore the Right Line $^\alpha\beta$ is the Chord of the Arc of the Parabolical Trajectory, gone thro' by the Comet between the first and third Observation, and $S^\alpha S^\beta$ are respectively the Distances of a Comet from the Sun at the first and third Observation, that is, the true Distance of every Point of the Comet's Trajectory, from the Sun, is the Hypothenuse of the rectangular Triangle, one Side of which is the Distance from the Sun of the Trace of that Point in the Plane of the Ecliptic, and the other a Per-

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pendicular drawn from that Point on the Plane of the Ecliptic, to the Point which terminates the Trajectory. As some of these Perpendiculars are longer, as B^c , and some shorter, as A^a , take a middle one between those, as for Example, M^u ; and commonly speaking, the Distance of any Point of the Comet's Trajectory, from the Sun, will be nearly the Hypothenuse of a Rectangular Triangle, one Side of which is the Distance of the analogous Point describ'd in the Trace of the Trajectory, and the other the Right Line M^u itself. Which being done, in MA (or MA produc'd) take MH equal to $SV + \frac{2}{3}IK$, that is, equal to SR , supposing LR equal to LV , and draw H^u ; this Line will be nearly equal to the Distance from the Sun, of a Point of the Trajectory (whose Mark is V) increas'd by the Double of the Right Line, intercepted between the Point and the Chord of the Arc of the Trajectory, namely, $^a c$ in the erected Trapezium AB^c^a , that is, the Right Line H^u is equal to a Right Line in the Plane of the Comet's Trajectory, analogous to SR in its Trace in the Plane of the Ecliptic, that is, H^u is equal to the Right Line SR , in the Parabola of *Prop.* 20. By the Corollary of the 17th Proposition, let the Velocity of a Comet, when it moves in its Parabolical Trajectory, at a Distance from the Sun, equal to the Right Line H^u , be compar'd with the Velocity of the Earth towards the Sun, and the Line which the Comet, moving equably with the said Velocity, wou'd go thro' in the whole Time that the Earth describes the Arc $Tt\tau$, or in the whole Time that a Comet goes thro' the Arc ACB in the Ecliptic, be defin'd in Parts of the Arc $Tt\tau$ describ'd by the Earth at the same Time; which

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Book V. of ASTRONOMY. 775

may be easily done in the following Manner. By Calculation, subtract the Length of the Arc $Tt\tau$, describ'd by the Earth between the first and third Observation, supposing any round Number for the mean Distance of the Earth from the Sun; a Length (as for *Example Z*,) which is to this Arc in a subduplicate Ratio of the Diameter of the *Magnus Orbis*, to the known Right Line $H\nu$, and which therefore is given, is the Length required; namely, that which a Comet equably carried with the same Velocity that it has, when it describes its parabolic Trajectory at a Distance from the Sun, equal to $H\nu$, wou'd go thro' in the Time that a Comet does really go thro' an Arc, whose Chord is $\alpha\beta$: For (by *Prop. 17.*) the Velocity of a Comet, at the said Distance $H\nu$, is to the Velocity of the Earth in the above-mention'd Ratio. But (by *Prop. 20.*) the said Length Z is equal to the Chord of an Arc which a Comet does really describe in that Time: Therefore if Z be found equal to the Chord $\alpha\beta$, it is certain that the Point C in the Longitude observ'd the second Time, was rightly taken for the Comet's Place; therefore A, C and B , are the three Places of the Comet in the Plane of the Ecliptic.

If Z be not found to be equal to the Right Line $\alpha\beta$, that is, if the Point C be not rightly taken in tC the Longitude observ'd the second Time; in $\alpha\beta$ (or $\alpha\beta$ produc'd if Occasion require) take $\alpha\varphi, \beta\pi$ equal to Z , take also AF, BP , equal to $\alpha\varphi, \beta\pi$, in such Manner, that F and φ may be on the same Side of the Right Line $A\alpha$, and P and π on the same Side of the Right Line $B\beta$. Then, by the same Method, that from the assum'd Point C , were found the Points A, B, D, F, P , from other assum'd Points $2C$ and $3C$, find the

new ones $2A, 2B, 2D, 2F, 2P$, and $3A, 3B, 3D, 3F, 3P$, as in the 26th Figure; where the same Things remaining, and the same Letters being us'd as before, those Things are only left out, of which we have no more Occasion to speak. If Z be less than $^{\alpha\beta}$, or AF , or BP less than AB (as it happens in this 26th Fig.) the Point $2C$ must be taken nearer to the Point σ (in which τA and τB meet;) and so on, if at least $3A, 3F$, be greater than $3A, 3B$: But if on the Contrary, AF be greater than AB , $2C$ must be taken greater than tC ; and so on, if at least $3A, 3F$, be less than $3A, 3B$. Thro' the Points $F, 2F$ and $3F$ draw a Circle, which cuts the Right Line τB between B and $3B$; as for Example in O , if the new Points $2C, 3C$, be taken in the Manner before shewn. Likewise thro' the Points $P, 2P$ and $3P$, draw a Circle cutting the Right Line τA in X ; the Points O and X will be the Places of a Comet reduc'd to the Ecliptic, or the Traces of the Comet at the first and third Observation. For it has been above demonstrated, that the Points A and B are the Traces of the Comet in the Plane of the Ecliptic, at the first and third Observation, if C be its Trace in the second: The same does likewise obtain in $2A, 2B$ and $2C$; and also in $3A, 3B$ and $3C$. It is also demonstrated that C is truly the Mark of the Comet's Place in the second Observation; if Z be equal to $^{\alpha\beta}$; that is, if either of the Right Lines AF and BP be equal to AB ; that is lastly, if the Points B and F coincide, and A and P . And therefore, if (Things remain as before) the Points B and F coincide, such a Point B will be the Mark of the Comet at the third Observation: Likewise when A and P coincide, such a Point A will be its Mark at the first Observation.

Book V. of ASTRONOMY. 777

servation. And, to make these coincide, thro' the three rightly found Points F , $2 F$, and $3 F$, a Circle is drawn, that is, a very simple Line going thro' three Points, cutting the Right Line τB in O . Since then the Point O is as well in the Place of the Point B , namely, in the Right Line τB , as in the Place of the Point F , namely, in a Circle; when the Point B happens to be at O , the Point F will also be there, that is, in that Case the Points B and F will coincide; and therefore the Point O is the true Mark of the Comet in the Plane of the Ecliptic at the third Observation, all the requisite Conditions agreeing with it. Likewise, for the same Reasons, the Point X is the true Mark of a Comet in the first Observation. Therefore, if from the Point O to the Plane of this Figure, which represents the Plane of the Ecliptic, be drawn the Perpendicular $O \omega$, equal to the Tangent of the known Latitude in the third Observation for the Radius τO ; ω will be the true Place of a Comet in its Trajectory. Likewise $X \xi$ perpendicular to the Plane of the Ecliptic rais'd at the Point X , equal to the Tangent of the Latitude at the first Observation of the Radius τX , will shew the Point ξ , the other Place of the Comet, in its proper Trajectory. About the Focus S , thro' the Points ω and ξ (by *Prop. 24.*) describe a Parabola; and it will be the Comet's requir'd Trajectory. But because there are two Parabolas describ'd about the Focus S passing thro' the two Points ω and ξ , which of these is the Trajectory of our Comet, will appear from any Observation of the Comet: For, its Place, deduc'd from one of those Parabolas, will agree to the observ'd Place; but, taken from the other, will not. Therefore we have, from Observations,

778 *The ELEMENTS* Book V.

tions, determin'd the Trajectory of a Comet moving in a Parabola. *Q. E. F.*

Also in τA , or τB , one may choose Points at Pleasure, as was done here in τC , and work with the Points deduc'd from hence in the two remaining Lines; after the same Manner that we work'd with the Points in τA , τB .

In the foregoing Investigation, whether Arithmetical or Graphical, or compounded of both, there is no Regard had to the Monthly Parallax of the Sun arising from hence, *viz.* That it is not the Earth's Center, but the common Center of Gravity of the Earth and Moon, which is carried in the *Magnus Orbis*; for this Monthly Parallax of the Sun, three times greater than the Diurnal, is sensible in defining of the Orbits of some Comets: Therefore we must by it correct the Comet's observ'd Places.

If the Elliptic Orbit of a Comet be found to agree better, let it be thus describ'd: Find the Mark of a Comet in the Plane of the Ecliptic in the second Observation; and it will be the same in Order and Situation between the Points C , $2 C$, $3 C$, as the Point O was between B , $2 B$, $3 B$, or as X between A , $2 A$, $3 A$: From the Mark thus found, raise, perpendicularly to the Plane of the Ecliptic, a Line which is the Tangent of the Latitude in the second Observation, for a Radius equal to the Distance between t and the said Mark; the upper End of the said Perpendicular, or the End of it above the Plane of the Ecliptic, is the Place of the Comet in its Trajectory observ'd the second Time. An Ellipse drawn about the Focus S , thro' the Points e , ξ , and the Point lately found (by *Prop. 29. B. III.*) will be the requir'd Orbit of the Comet.

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Book V. of ASTRONOMY. 779

SCHOLIUM.

The foregoing Method for determining the Point O , or the Point X , is universal, and of great Use in several Physical Aproximations. For the most Part three Points, as here, A , $2A$, and $3A$, found, according to Art, from the three assum'd ones, *viz.* C , $2C$, and $3C$, must be so posited, that the requir'd Point may be between their Extremities A and $3A$, which is the Reason why, from the three Points, P alone must be plac'd on the other Side of the corresponding Point A : For thus will the requir'd Point be more certainly and nearly determin'd by the Intersection of a Circle with the Right Line A , $2A$, in which it must necessarily be, than if the requir'd Point, being beyond A and $3A$, was to be determin'd by the Intersection of a Right Line with the Circle beyond the Points A and $3P$; for the same Reason, from some observ'd Place of one Comet, (or of any other mov'd according to any Law) its Place, and its intermediate Time whatever, is more certainly defin'd, than its Place, or Time, before the first, or after the last Observation.

Sometimes it may be sufficient, in Investigations of this Nature, to determine two such as P and $2P$ are (*viz.* if one of them be nearly true) and join them by a Right Line, whose Intersection with the other A , $2A$, will determine the true Point sought: And here, again, it will be convenient, that the Points P , and $2P$, lie on contrary Ways, with Regard to the Point sought.

PROPOSITION XXVII.

TO determine the Point that is fittest to be taken, the first, second, and third Time, for the Comet's Tract in the Plane of the Ecliptic. [Fig. XXVII.]
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780 *The* ELEMENTS Book V.

The Construction of the foregoing Problem is universal: But, in Practice, we must not take any Points C , $2C$, for the Mark of the Comet in the second Observation, but we must take those Points nearly true, if we can by any Means know whereabout the true ones shou'd be. And that may be done by Help of the 12th *Prop.* by four Observations of a Comet. For a Right Line being drawn, whose three Parts, intercepted between four Right Lines given in Position, have the same Ratio as three Intervals of Time between four Observations, will not differ very much from that Part of a Comet's Trajectory which is then run through. Therefore its Interfection with a Right Line, that represents the Longitude in the second Observation, will very fitly be taken for the Mark of a Comet in that second Observation.

If you know, within a little, the Quantity of the Angle which the Trace of the Orbit of a Comet contains, with a Right Line joining the Earth and the Comet in the second Observation, or its Equal, (by *Prop.* 17. B. I. *Conic.*) namely, $A\delta t$, (*Fig.* XXVII.) which the Chord AB contains with the Right Line tC , which may be done by Help of the 9th *Prop.* or more accurately by the *Schol.* of *Prop.* 13; then the Point C , to be taken at first, will be determin'd after the same Manner: Draw the Right Line AB , terminated at each End by the Right Lines TA , τB given in Position, and cutting the Right Line tC , which is also given in Position at δ , in an Angle equal to the given Angle; in such Manner, that it may be to $\sqrt{2 \times T\tau}$, the Chord (that is, nearly to $\frac{2}{3}$ of $T\tau$) in a subduplicate Ratio of St to $S\delta$; and draw, thro' S , the Right Line SDC , whose Part DC , interjected between the Legs of the Angle $A\delta C$,

Book V. of ASTRONOMY. 781

$A \delta C$, must be equal to the Right Line $t N$, which is an univarsal Problem of the Ancients concerning *Inclinations*, and which is done by their Method, or by the 7th Section of the 5th *Optical Lecture*, or in the 3d Section of the 6th *Geometrical Lect.* of Dr. Barrow. The Point C , thus determin'd, is that very same which is conveniently taken, the first Time, for the Trace of the Comet in the Plane of the Ecliptic. Suppose that C is the Trace of the Comet in the Plane of the Ecliptic, and a parabolic Arc going thro' A , B , and C , is the Trace of an Arc of the Trajectory described between the first and third Observation: Therefore, supposing that the accelerating Gravity, towards the Sun, is the same, or equal, at the Distance of the Earth from the Sun, and at the Distance of a Comet from it, (that is, in *Galilæus's* Supposition of Gravity, which is nearly true,) $C D$ and $t N$ are equal, as being the Spaces in falling toward the Sun gone thro' at the same Time by a Comet, and by the Earth; AB , the Chord of the Parabola, will (by *Prop. 22.*) be to $\sqrt{2 \times T t}$ the Chord of the Circle, whose Center agrees with the Focus of a Parabola, in a subduplicate Ratio of the Right Line $S t$ to the Right Line $S D$; but AB is drawn to $\sqrt{2 \times T \tau}$ in a subduplicate Ratio of $S t$ to $S \delta$, and also AB cuts the Right Line $t C$ in a due Angle; and therefore $C D$ is equal to $t N$: Therefore, the Right Line AB has nearly all the requir'd Conditions to become the Chord of the Arc, which is the Trace of the Comet's Trajectory, intercepted between the first Longitude $T A$, and the third γB ; and therefore the Point C has all the Conditions requir'd, to be nearly the Trace of the Comet at the second Observation. Therefore the Point C is rightly taken for the Trace of the

the Comet (tho' it must afterwards be corrected) at the second Observation.

The Point C therefore, which is to be taken the first Time, being determin'd, the Points which are afterwards taken (namely, $2C$, and $3C$, [Fig. XXVIII.] are not to be taken without Consideration or Allowances; neither does the Caution given in the foregoing Proposition, namely, that the Point $2C$ be taken nearer the Point σ , when the Line Z is less than AB , and so, on the contrary, suffice; but when we have proceeded so far, that AB , rightly drawn, be equal to the Longitude Z agreeable to it: In tC take the Point $2C$, in such Manner, that the Distance $\sigma 2C$ may be to the Distance σC , in a Ratio compounded of the Ratio of Z to AB , and a subduplicate Ratio of SC to $S 2C$. The Cause why the first compounding Ratio is made Use of, is manifest from what is said before: For, since AB is to be found so as to be equal to Z agreeable to it, if it be greater, or less, it must be diminish'd, or encreas'd till they become equal: Now, they wou'd be equal by the first compounding Ratio alone, if Z was constant and invariable; but as the Longitude Z is always different, as AB is different, therefore the second compounding Ratio is necessary. Now, it is plain, (from *Prop. 23.*) that in Parabolas the Chords of the Arcs ACB , $2A$, $2C$, $2B$, (in which CD , $2C$, $2D$, tending to the Focus, and intercepted between the Vertex and the Chord, are equal) are in a subduplicate Ratio of the Right Lines SC , $S 2C$; and it has been shewn above, that the Right Lines CD , $2C$, $2D$, are nearly equal, (namely, being the Spaces which a Comet falling towards the Sun wou'd describe at different Distances from it in the same Time) and also that the

Book V. of ASTRONOMY. 783

the Trace of a Comet, in the second Observation, is near to the Vertex of the Arc ACB , that is, of a Part of the Trace of the Trajectory of a Comet describ'd between the first and third Observation. But also the Point $2C$, found after that Manner, is not said to be the accurate Trace of the Comet in the Plane of the Ecliptic, but it is nearly true, and but very little remov'd from a Point truly accurate.

After the same Manner may be determin'd the Point $3C$ from $2C$, as $2C$ was determin'd from C , if we find three Points, as was done in the foregoing Proposition; but, if we work this Way, it will hardly be needful to make Use of a second Operation: And a Right Line joining the two Points F and $2F$, determin'd from the two assum'd Points C and $2C$ by the foregoing, will cut τB in the Point O , (to which they are very near) the accurate Trace of the Comet in the third Observation, and the Right Line $P, 2P$, will likewise cut TA in the Point X , the Trace of the same Comet, at the first Observation, as we have shewn before, that it must happen when C , and $2C$, are near to the true Point. Therefore we have determin'd the Point, which (in the Operation of the foregoing *Prop.*) is most conveniently taken for the Trace of the Comet in the Plane of the Ecliptic, the first, second, and third Time.

In nearly determining the Points C , and $2C$, after the aforesaid Manner, in Graphical Operations, we must often proceed by Trials, where a more accurate, or a Geometrical, Method is wanting; neither is it needful here to make Use of Curves very difficult to be drawn, which the Construction wou'd require.

PROPOSITION XXVIII.

TO determine the Position of the Line of Nodes of the Trajectory of a Comet, and the Time when a Comet is in the Nodes. [Fig. XXIX.]

Things remaining the same, and being denoted by the same Letters as in the foregoing Propositions, the Right Lines OX , $\omega\xi$, must be produc'd till they meet in Ω : Draw SO , SX , $S\Omega$. All the Points were before given, namely, S , O , and X ; and therefore, as well the Sides, as the Angles of the Triangle SOX ; and therefore the Angle $SX\Omega$, adjoining to one of them. Moreover, if thro' ξ you draw ξo parallel to XO , meeting with the Right Line $O\omega$ at o , the Triangles $\omega o\xi$, and $\xi X\Omega$, will be equiangular; and therefore from the known Sides $o\xi$ (equal to OX) and $o\omega$ (the Difference of the known Right Lines ωO , and ξX) together with the Right Angle $\omega o\xi$, the remaining Sides and Angle will be known, and also the Angles of the Triangle $\xi X\Omega$: But in this Triangle, one Side $X\xi$ is given, therefore the other Sides will also be given; namely, $X\Omega$ and $\xi\Omega$. Then in the Triangle ΩXS , you have the Sides XS , $X\Omega$, with the Angle between them $SX\Omega$; therefore you will also know the Angle $XS\Omega$. But as was said before, the Position of the Right Line SX is given, or the Angle which it makes with TX , whose Position is known by Observation, (for in the Triangle XTS , the Sides TS and TX are given, and also the Angle XTS , which is the Distance between the known Place of the Sun, and the Place of the Comet observ'd the first Time, whence TXS is known) and therefore the Position of the Right Line $\Omega S\Omega$ will be known, or the Places of the Nodes seen from the Sun. If the Right Lines $O\omega$, $X\xi$, are equal,

the

Book V. of ASTRONOMY. 785

the Line of the Nodes is parallel to $O X$, and therefore known in Position.

To determine the Time when a Comet is in the Node, let $\omega \xi$ [Fig. XXX.] be the Trajectory of a Comet describ'd by *Prop. 26.* thro' the Points ω and ξ , in whose Plane are found the Right Lines $S\omega$, $S\xi$, and let the before-found Line of Nodes $\delta\delta S\mathcal{U}$ meet with the Trajectory in $\delta\delta$ and $\mathcal{U}$; and (by *Prop. 14. B. I.*) the Interval of the Time between the first Observation, and the Moment that the Comet comes to the Node $\delta\delta$, will be to the Interval of Time between the first and third Observation, as the Area $\xi\delta\delta S$, to the Area $\omega\xi S$: But the Ratio of the Areas $\xi\delta\delta S$, and $\omega\xi S$, is known; (for the Areas themselves are known, by the 24th *Prop. Archim. of the Quadrature of the Parabola*) therefore also the Time at which a Comet is in the Node $\delta\delta$. After the same Manner will be known the Time that a Comet comes to the other Node $\mathcal{U}$. Q. E. F.

PROPOSITION XXIX.

TO determine the Inclination of the Plane of the Trajectory of a Comet, to the Plane of the Ecliptic. [Fig. XXXI.]

Things remaining as before, from the Point O , to $\mathcal{U} S\mathcal{U}$, the Line of Nodes, draw the Perpendicular $O\mathcal{U}$, and join $\omega\mathcal{U}$. In the Triangle $O S\mathcal{U}$, besides the Right Angle at $\mathcal{U}$, you have (by the foregoing *Prop.*) the Angle $O S\mathcal{U}$, and the Side $O S$; therefore $O\mathcal{U}$ is known. Then, in the Rectangular Triangle $\omega O\mathcal{U}$, the Sides about the Right Angle, namely, $O\omega$, and $O\mathcal{U}$ are known, and therefore the Angle $O\mathcal{U}\omega$, will be known, which (by *Def. 6. El. 11.*) is the Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic.

X x x

P R O-

PROPOSITION XXX.

TO determine the principal Latus rectum of a Comet's Trajectory, the Comet's Perihelium, and the Time that a Comet is in the Perihelium. [Fig. XXXII.]

About the Focus S , thro' the Points ω and ξ , describe the Trajectory of a Comet, by *Prop. 24.* then about S as a Center, thro' either of the said Points (as for $Ex.\omega$) draw a Circle meeting again with the Trajectory at V ; draw ωV , to which from S , draw the Perpendicular $S\pi$, which produce, till it meets with the Parabola at π ; π will be the Perihelium of the Trajectory, and therefore the Quadruple of $S\pi$ is the principal *Latus rectum*.

The first is plain; for since the Focus S is found in the Axis of the Parabola, a Circle, describ'd about the Center S , will cut the Parabola (if it cuts it at all) in two Points equally distant from the Axis, and therefore the Right Line ωV , joining the Intersections, will be perpendicular to the Axis, and π the Vertex of the Parabola, or the Perihelium of the Trajectory, and the Quadruple of $S\pi$, the Parameter of the Diameter, whose Vertex is π (that is, of the Axis itself,) or the principal *Latus rectum*.

Take a Time, whose Interval from the first Observation, when the Comet was in ξ , is to the Interval of the Time between the first and third Observation, as the Area $\xi\pi S$, to the Area $\omega\xi S$; and that will be the Time that the Comet is in the Perihelium.

The Variety of Cases, namely, when the Time, that the Comet is in the Perihelium, is before the first Observation, and when it is after it, we leave to the Judgment of the Astronomer, as also innumerable other Things relating to

Book V. of ASTRONOMY. 787

to this Problem, about defining a Comet's Trajectory, and the Parts of it.

SCHOLIUM.

Hence also will a Comet's Perigæum, and the Time of it, be determin'd: For since the Time (namely, that between the first and third Observation) that the given Area $\xi \omega S$ is describ'd by a Radius drawn from the Comet to the Sun, is given, you will also have the Area describ'd in one Day; and since ξ is given, which is the Place of a Comet in the first Observation, the Places of a Comet, for each Evening, will be given in its proper Trajectory. But the Place of the Earth in its Orbit is given for every Day, and (from what has been said before) the Situation of the Earth's Orbit, and a Comet's Trajectory, in respect to each other; whence the Place where the Comet is nearest to the Earth, that is, the Comet's Perigæum will easily be known, and also the Time when it happens.

PROPOSITION XXXI.

TO correct the Trajectory of a Comet which was found by Prop. 26.

Let the Position of the Plane of a Comet's Trajectory, determin'd by Prop. 26. be assum'd, then choose three Places of the Comet, nicely observ'd, different from those three by which the Trajectory was first determin'd, and very far distant from one another: It will be convenient for one of them to not be far distant from the Comet's Perihelium. Let the Time, between the first and third of those Observations, be called S : Take a Number C , which may be to 1 as the Time between the first and second Observation, to the Time between the second and third: From these apparent Places, by a trigonometrical Calculation, after the Method us'd before,

X x x 2

find

788 *The ELEMENTS* Book V.

find three Places of the Comet in the foresaid Plane of the Trajectory, taken as very accurate; that is, find three Points of the before defin'd Plane, in which the Comet has the same Longitude and Latitude, as it is really found to have; and (by *Prop. 29. B. III.* or perhaps by *Prop. 24. B. V.*) determine, by Calculation, what Conic Section goes thro' those three Points, whose Focus coincides with the Sun, and whose Areas, terminated by Right Lines, drawn to the Sun, and Portions of the Curve, are determin'd: and therefore their Ratio will be also determin'd: As for Example, let its Area, between the first and second Observation, be to its Area between the second and third Observation, as G to 1 ; and that same will be the Ratio between the Times, in which these Areas wou'd be describ'd by a Comet, from which a Radius is drawn to the Sun. Moreover, from the known Magnitude of this Track of the Comet, and the Magnitude of all its Parts, in respect of the Track of the Earth and its Parts, may (by *Prop. 41. B. I.*) be known the Velocity with which a Comet describes it; therefore the Time that a Comet wou'd describe the two above-found Areas; let it be call'd T . Let $T=S$, and $G=C$, the Position of the Plane of the Trajectory hitherto found, will be true and accurate, and want no Correction; but if it shou'd, then $T-S$ will be the Error in the whole Time between the first and third Observation, arising from the less accurate Position of the Plane of the Trajectory, and $G-C$ will also be the Error arising from it in the Ratio of the Time between the second and third Observation; which appears to be so, because in each Case Unity is taken for the Consequent of two Terms, which expres the Ratio between the two Times.

Let

Book V. of ASTRONOMY. 789

Let the Longitude of either Node of the Trajectory (found by *Prop.* 28.) call'd K , be increas'd by 20 or 30 Minutes, which Minutes call P , keeping the foremention'd Inclination of the Plane of the Trajectory to the Plane of the Ecliptic; and after the Manner before made use of, find in this new Place three Points, which may accurately answer to the three foresaid most accurate Observations of the Comet, and the Trajectory going thro' them, whose Focus must coincide with the Sun's Center, and let its two Areas between the three Observations, describ'd by a Right Line drawn to the Sun, whose Ratio is express'd by that of g to L , and also the whole Times which the Comet will take up, in describing each of those Areas, be express'd by t . If the Time t be found to be equal to S , and g equal to C , this assum'd Position of the Plane will be the true one; if not, $t - S$ will be the Error in the whole Time between the first and third Observation, and $g - C$ the Error in the Ratio of the Time between the first and second Observation, to the Time between the second and third, each arising from the less accurate Position of the Plane of the Trajectory to the Plane of the Ecliptic.

Let the found Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic call'd I , be (by *Prop.* 29.) increas'd 20 or 30 Minutes, which must be call'd Q , the Longitude of the Node first found by *Prop.* 28. remaining the same; and, as was done before, find, in this new Plane, three Points answerable to the three foresaid Observations of the Comet, and also the Trajectory going thro' them, whose Focus is the same as the Sun's Center, and its two Areas between the three Places observ'd, describ'd by a

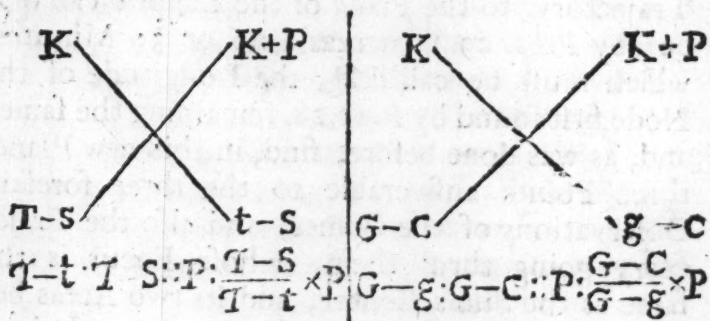
X x x 3

Right

Let

Right Line drawn to the Sun, whose Ratio is the same as that of γ to I : Find also the whole Time τ , which the Comet wou'd take up in describing each Area at the same Time, by a Right Line drawn to the Sun. If τ be found to be equal to S , and γ also equal to C , this assum'd Position of the Plane of the Trajectory will be the true one, and as it shou'd be; if not, $\tau - S$ will be the Error in the whole Time between the first and third Observation, and $\gamma - C$ likewise the Error in the Ratio of the Time between the first and second Observation, to the Time between the second and third Observation.

Let the Operation therefore be perform'd by the Rule of False Position, the Scheme of whose Operation is here annex'd, where K is one Position of the Longitude of the Node, and $T - S$ the Error arising from thence in the whole Time between the first and third Observation; $K + P$ the other Position of the Longitude of the Node, and $t - S$ the Error of the said whole Time arising from thence. According to the said Rule, as the Difference of Errors to the Difference of Positions, so let either of the Errors be to a fourth Quantity; and this will be the Correction of the Position, from which flow'd the assum'd Error.



Af

Book V. of ASTRONOMY. 791

After that Manner the Error of the posited Longitude of the Node, that is, of K itself, in the whole Time between the first and third Observation, is $\frac{T-S}{T-t} \times P$; and the Error P of the same Longitude K in the Ratio between the two Times between the three Observations is $\frac{G-C}{G-g} \times P$; therefore the true and corrected Longitude of the Node is $K + \frac{T-S}{T-t} \times P$, or $K + \frac{G-C}{G-g} \times P$.

Likewise the Error of the posited Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic, viz. I , from the first Head is $\frac{T-S}{T-\tau} \times Q$; and from the second it is $\frac{G-C}{G-\gamma} \times Q$; and therefore the true and corrected Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic, is $I + \frac{T-S}{T-\tau} \times Q$, or $I + \frac{G-C}{G-\gamma} \times Q$. And the corrected Longitude of the Node, and Inclination of the Plane of the Trajectory to the Plane of the Ecliptic, will become the same, if in the Operation we make Use of the Errors arising from the second Position.

$$\begin{array}{c|c}
 \begin{array}{ccc} I & & I+Q \\ & \diagdown & \diagup \\ T-S & & T-S \end{array} &
 \begin{array}{ccc} I & & I+Q \\ & \diagdown & \diagup \\ G-C & & \gamma-C \end{array} \\
 \hline
 T-\tau : T-S :: Q : \frac{T-S}{T-\tau} \times Q &
 G-\gamma : G-C :: Q : \frac{G-C}{G-\gamma} \times Q
 \end{array}$$

X x x 4

And

And because both the Errors come under Consideration, as well that in the whole Time, as that in the Ratio between the two Times between the three Observations, let us suppose as

well $\frac{T-S}{T-t} \times P$, as $\frac{G-C}{G-g} \times P$, to be separately

equal to $m \times P$; that is, $\frac{T-S}{T-t} = m$ and $\frac{G-C}{G-g}$

$= m$: Likewise let us suppose as well $\frac{T-S}{T-\tau} \times Q$

as $\frac{G-C}{G-\gamma} \times Q = n \times Q$; that is, $\frac{T-S}{T-\tau} = n$, and

$\frac{G-C}{G-\gamma} = n$. Whence $mT - mt$ will $= T - S$,

and $mG - mg$ will $= G - C$; and also $nT - n\tau = T - S$, and $nG - n\gamma = G - C$: Whence the Double of $T - S = mT - mt + nT - n\tau$, and the Double of $G - C = mG - mg + nG - n\gamma$; and therefore, going backwards, if m and n be taken in such Manner, that the Double of $T - S$ may be $= mT - mt + nT - n\tau$, and the Double of $G - C = mG - mg + nG - n\gamma$,

then as well $\frac{T-S}{T-t} \times P$, as $\frac{G-C}{G-g} \times P$ will be

$= m \times P$, and likewise as well $\frac{T-S}{T-\tau} \times Q$ as $\frac{G-C}{G-\gamma} \times Q$ will be $= n \times Q$; therefore the Er-

ror of the Longitude of the Node will be $m \times P$, and the Error of the Inclination of the Plane of the Trajectory will be $n \times Q$: Whence the corrected Longitude of the Node will be $K + mP$, and the corrected Inclination of the Plane of the Trajectory to the Plane of the Ecliptic is $I + nQ$.
The

Book V. of ASTRONOMY. 793

The Plane of the Trajectory being corrected, as well in respect to its Interfection with the Plane of the Ecliptic (that is, the Longitude of either Node) as its Inclination with the Plane of the Ecliptic; find (as before) in the Plane thus corrected, and now most accurately posited, three Points, which may agree with the three aforesaid most accurate Observations, and (as before) taking the Sun's Center for your Focus, thro' the three said Points, draw a Conic Section: And this, being corrected, will be the Comet's requir'd Trajectory.

SCHOLIUM.

Having found the correct and accurate Trajectory; its Axis, *Latus rectum* to the Axis, the Time in which the Comet is in the Perihelium, and the other Things belonging to it, will also be accurately defin'd, according to the Rules foregoing. If it happens that an Ellipsis describ'd about the Sun's Center, as its Focus, (and not a Parabola) goes thro' those three Points: That is, if that Part of the Comet's Trajectory, which comes under our Observation, does evidently shew that the Comet returns in an Orbit, and does not run out *in infinitum*, then from knowing the greater Axis of this Orbit, you will know the Comet's Periodical Time, for (by *Prop. 40. and 42. B. I.*) the Square of the Periodical Time of a Comet is to the Square of the Periodical Time of the Earth about the Sun, as the Cube of the greater Axis of the Orbit of the Comet to the Cube of the greater Axis of the Orbit which the Earth describes about the Sun, or to the Double of the mean Distance of the Earth from the Sun.

If, from the known principal *Latus rectum*, as, for Example, of the three Trajectories describ'd

in

794 *The* ELEMENTS Book V.

in the three assum'd Planes before the full Correction, viz. R the first, r the second, and s the thrd, you wou'd find the principal *Latus rectum* of this last Trajectory describ'd in the Plane accurately posited; it may be found by the *Rule of False* according to the Method of the Things above; and it will be equal to $R + m r - m R + n s - n R$. For, in the true and accurate Longitude of either of the Nodes, you add to its Longitude in the first Plane (determin'd by *Prop. 28.*) the Excess of the assum'd Longitude in the second Plane above the other Longitude multiplied into m ; and in the accurate Inclination of the Plane of the Trajectory, you add to the Inclination of the first Plane (determin'd by *Prop. 29.*) the Excess of the assum'd Inclination in the third Plane, above the foregoing Inclination multiplied into n . And since the Trajectory and its *Latus rectum* changes, as well upon Account of the Longitude of the Node changing, as of the Inclination of the Plane to the Plane of the Ecliptic changing, to the *Latus rectum* of the Trajectory in the first Plane, or to R must be added $m r - m R$, that is, the Excess of the *Latus rectum* in the second Plane above the *Latus rectum* in the first Plane multiplied into m ; and also you must superadd $n s - n R$, namely, the Excess of the *Latus rectum* in the third Plane, above the *Latus rectum* in the first drawn into n . And after that Manner you will have the true *Latus rectum*, viz. $R + m r - m R + n s - n R$. Much after the same Manner will be corrected the *Latus transversum*, if it be defin'd in Magnitude.

SEC-

SECTION IV.

Of the Manner of finding a Comet's Helio-
centric or Geocentric Place in its Traje-
ctory before determin'd for a given Time;
and of the Tables necessary for readily
defining the Comet's Motions.

PROPOSITION XXXII. LEMMA.

FROM the Focus *S* of the given Parabola *VP*,
whose Vertex is *V* and Axis *VE*, to draw the
Right Line *SA* in such Manner, that the parabolic
Area *VRAS* comprehended under the Right Lines
VS, *SA*, and the parabolic Curve *VRA* may be e-
qual to a given rectilineal Figure; as for Exam-
ple, to the Square of the Right Line *B*; and to de-
fine, by Computation, the Length and Inclination
to the Axis *VE* of the Right Line *SA* so drawn.
[Fig. XXXIII.]

Suppose it done, and thro' *V* draw the Right
Line *VF* perpendicular to *VE*, and let the Point
C be in it, thro' which *CA*, being drawn parallel
to *VE*, meets with the Parabola in the Point *A*,
such as *SA* being drawn, may give a Solution to
the Problem. Thro' *A* draw *AD* perpendicular
to *VE*, meeting with the Axis at *D*. Let *VS*,
the Distance of the Focus from the Vertex, be
call'd *f*; and (as is well known) the principal
Latus rectum of the Parabola will be $4f$. Let the
unknown Quantity *VC* be call'd x ; and *AD* will
be also $= x$, therefore (from *El. Conic.*) *VD* or
its equal *CA* will be $\frac{x^2}{4f}$: Whence the rectangu-

lar Parallelogram *VCAD* is equal to $\frac{x^3}{4f}$, and the
parabolic

796 *The ELEMENTS* Book V.

parabolic Space $VRAD$ (which, from *Archimedes's Dimension of the Parabola*, is subfesquialteral

to the forefaid *Parallelogram*) will be $\frac{x^3}{6f}$, name-

ly, $\frac{2}{3}$ of $\frac{x^3}{4f}$. Moreover, ſince VD is $= \frac{x^2}{4f}$ and VS
 $= f$, SD will be $= \frac{x^2}{4f} - f$, if S be between V

and D , as in the 1st *Fig. XXXIII*; or if D be
between V and S , as in the 2d *Fig. XXXIII*, SD

will be $= f - \frac{x^2}{4f}$; that is, in the first Caſe,
 $SD = \frac{x^2 - 4f^2}{4f}$, and in the ſecond $SD = \frac{4f^2 - x^2}{4f}$

Whence the Rectangle, under AD and DS , will
be $= \frac{x^3 - 4f^2x}{4f}$ or $\frac{4f^2x - x^3}{4f}$; and therefore

half of it, namely, the Triangle ASD will be
 $\frac{x^3 - 4f^2x}{8f}$ or $\frac{4f^2x - x^3}{8f}$. And therefore the Space

$VRAS$ (for it is in the firſt Caſe equal to $VRAD$,
the Triangle ASD being taken from it, in the
ſecond equal to $VRAD$, increas'd by ASD)

will be $\frac{x^3}{6f} - \frac{x^3 - 4f^2x}{8f}$ in the firſt Caſe, and
 $\frac{x^3}{6f} + \frac{4f^2x - x^3}{8f}$ in the ſecond; and therefore
in every Caſe $VRSA$ is equal to $\frac{x^3 + 12f^2x}{24f}$.

And this muſt be equal to b^2 , therefore $24fb^2$
 $= x^3 + 12f^2x$. And having diſpoſed the $\mathcal{A}$ -
quation, as is uſual, $x^3 = -12f^2x + 24fb^2$.
Therefore, by *Cardan's Rule*, the Value of
 x , or AD , expreſs'd in known Quantities, will
be

be $\sqrt{C:12fb'} + \sqrt{144f^2b' + 64f^6} - \sqrt{C:12fb' - \sqrt{144f^2b' + 64f^6}}$. But when AD is known, VD will also be known; for it is equal to the Square of AD apply'd to the Quadruple of the known Quantity VS . Therefore (by the 3d *Datum*) you have the Line compounded of DV and VS , to which the Right Line SA is equal, by *Corol. 2. Prop. 14.* and therefore this Line SA will be also given in Magnitude; but DA is also given, therefore (by 1 *Dat.*) the Ratio between SA and AD , that is, between the Radius and the Right Sine of the Angle ASD , which SA contains with the Axis. Therefore that Angle ASD , or $VS A$, is known: And SA was before known in Magnitude. Q. E. F.

On the contrary also in the given Parabola, you may find the Area $VRAS$, which will make the Right Line SA equal to the given Right Line; that is, you may find the Ratio of the given Area $VRAS$ to the Square of the Right Line VS , which will make SA to be in a given Ratio to the Right Line VS .

SCHOLIUM I. (Fig. XXXIV.)

We have before shew'd the Method of the Calculation made use of in Astronomy, for determining the Magnitude and Position of the Right Line AS . But if you like a Geometrical Construction better, it may thus be easily had, if according to the universal Method of Geometricians, the before found Cubic Equation $x^3 + 12f \cdot x - 24fb' = 0$, be constructed by the propos'd Parabola RVP and a Circle. Let the *Latus rectum* of the Parabola propos'd (which is equal to $4f$) be call'd L ; $12f = q$, and $24fb' = r$, that the propos'd Equation may be in this usual Form $x^3 + qx - r = 0$. Take (ac-

cording

798 *The ELEMENTS* Book V.

cording to the commonly known Precepts of this Construction) $VG = \frac{1}{2}L$ in the Axis of the Parabola, and $GH = \frac{q}{2L}$ (and that backward towards the Vertex, because it is $+q$) that is, in our $\mathcal{A}$ equation $GH = \frac{12f^2}{8f}$ or $\frac{3}{2}f$, whence it happens that the Point H will come to be in the middle Point between V and S : For because $VG = \frac{1}{2}L$, VG will be the double of VS , and taking from hence $GH = \frac{1}{2}VS$, VH , will be $= \frac{1}{2}VS$. Moreover, suppose HK perpendicular to $VS = \frac{r}{2L}$; that is, in our Case $= \frac{24fb^2}{32f^2} = \frac{3b^2}{4f}$; that is, equal to $3m$, if $m = \frac{b^2}{4f}$; (or to the Right Line found by this *Prop.*) if (by *Prop. 45. El. 1.*) the propos'd rectilinear Figure be apply'd to the *Latus rectum* of the Parabola) and on those Parts of the Right Line VS , towards which AS is to be drawn. And a Circle, describ'd with the Center K thro' V , will cut the Parabola RVP in the requir'd Point A , and AD , which is perpendicular to VS , will be x , or the Root of the foregoing $\mathcal{A}$ equation. This Construction has no Need of a Demonstration, because of the common Construction of the $\mathcal{A}$ equation $x^3 + 12f^2x - 24fb^2 = 0$, arising from the Nature of the Problem: But it is easie to construct it, running over the Steps of the foregoing Analysis.

After the same Manner may be defin'd the Magnitude and Position of the Right Line AS , drawn from the Focus S , such that the parabolic Space $\mathcal{Q}RAS\mathcal{Q}$, comprehended between that

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that Line, and another SQ given in Position different from SV , and that Part of the Parabolic Curve between them QRA , may be equal to the given Rectilineal Figure B^q . For the Axis SV being drawn, you must draw the Right Line SA in such Manner, that the Space $VRAS$ may be equal to the Sum or Difference of the Spaces B^q , and QVS , according as SQ lies towards the same Parts of SV to which SA must be drawn, or towards contrary Parts.

SCHOLIUM 2 [Fig. XXXV.]

Supposing VP and $v\sigma$ any two Parabolas, whose Foci are S and σ , and Vertices V and v , and supposing the Areas VAS , $v\alpha\sigma$, equal to the respective Squares of B and β ; if B be to VS , as β to $v\sigma$, then AS , $\alpha\sigma$, VS , $v\sigma$, will be proportional, and the Angles VSA , $v\sigma\alpha$, equal. For, because all Parabolas are similar Figures, and the Foci, and Vertices, are Points similarly posited in them; and besides, as the Figures VAS , $v\alpha\sigma$, are in a duplicate Ratio of the homologous Sides VS , $v\sigma$, (for their Equals B^q , and β^q are in a duplicate Ratio of B to β , that is, by Supposition, in a duplicate Ratio of VS to $v\sigma$;) it is also plain, that these Lines are similar to one another, and therefore the Angles VSA , $v\sigma\alpha$, equal, and that AS is to $\alpha\sigma$, as VS to $v\sigma$. And therefore from hence, when you have laid down SV , the Distance of the Vertex of the Parabola from its Focus, express'd in any round Number, you will find the Angle VSA , and the Length of the Right Line SA , according to the different Ratio between the Rectilineal Figure B^q , to which VSA is suppos'd equal, and the Square of VS . And the Magnitudes of the Right Lines AS , and of the Angles

Angles VSA dispos'd in Tables, serve for any Parabola, since (as was before shewn) the Right Line AS will, in every Parabola, still have the same Ratio and Position to VS , the same Ratio remaining between B and VS .

PROPOSITION XXXIII.

TO find a propos'd Comet's Heliocentric Place in Longitude and Latitude, and its Distance from the Sun at a given Time. [Fig. XXXVI.]

Let $\delta\delta V$ be a Comet's Trajectory, describ'd by Prop. 26. and corrected by Prop. 31. in whose Focus S , let the Sun be plac'd; and let ω be a Point in it, in which the Comet is at any one of the Observations by which the Trajectory was determin'd: Let VX be the Axis of this Trajectory, whose Position is known by Prop. 30: By the same Prop. will also be known the Time that the Comet is in the Perihelium V ; and let $\delta\delta$ be the Line of Nodes known in Position by Prop. 28. If ωVR , the corrected Trajectory of the Comet, be found to be parabolical, take such a Space as may be to the Space ωVS , (known from Archimedes's Quadrature of the Parabola) as the Interval between the given Time and the before-found Moment that the Comet is in the Perihelium, to the Interval between the said Moment and the Observation of the Comet in ω , call such a Space B^9 . By the foregoing Prop. Let the parabolic Space $URAS$ be suppos'd equal to B^9 , and find as well the Position as the Length of the Right Line SA in Respect of SV , whose Position in Length, in Respect of the Distance of the Earth's Aphelium from the Sun, is known from what has been said before. But if a Comet's Trajectory be found to be Elliptical, draw (by the 3d, 6th, or

Book V. of ASTRONOMY. 801

7th Prop. of B. III. as was done in the Planet's Orbits) the Right Line SA , in such Manner, that the Area $VRAS$ may be to the whole Area of the Comet's Orbit, as the Interval between the given Time, and the Moment that the Comet is in the Perihelium, to the whole Periodical Time of the Comet, known from knowing its greater Axis; and you will have the Right Line SA , as well in Position as Length. In either Case, from A , to the Line of Nodes $\Omega\varnothing$, draw the Perpendicular AN meeting with $\varnothing\Omega$ at N ; from the same A , to the Plane of the Ecliptic, let fall a Perpendicular meeting with it at α ; join αN , αS ; and the Angle $AN\alpha$ will be the Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic, and therefore known by Prop. 29. Because the Angles VSA and VSN are known, the Angle NSA , which is their Sum or Difference, will also be known: Therefore, in the Triangle $SN\alpha$ rectangular at N , having (by what foregoes) the Side SA express'd in Parts of the Mean Distance of the Earth from the Sun, and the Angle NSA , you will have the Sides SN and NA . In the Triangle $N\alpha A$ rectangular at α , knowing the Side NA , and the Angle $AN\alpha$ (which is the Inclination of the Comet's Trajectory, to the Plane of the Ecliptic, found by Prop. 29. and corrected by Prop. 31.) you will know the other two Sides $N\alpha$ and $A\alpha$. Again, in the Triangle $SN\alpha$ (in the Plane of the Ecliptic,) besides the Right Angle at N , you have the Sides SN and $N\alpha$; therefore you will have the Side $S\alpha$ and the Angle $NS\alpha$: But (by Prop. 29.) the Position of the Right Line SN is given; therefore also the Position of the Right Line $S\alpha$ will be given; that is, the Heliocentric Place of a Comet reduc'd to the Ecliptic, or

the Comet's Heliocentric Longitude. Lastly, in the Triangle $S^a A$, rectangular at a , you have all the Sides, as being found before; therefore you will also know AS^a the Comet's Heliocentric Latitude, if there is any Occasion of reducing the Place of the Phænomenon, seen from the Sun to the Plane of the Ecliptic, rather than any other Plane.

And on the Contrary, this shews you how to find the Time that a Comet will be in a given Place of its Trajectory.

PROPOSITION XXXIV.

TO find the Heliocentric Place, as to Longitude and Latitude, and the Distance from the Earth, of a propos'd Comet at a given Time. [Fig. XXXVII.]

The same Things being laid down, as in the foregoing Prop. besides them let BT be the *Magnus Orbis*, and in it let T be the Place of the Earth for the Time given, found by Prop. 18. B. III. draw TA , T^a : And (by Prop. 18. El. 11.) let the Plane of the Triangle TA^a be perpendicular to the Plane of the Ecliptic. In the Triangle TS^a (in the Plane of the Ecliptic, you have the Sides S^a , ST , the first of them found by the foregoing Prop. and the last from the Theory of the Earth (each express'd in Parts of the Mean Distance of the Earth from the Sun) and the Angle TS^a comprehended by them, (for each of them is given in Position;) therefore you will also have the Side T^a , and the Angle ST^a : But TS is given in Position, namely, the Place of the Sun for a given Time; therefore the Position of the Right Line T^a is given, that is, the Place of the Comet in A seen from the Earth, reduc'd to the Ecliptic, or the requir'd Geocentric Longitude of the

Book V. of ASTRONOMY. 803

the Comet. Moreover, in the Triangle A^aT , besides the Right Angle A^aT , you have the two Sides A^a , T^a , this last lately found in the Parts of the Mean Distance of the Earth from the Sun, and the first of them found by the foregoing *Prop.* in the same Parts; therefore the Angle AT^a , the requir'd Latitude of the Comet, seen from the Earth, or its Geocentric Latitude, is known; as also the Hypothenufe TA , being the Comet's Distance from the Earth. Q. E. F.

Thus may be known when the Motion of the Comet will be retrograde, in respect of its Motion before, because the Earth, together with the Eye, has mov'd in the mean Time, as it happens in respect of the Planets, when at the same Time the Comet, seen from the Sun, appears always to go on the same Way. And, conversly, that Phænomenon of the Retrogression of a Comet, observ'd from the Earth, shews that Comets are not very far distant from the Earth, but that when they are seen by us, they come down into the Region of the Planets. Moreover, as the Eye and the Comet are carried about the Sun, in different Planes, the Comet seen by the Eye, and referr'd to the Fix'd Stars, will, from the Laws of Optics, appear not to describe a great Circle among them, but to deviate differently from it; when at the same Time, if it was seen from the Sun, which is plac'd in the Plane of the Trajectory, it would accurately describe a great Circle among the Fix'd Stars. This Deviation from a great Circle (which also agrees with the Planets, as well as the Retrogradation mention'd before) puts it out of all Doubt, that the Comets are in the Region of the Planets.

PROPOSITION XXXV.

TO describe Tables necessary for readily and exactly finding the Places of Comets.

The Trajectory of any Comet being defin'd, if it be found to be Elliptical, (such as we suppose it will be found, as being the only one of the Conic Sections which agrees with the Phenomena of Comets, if they are Bodies coeval with the World, revolving in an Orbit like the Planets) Tables, for readily finding any Comet's Place for a Time given, will be of the same Kind (and constructed after the same Manner) as the Tables of the Primary Planets, and their Use the same, concerning which we have spoken in the 39th and 40th Propositions of the 3d Book.

If the Orbits of all those Comets, of which we have Observations accurate enough, are pretty nearly determin'd, their Periods are also known (as is shewn in the Scholium of *Prop. 31.*) and the Times that they will again be seen, and they may be corrected from Observations which will be made then; and at Length all Comets will be reduc'd to a Rule, perhaps in a shorter Time than has been done concerning the Planets, after their Number and their revolving in Orbits were known. I hear that it is done in 24 Comets, (whose Places have been observ'd by Astronomers) already by Dr. *Halley*; and I have Reason to believe, that the Number of the Comets is not vastly great: But it is likely that the Periods of some of them are so great, from the immense Interval between Saturn, and the nearest Fix'd Stars, that they are not yet descended into the Regions of the Planets (where only they are visible to us) since Astronomers have begun to look upon them as cœlestial Bodies, and more

accu-

accurately to determine their Places; therefore the Determination of the Orbits, in which those Comets move, must be left to Posterity. But sometimes it will happen, that some Comet will finish a whole Period, and yet not be observ'd to do so by Astronomers, upon Account of its Nearness to the Sun; which, for the same Reason, does also sometimes happen to *Mercury* itself.

Besides, the greater Axes of the Comet's Orbits, upon Account of the Disturbance arising from the Attraction either of other Comets or of Jupiter and Saturn, may a little change their Situation, and so the same Comet may be mistaken for a different Comet: But after several Revolutions the Thing will be plain enough; for such a Disturbance will not produce very great Effects; and tho' the Orbit of the Moon is every Day chang'd, yet it is reduc'd to an Astronomical Calculation, and consider'd as if it was always the same.

But tho' the Orbits of the Comets are not yet determin'd, some Tables may be constructed very useful for the more easie defining of their Motions, and a more ready Way of describing their Trajectories: As for *Example*, a Table, by which, from the given Distance of a Comet from the Sun, in its Perihelion, express'd in Parts of the Mean Distance of the Earth from the Sun, may be readily defin'd the Time that a Comet, after its Perihelion, as it recedes from the Sun, will get out of our Sight. For supposing the greater Semiaxis of the Orbit of the Earth to consist of 100000000 Parts, an Arc of $59' \frac{1}{2}$ (which the Earth goes thro' by its mean diurnal Motion) will (by the Dimension of a Circle) contain 1720212 of those Parts, and the twenty fourth Part of it, or an Arc describ'd by the

Earth in its mean horary Motion, will have 71675 of those Parts : Therefore to the different Distances of a Comet from the Sun, when in its Perihelium, must respectively be ascrib'd Parts, gone thro' by a Comet in its Perihelium, in the Space of a Day or an Hour ; for these Parts will be to the former Parts, as the Velocity of a Comet in its Perihelium, to the mean Velocity of the Earth, that is (by *Prop.* 17.) in a subduplicate Ratio of 200000000 the greater Axis of the Orbit of the Earth, to the Distance of a Comet in its Perihelion from the Sun ; or Parts must be ascrib'd, expressing the Area which a Comet, by a Radius drawn to the Sun, sweeps thro' (as being made from multiplying the former in half the Distance of the Perihelion from the Sun.) Moreover, since by Observation we pretty nearly know at what Distance from the Sun (in respect of the Distance of the Earth from it) a Comet must be, to cease to be seen with the naked Eye ; that is, when the Ratio of that Distance, to the Distance of a Comet from the Sun in its Perihelion, is given, you will also (by the Inverse of *Prop.* 33.) have the Area which a Comet, with a Radius drawn to the Sun, must describe after it has pass'd the Perihelion, that it may come to that Distance from the Sun, that is, that it may cease to be seen ; and consequently the Time after the Perihelium ; as for *Example*, (by *Prop.* 11. B. I.) for one Day as the lately found Area, to the before-found Area describ'd in one Day by a Comet, by a Radius drawn to the Sun. Therefore to every Distance of the Perihelium, ascribe an agreeable Time, in which the Comet, after it has pass'd the Perihelium, as it goes from the Sun, gets out of our Sight.

Just

Book V. of ASTRONOMY. 807

Just in the same Manner, and from the same Principles, may be calculated a Table, readily shewing the Time that a Comet, after it has pass'd the Perihelium, comes to a Distance from the Sun, equal to the Distance of the Earth from the Sun, if the Perihelium of the Comet is less distant from the Sun, than the Earth itself. And this Table may be join'd to the first Table, and will, as it were, make but one Table; namely, by writing over-against every Distance of the Perihelium of a Comet, express'd in Parts of the Semidiameter of the *Magnus Orbis* (besides the Parts gone thro' by a Comet in its Perihelium, in the Space of a Day, and the Parts of the Area which a Radius sweeps thro' in the same Time, of which we spoke above,) in one Column, the Time after the Perihelium in which it comes to a Distance from the Sun, equal to the Earth's Distance from it; in the other, the Time that it comes to a Distance four Times or five Times greater than the former, namely, that in which it ceaseth to be seen by us.

Thirdly, The Table calculated by the second Scholium of *Prop. 32.* will be very convenient, in which, putting a round Number for the Distance of a Comet in the Perihelium from the Sun, over-against the different Bigness of the Area *VAS* (in the Fig. to which the said Scholium refers) express'd in Parts of the Square of the Right Line *SV*, set down the Quantity of the Angle *SVA* agreeable to it, and the Length of the Right Line *SA* express'd in Parts of the Right Line *SV* also agreeable to it. For from that and the foregoing Table, and the known Distance of a Comet in its Perihelium from the Sun, you may readily know the Longirude of a Comet from the Perihelium when seen from the Sun, and the

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808 *The* ELEMENTS Book V.

Distance of a Comet from the Sun's Center, agreeable to the Time before or after the Perihelium. For SV being given express'd in Parts of the mean Distance of the Earth from the Sun (from the first Table) you will readily have the Area which is describ'd (a Radius being drawn to the Sun) by the Comet in the Space of one Day or one Hour: Therefore it will be given, express'd in Parts of the Square of the Right Line VS , and fit for this third Table; and therefore from the given Time before, or after the Perihelium, will be given the Area VAS , proportionable to the said Time in the said Parts; therefore, from the third Table, you will immediately have the Angle VSA , which is the heliocentrical Longitude of the Comet from the Perihelium, agreeable to the propos'd Area, and therefore proportionable to the propos'd Time, and also you will have in Parts, of SV , the Length of the Right Line SA agreeable to this, which, by Help of the Rule of Three, may be express'd in Parts of the Semidiameters of the *Magnus Orbis*.

But from what has been said in former Propositions, other Tables also may be constructed, for making easy the Calculation of the Comet's Motions, which we must leave to the Industry of Astronomers.

S C H O L I U M.

By the foresaid Method, or one like it, Dr. Halley has determin'd the Orbit of the Comet, which appear'd about the End of the Year 1680, and some of the following Months, which he assum'd as Parabolical, and found (as it appears from his Letters to Sir Isaac Newton) that the Perihelium of its Orbit was in $\nearrow 27^{\circ}. 22'. 30''$; and therefore its Aphelium in $\Pi 27^{\circ}. 22'. 30''$,
that

Plate 1. Book 5.

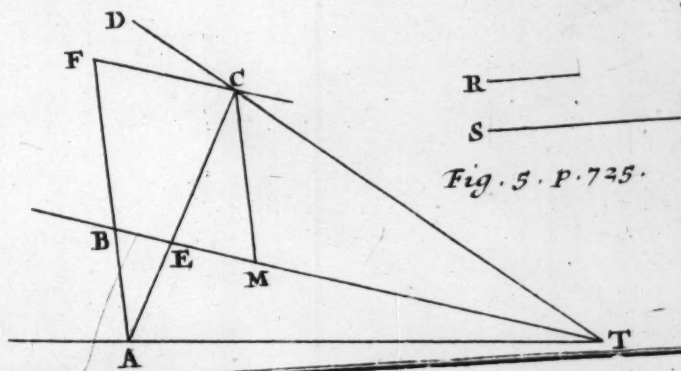
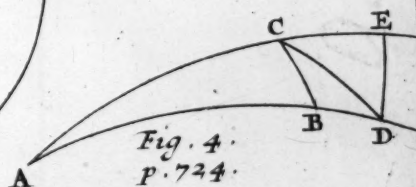
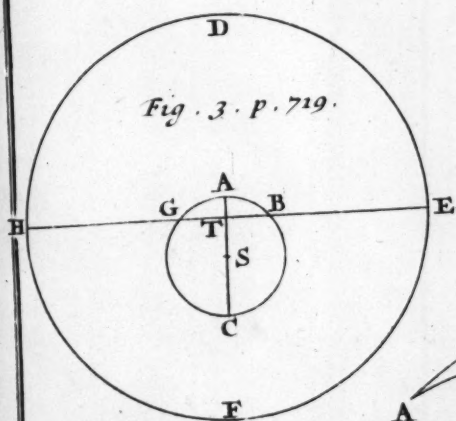
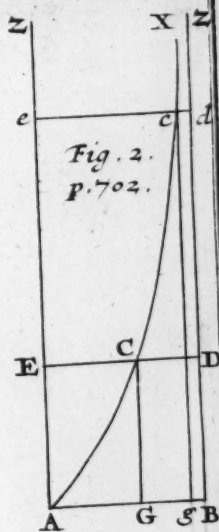
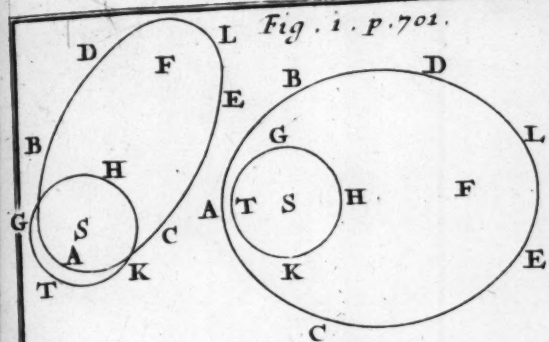


Plate 2. Book 5.

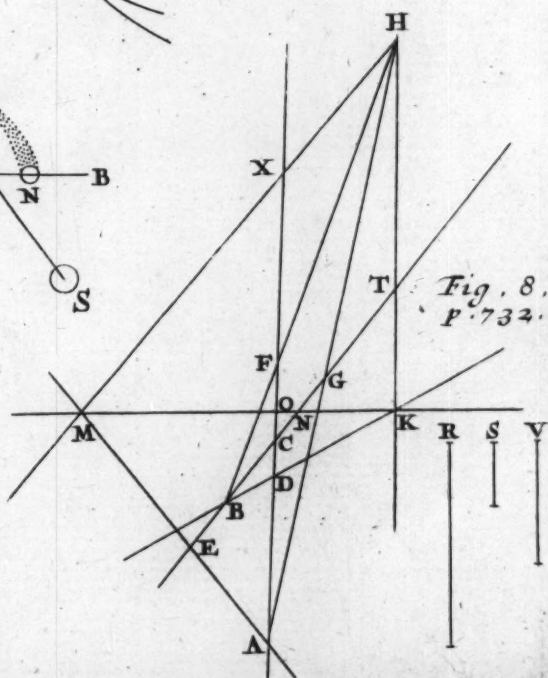
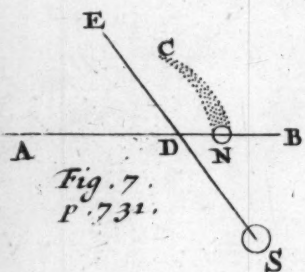
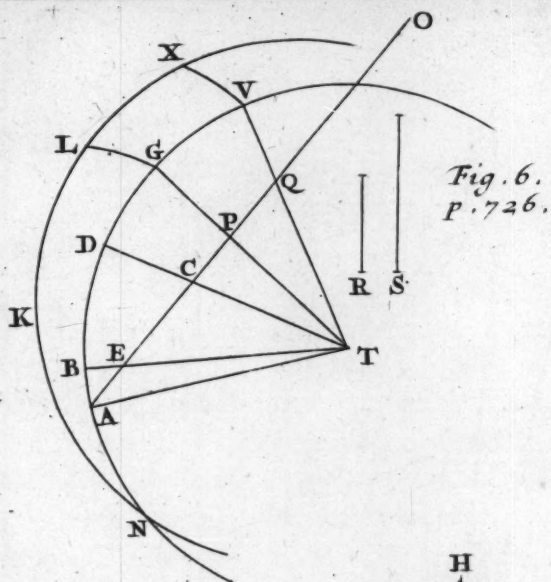
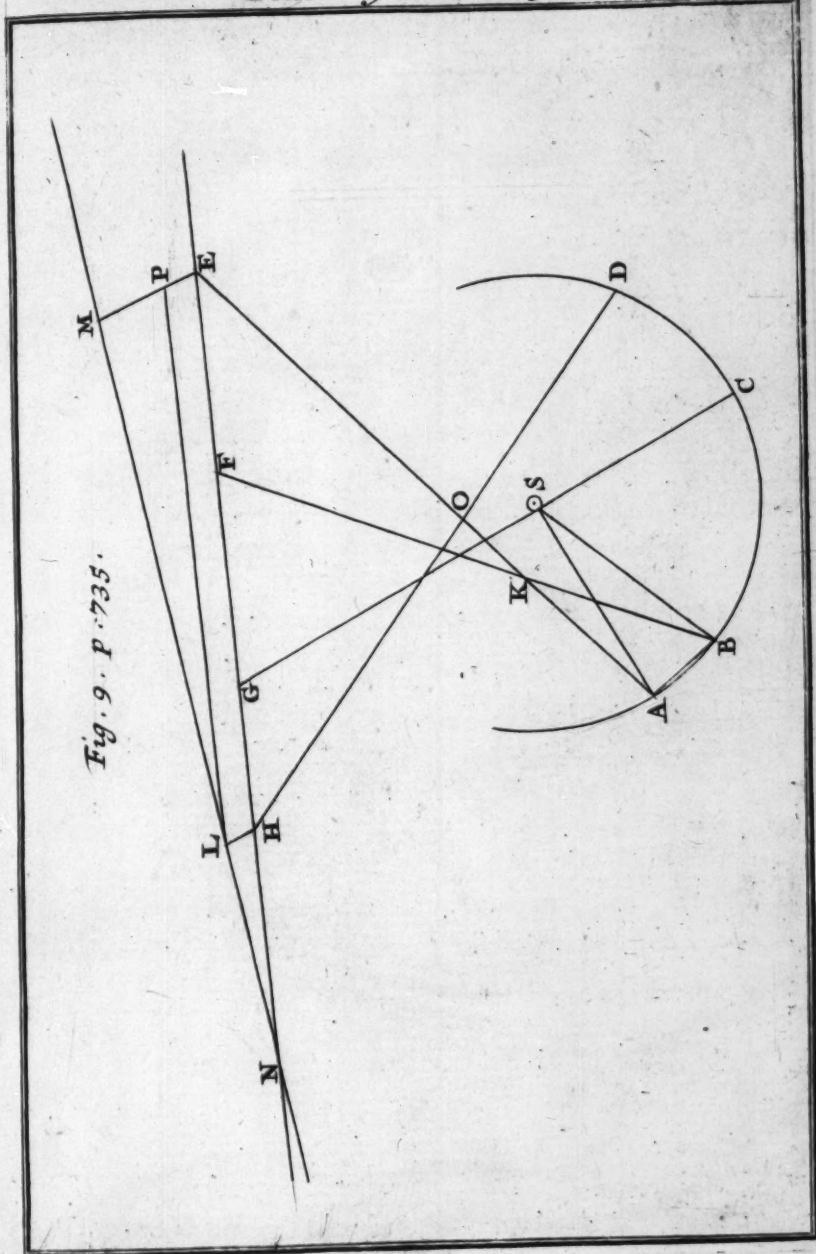


Fig. 9. P. 735.



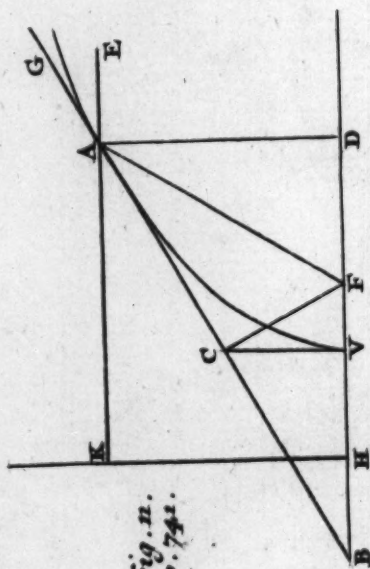
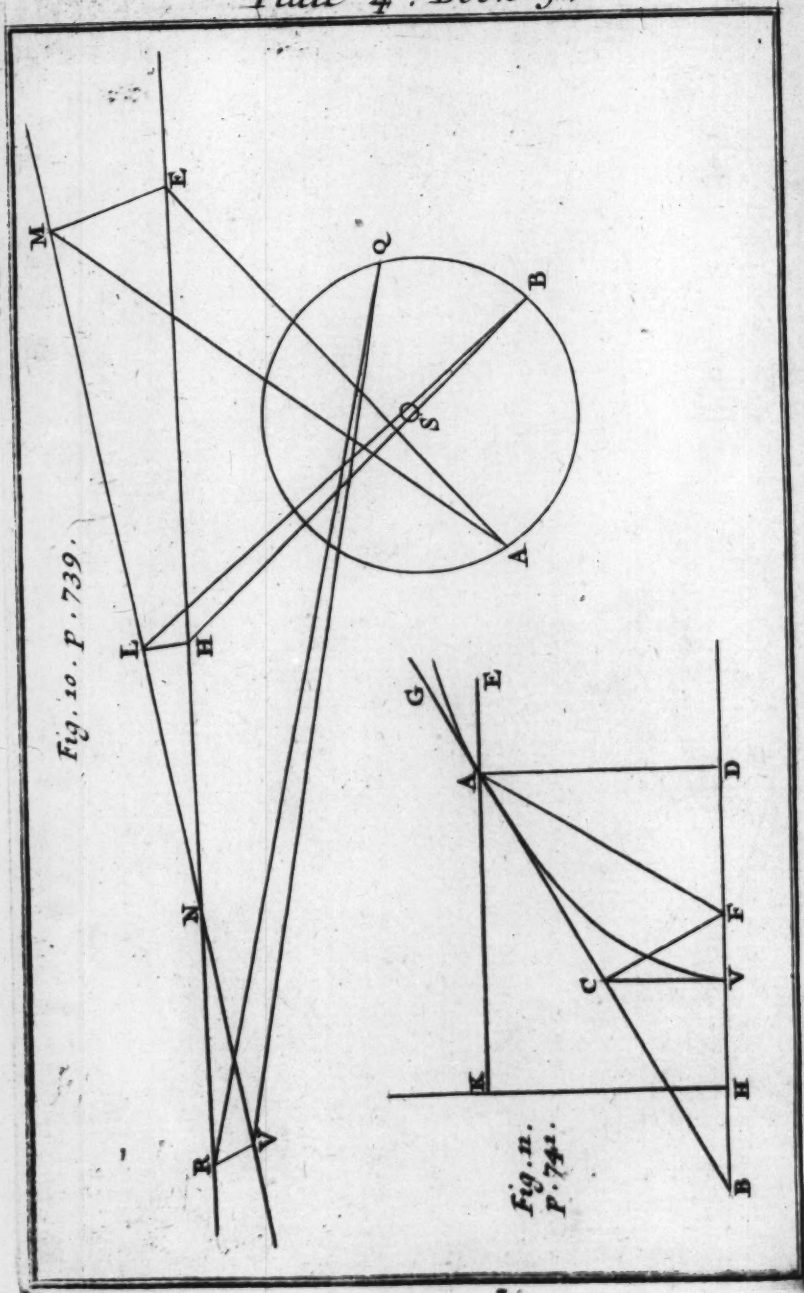


Plate 5. Book 5.

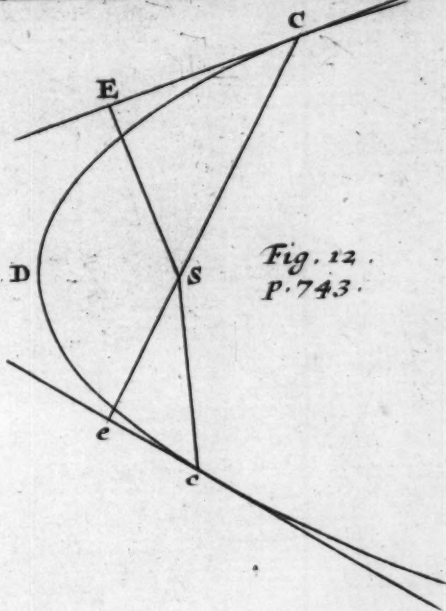


Fig. 12.
P. 743.

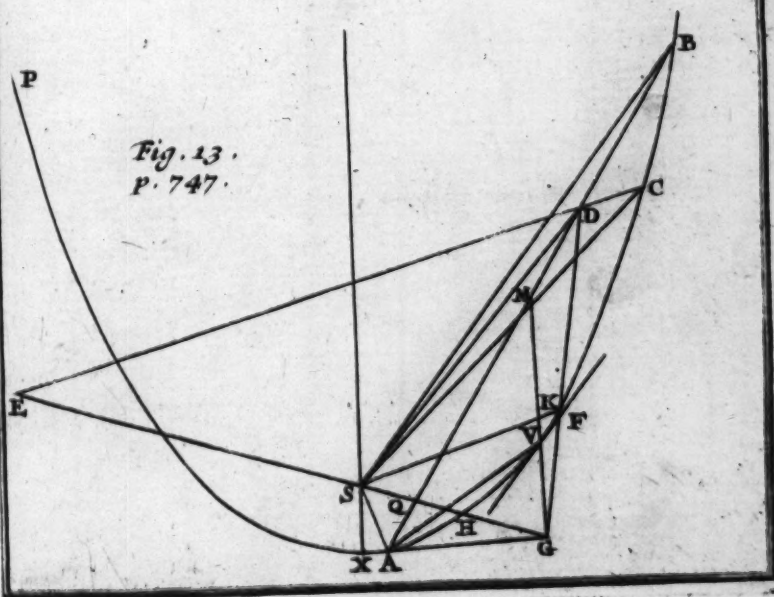


Fig. 13.
P. 747.

Plate 6. Book 5.

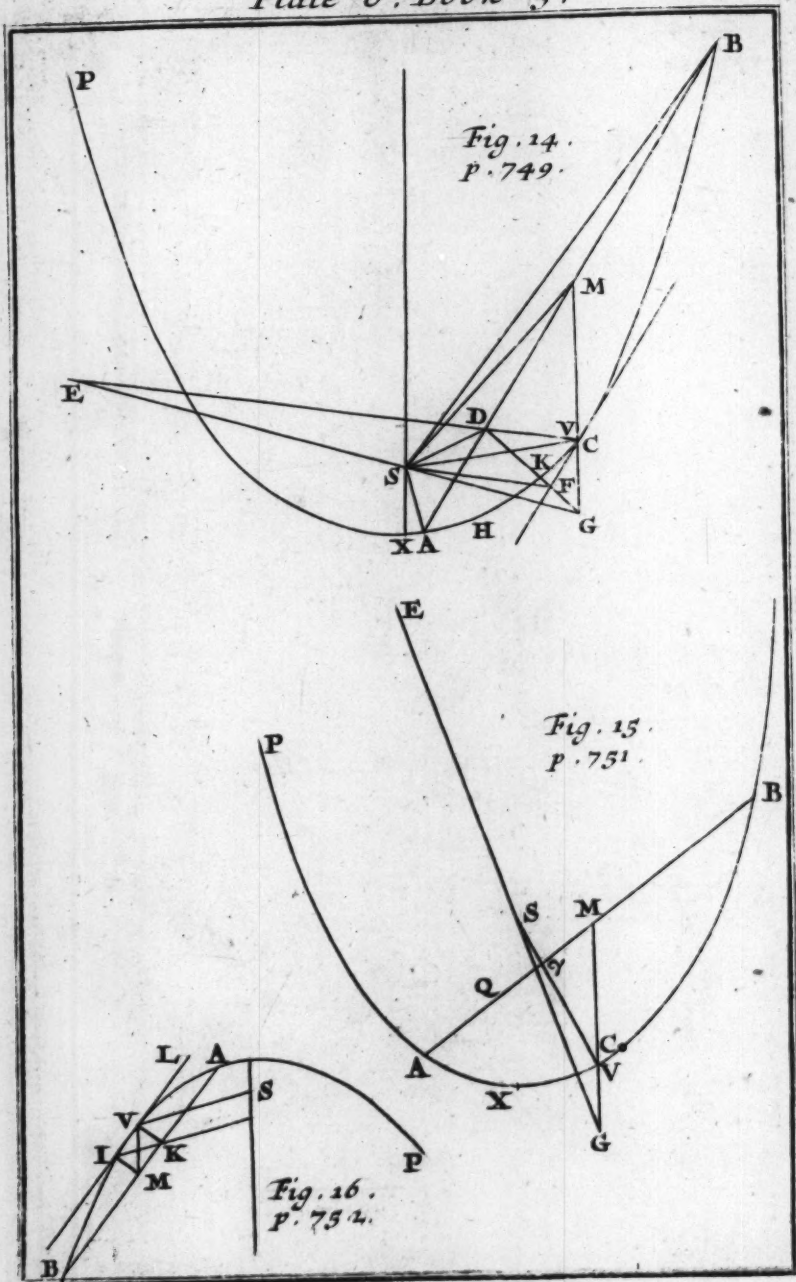


Plate 7. Book 5.

Fig. 17.
p. 753.

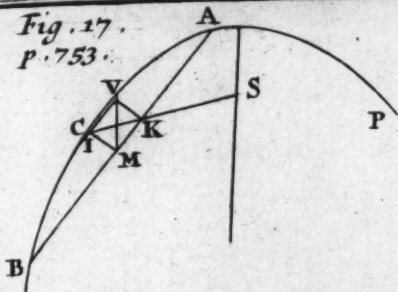


Fig. 18.
P. 753.

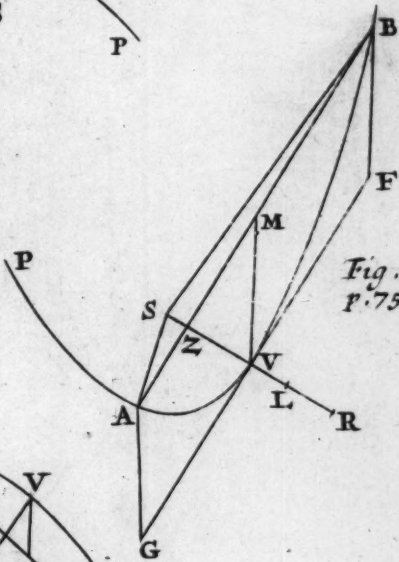


Fig. 19.
p. 758.

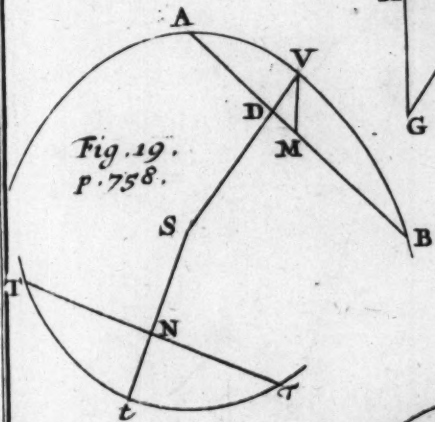


Fig. 20.
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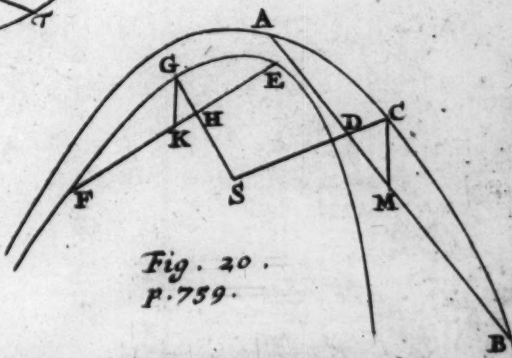


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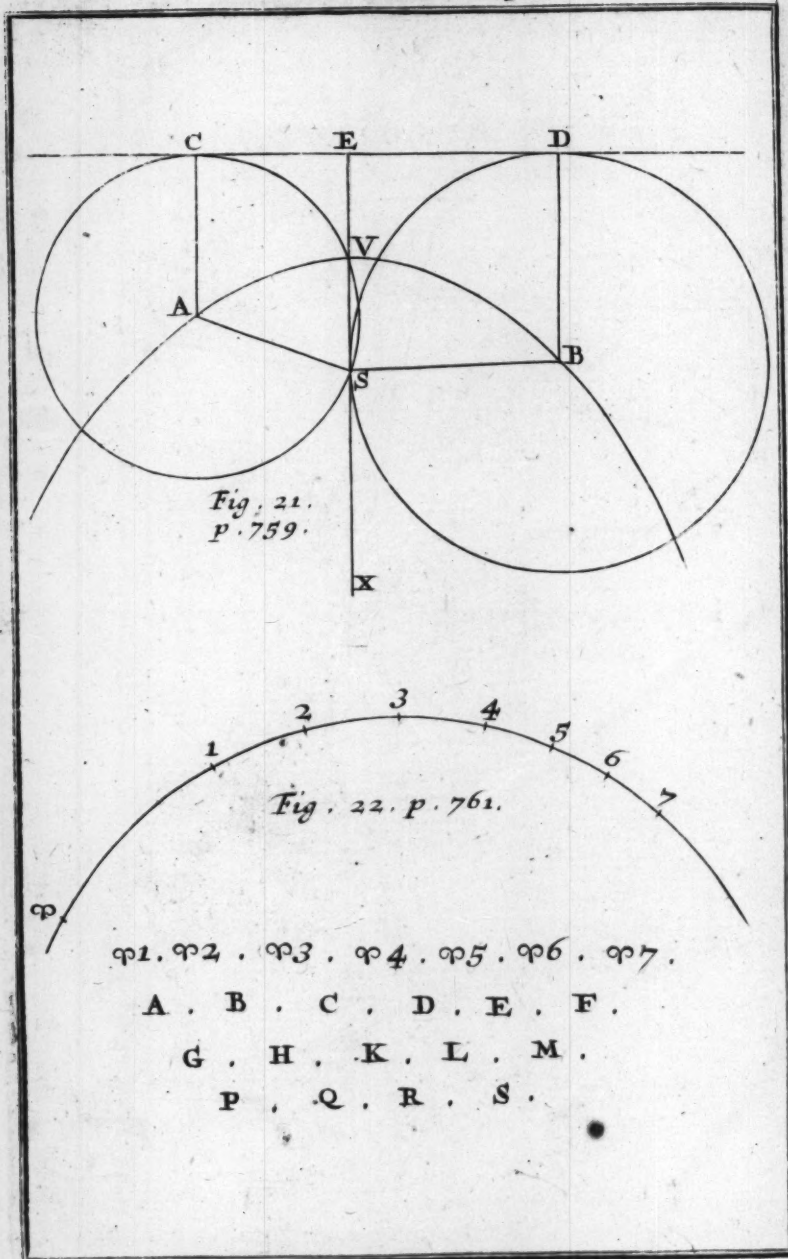


Plate 9. Book 5.

Fig. 23. p. 765.

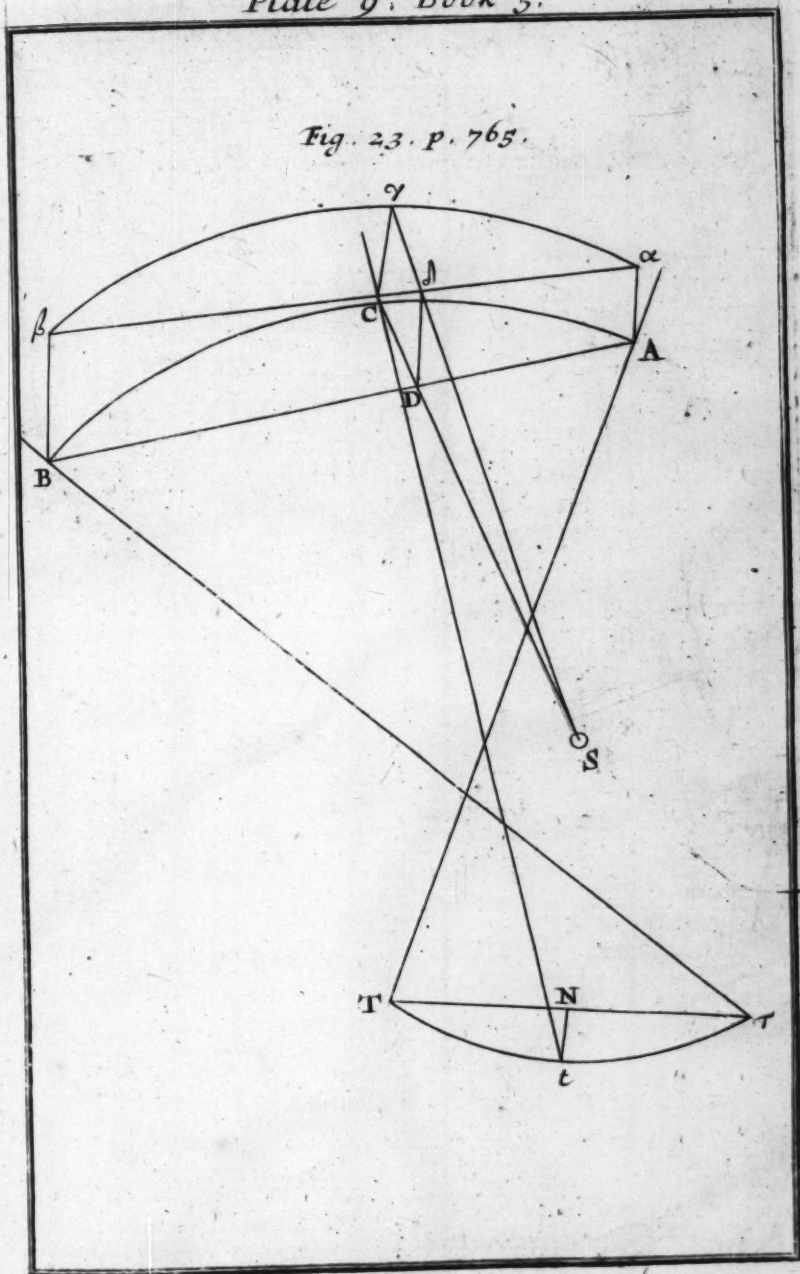


Fig. 24. P 768

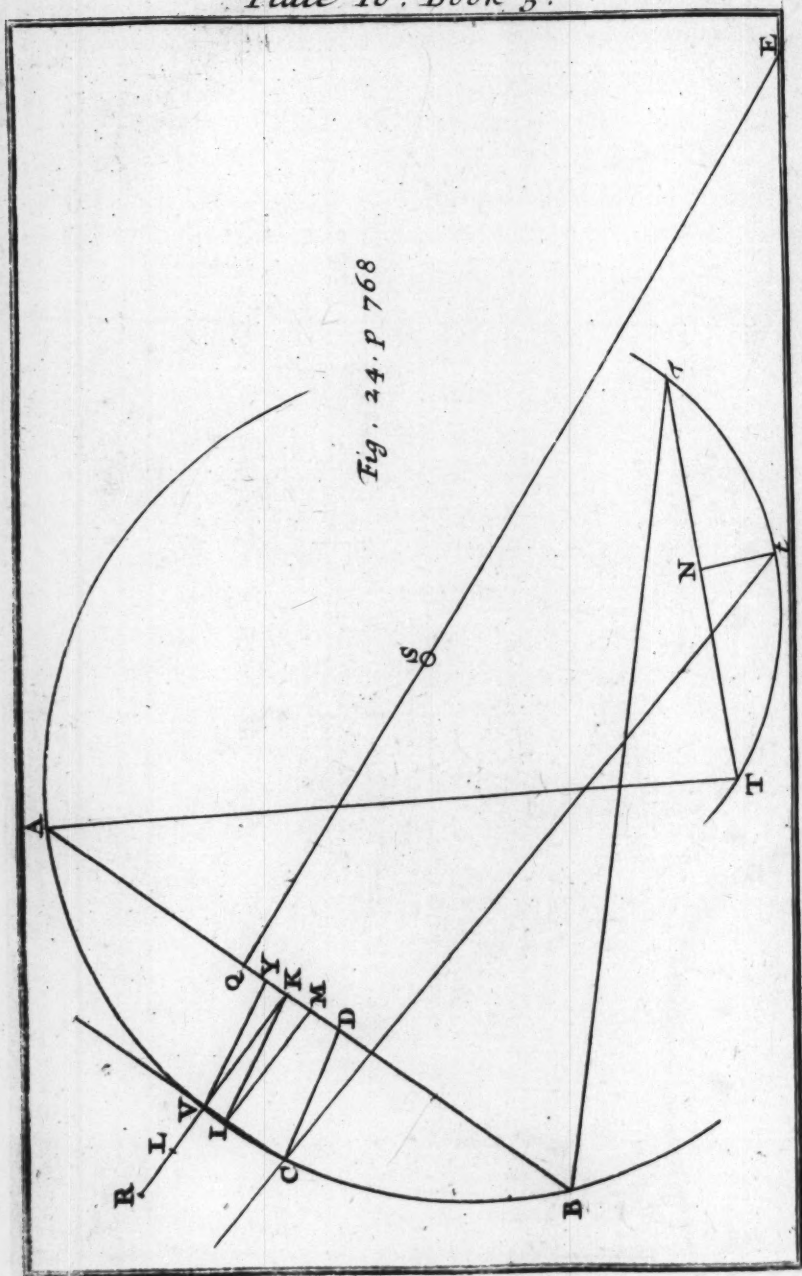
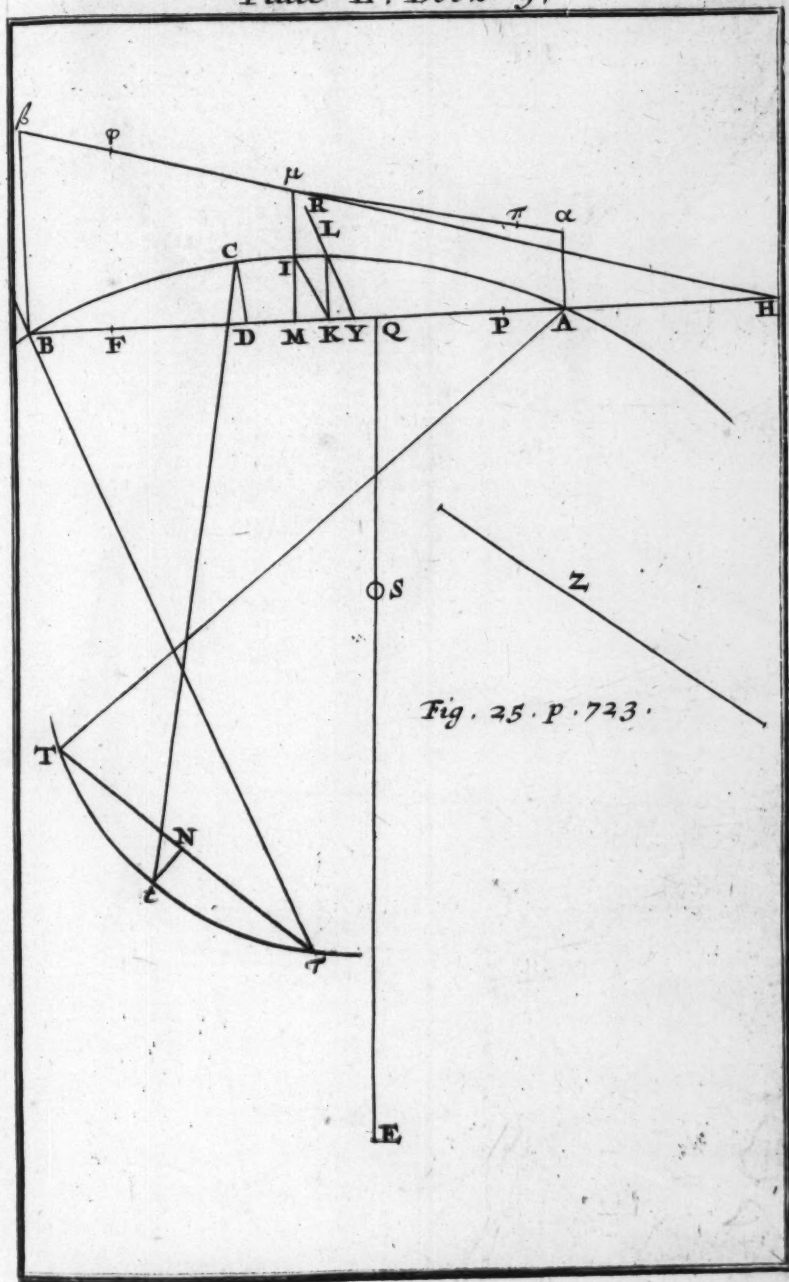


Plate 11, Book 5.



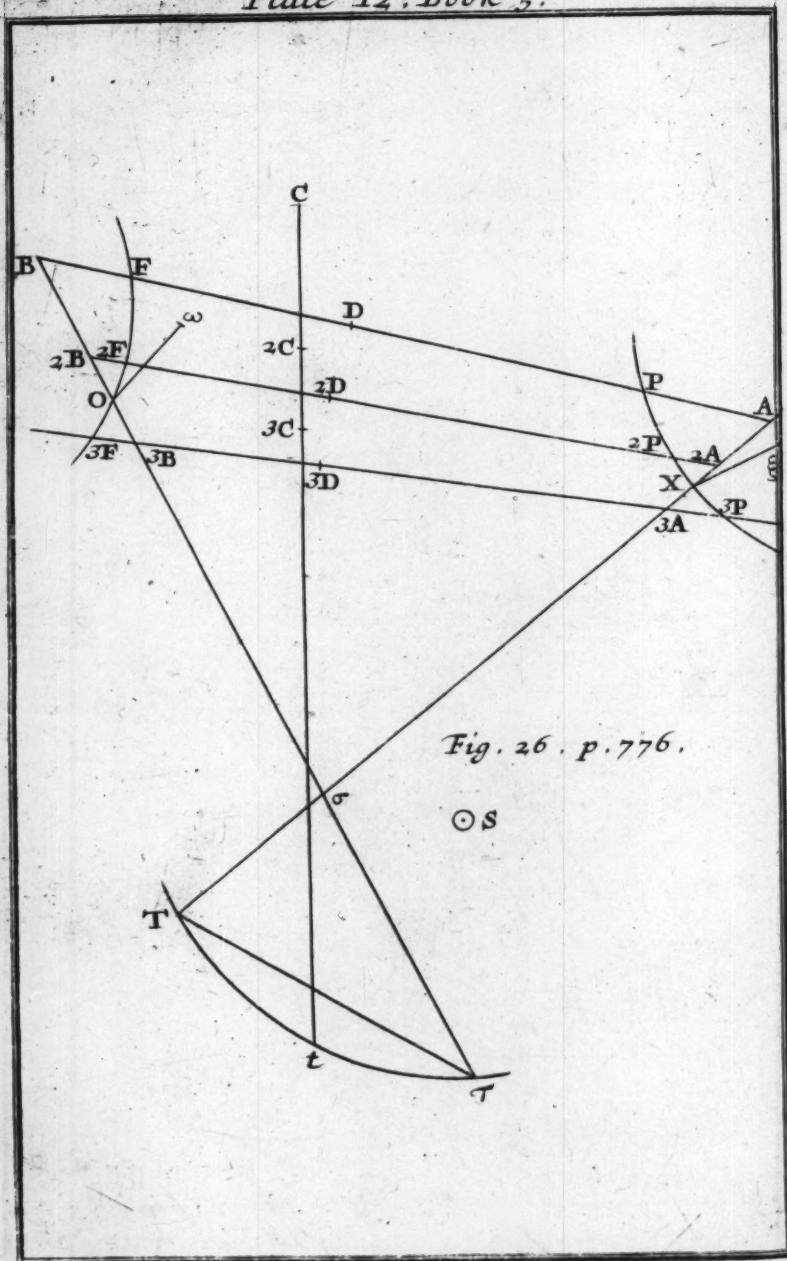


Plate 13. Book 5.

Fig. 27. p. 779.

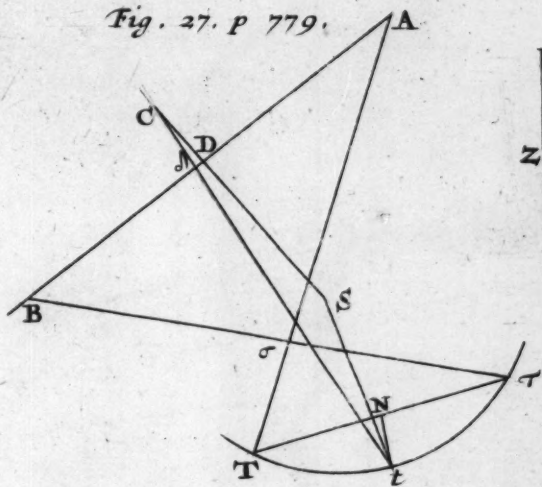


Fig. 28. p. 781.

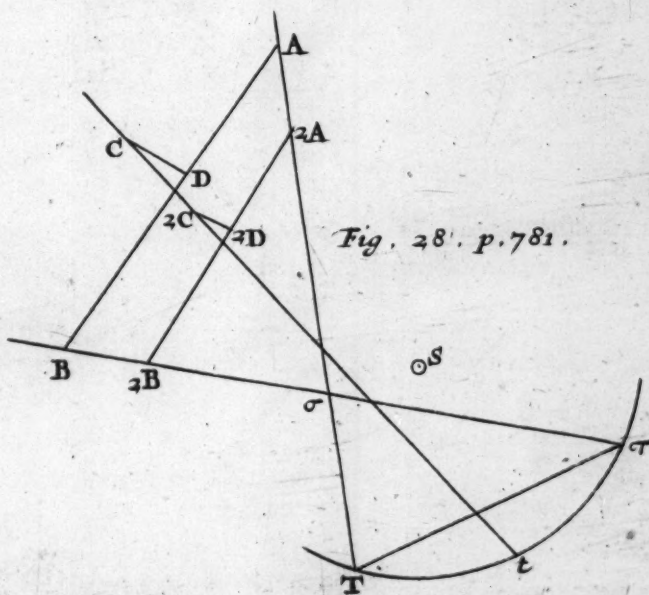


Plate 14. Book 5.

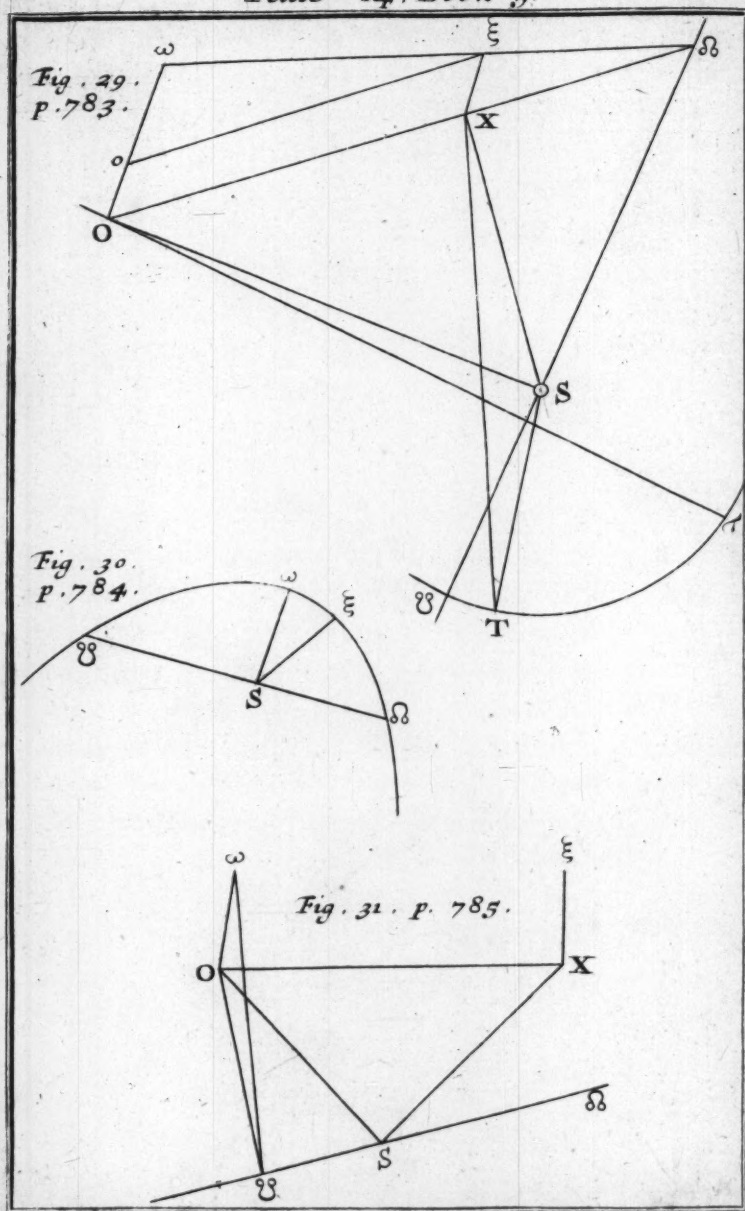


Plate 15. Book 5.

Fig. 32. p. 785.

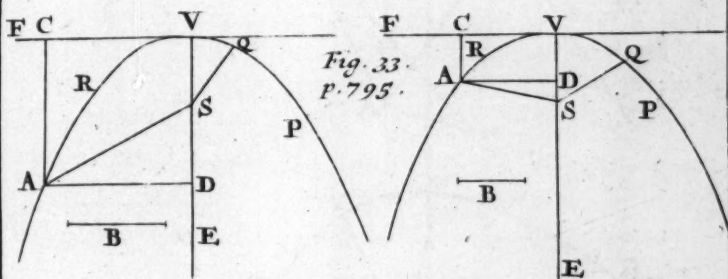
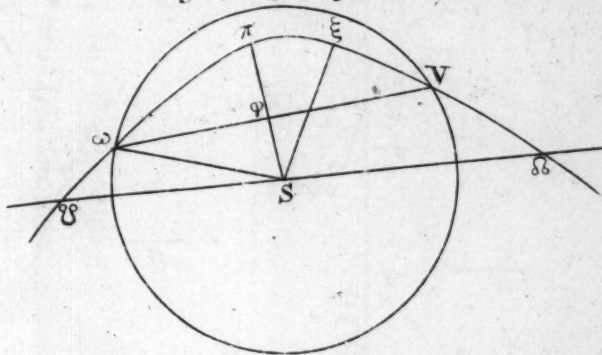
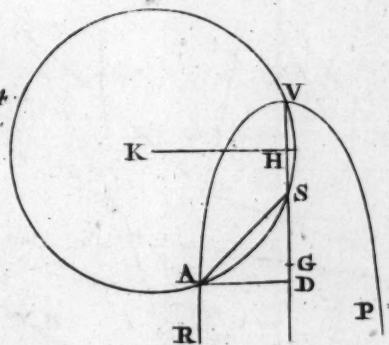


Fig. 34.
p. 797.



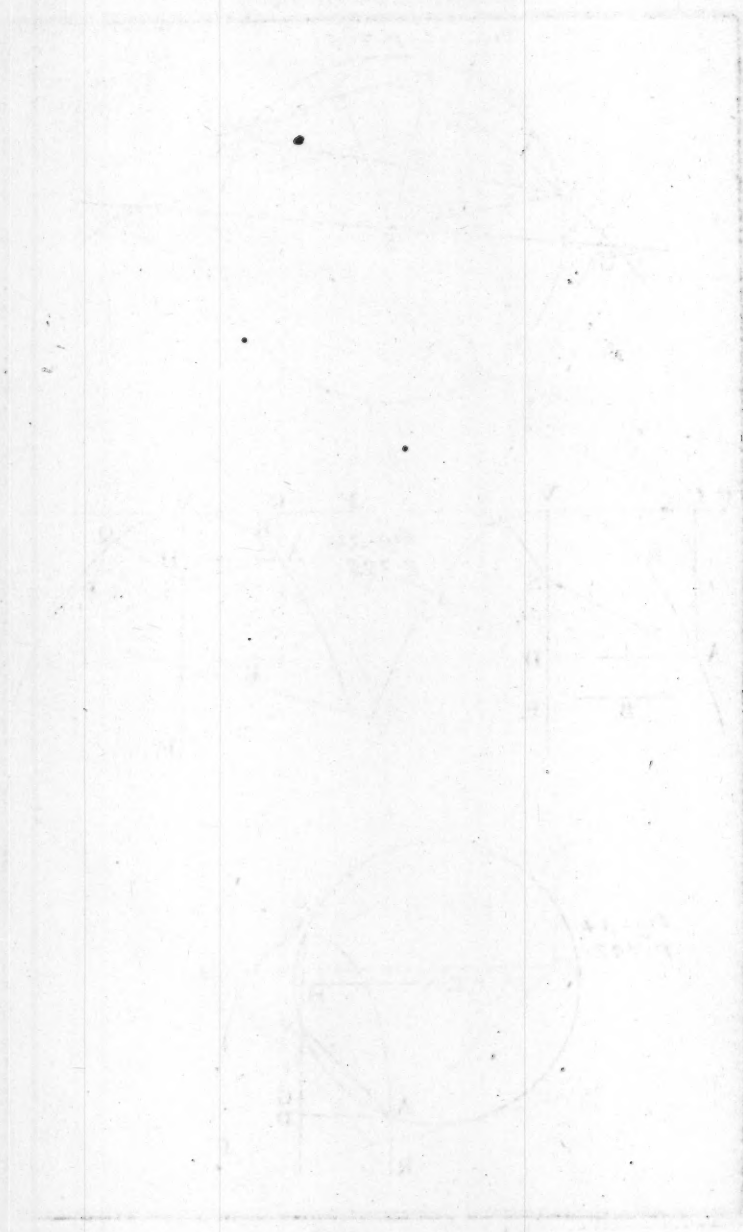
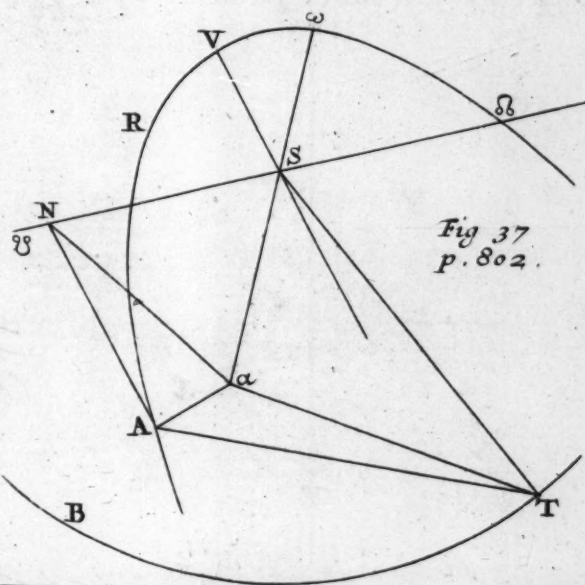
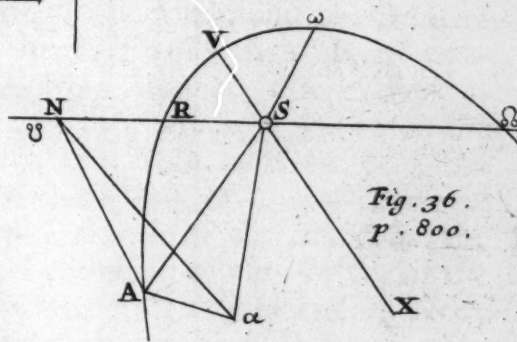
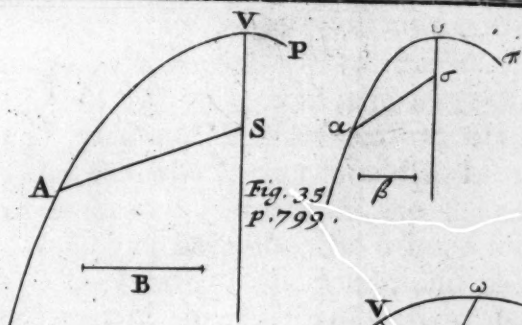


Plate 16. Book 5.



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Book V. of ASTRONOMY. 809

that the principal *Latus rectum* of its Orbit was 0,0243 Parts, of which the mean Distance of the Earth from the Sun is 1. Besides, he found this Orbit so plac'd in respect to the Plane of the Ecliptic, as to be inclin'd to its Plane in an Angle of $61^{\circ}.20'.20''$, and the ascending Node in $1^{\circ}.53'$, and the descending one in $1^{\circ}.53'$, and that the Comet mov'd so in this Orbit, as to be in its Perihelion in the Year 1680, on the 8th Day of *December* Old Stile, 0 Hour 4 Minutes after Noon. He found also the Orbit of the Comet of the Year 1683. figured and posited after the following Manner, *viz.* its Perihelium in $25^{\circ}.29'.30''$, which it was in on the 3d Day of *July*, Old Stile, 2 Hour. 50'; the *Latus rectum* of this Orbit of 2,2408 Parts of which the mean Distance of the Sun from the Earth is 1, the Inclination of this Orbit, to the Plane of the Ecliptic $83^{\circ}.11'$, ascending Node in $23^{\circ}.23'$, and descending Node in $23^{\circ}.23'$.



THE



THE
ELEMENTS
OF
Physical and Geometrical
ASTRONOMY.

The SIXTH BOOK.

Of Comparative Astronomy.

IN the Five foregoing Books, we have explain'd Astronomy, as it is agreeable to the Inhabitants of the Earth; that is, we have describ'd the *Phænomena* of the Fix'd Stars, of the Sun, of the Planets and Comets, as they are seen from the Earth, and we have taught how to find the respective Places of every one of them for a given Time, and other Things of that Nature, each in its proper Place: And now we shall, in a few Words, shew what Kind of Astronomy the Inhabitants of the rest of the Planets

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Book VI. of ASTRONOMY. 811

nets, and other vast Bodies which are dispers'd over the Universe, wou'd establish. Not that we believe, and much less affirm, that there are any such Astronomers any where; but by supposing the Eye to be plac'd sometimes in one, and sometimes in another of the Cœlestial Bodies, and explaining what Sort of Astronomy a Spectator wou'd establish according to his different Station, we hope to give no small Light for the thorough Understanding of those Things which we have said of this our Earth: For, we wou'd not speak of such empty Appearances, and never seen, or likely to be seen, in any one, unless it was for this End. Therefore, as we go on, we shall not only suppose the Eye in the Planets which move about the Sun, and their Satellites, where several famous Men have formerly, and do now believe, that it is probable that there are Inhabitants endu'd with Sense, but we shall suppose it also in some one of the Comets, in the Sun itself, and in the Fix'd Stars, where no body has ever suppos'd Inhabitants.

A great many eminent Men have taken Pains to illustrate this. *Plutarch* says many Things with Regard to this, in his Book of the *Face in the Orb of the Moon*. *Kepler* wrote a Book of *Lunar Astronomy*, publish'd after his Death, under the Title of *The Astronomical Dream*, in which he speaks of several Physical and Astronomical Matters, as he always does every where else. But in explaining the Phænomena seen from the Moon, he has made some few Errors, because he took no Notice of the Libration of the Body of the Moon, or else he did not refer it to its proper Causes: Therefore I have been forc'd to alter it quite. The famous *Christian Hugen*s has lately publish'd a *Theory of the World*, full of learned
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Conjectures, in which he has shewn several Astronomical and Physical Considerations worthy of his Genius. But these two ingenious Men were in earnest about the Inhabitants of the Planets; especially *Hugens*, who did not at all doubt, but that in every one of the Planets were Inhabitants like, or, at least, not very unlike those of the Earth; and to this Purpose it was, that he wrote a Book, rather of Physical, than of Astronomical, Considerations. But I, who have only undertaken to treat of Astronomy, have said nothing concerning the Things necessary, or convenient, for the animal or sensitive Life, which may be found, or are wanting, in particular Planets: I have only plac'd the Eye (which, with the Reader's Leave, I shall sometimes call a Spectator) as beholding Things above and round about it.

Beginning at the Sun, where the Astronomy is most simple, I shall go on, in Order, to Mercury, and the other Planets: The most remarkable Phænomena, belonging to every one of them, shall be treated of by themselves; but those that are common to many, are referr'd to one and the same Proposition. Then transferring the Eye to some Comet, I shall observe what would be seen from such a Place; and, by Way of Corollary, we describe how our Solar System wou'd appear, if seen from some Fix'd Stars. Lastly, several Phænomena which will appear to a Spectator in the Earth's Attendant, the Moon, and besides, several Phænomena, that would appear from one of the Satellites of Jupiter and Saturn, are shewn.

I mention, by the Bye, what will be the true, and what the apparent, Astronomy, in these different Stations; what Conveniencies they will have, and what Inconveniencies they will be
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Book VI. of ASTRONOMY. 813

subject to, who, by Observations made from these Stations, wou'd endeavour to find the true System of the Circumsolar Planets, and measure their Distances and Magnitudes. And lastly, I shew which is the most convenient Situation for Astronomy, as well among the Primary, as among the Secondary Planets.

The Phænomena, that are look'd upon by the Inhabitants of the Earth as principal, I also every where hold as principal, *viz.* the Vicissitude and Measure of Day and Night ; the Order of the Fix'd Stars ; the apparent Magnitude of the Sun, the Quantity of its Light, and Intensity of its Heat ; the Length of the Years and Months ; the Summer and Winter ; the Phases of the Moon ; the Eclipses of the Luminaries ; the Number, Directions, Stations, and Retrogradations of the Planets ; the Aptitude of Phænomena for the finding out of the true System ; the exact Accuracy that is requir'd in Observations that are of Use in Astronomy, and other Things of that Nature.

PROPOSITION I.

TO describe what will be the chief Phænomena, when the Eye is in the Sun.

If the Eye is suppos'd in the Sun's Center, looking round about every Way, the Fix'd Stars, and all the Planets, will appear to be in a Sphere concentric to the Eye, as it happens to us when we look from the Earth ; and, indeed, the Fix'd Stars, and the Milky Way, will appear in the same Order, and distributed into the same Constellations, as they appear to us Inhabitants of the Earth. For, since the Order of the Fix'd Stars, and the Figures of the Constellations, are not chang'd, in respect to us who are carried with

with the Earth about the Sun, it is plain that it remains the same to the Eye placed any where within that Circle, and therefore to it in the Sun's Center. If the Representation of Things, which give Names to the Constellation, be drawn forward and concave, they will become the Schemes, or Constellations, of the Ancients; but, if they be drawn backwards, and convex, they will become the modern Figures of the Constellations, more fit for artificial Globes, than for the Heaven itself.

It is also plain, that the Order of the Fix'd Stars will remain the same, if the Eye be on the Surface of the Sun, or in Mercury, or in Venus, because they are contain'd within the Orbit of the Earth. But also because of the immense Distance of the Fix'd Stars, upon Account of which it happens, that tho' the Place of the Observer is mov'd, the whole Diameter of the *Magnus Orbis*, the Order of the Fix'd Stars does not appear chang'd in the least; if the Eye be in Saturn, the outmost of the Planets of the solar System, this Order will remain the same, or, at least, not sensibly chang'd. For the Change of Distance, which is insensible to us, if it be ten times greater, (which may happen, because the Distance of Saturn from the Sun is ten times greater than that of the Earth from it, will not be very sensible.

Tho' the Planets seem plac'd among the Fix'd Stars, our Observer will, in a little Time, distinguish them from the Fix'd Stars, observe the Period of each of them, and compare them together, taking the Period of one for a Measure, to which the Period of the rest must be referr'd. And, altho' we suppose the Observer unmov'd, (because he is suppos'd in the Sun's Center)

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Book VI. of ASTRONOMY. 815

and, for that Reason, liable to no Change of Situation (or any thing like it) by which he may measure the Distances of the Planets, he will naturally conclude, that of two Planets compared together, that is the remotest which takes up more Time to return to the same Fix'd Stars; and therefore, that Saturn is the remotest, Jupiter nearer, then Mars, then the Earth below it, then Venus, and lowest of all Mercury.

Neither is there Reason to doubt, but that the first Astronomers made Use of that Rule for finding out the Order of the Planets: (For the Opinions which *Plutarch*, in his 2d *Book of the Opinions of Philosophers*, makes Mention of that of *Zenocrates*, who suppos'd the Stars to move in one and the same Surface; and that of *Metrodorus of Chios*, and of *Crates*, who believ'd the Sun to be plac'd highest of all, and after it the Moon, then the fix'd and wandering Stars, were either the Notions of ignorant Men, or, as I am more inclin'd to believe, of such Men as had a Mind to hide the Truth :) Tho' it be more difficult to apply it from the Earth than the Sun, by Reason of the Stations, and Retrogradations, of the Planets seen from the Earth; therefore we have been oblig'd to observe several of their Stations and Retrogradations, to know from their mean Motions, what their Periods are. But the whole Revolution of each of them, was not so easily known; because, when seen from the Earth, they did not, after one whole Revolution, return to the same Fix'd Stars, because the Observer's Situation is not then in the same Place. Notwithstanding these Inconveniencies, they believ'd Saturn the remotest, because the most slow, then Jupiter, then Mars, and the Moon lowest of all. But, concerning the three remaining

816 *The* ELEMENTS Book VI.

maining ones, namely, the Sun, Venus, and Mercury, there was still left Room to dispute, because they found their mean Motions equal thro' the whole Zodiac. Whence, some plac'd Venus next to Mars, Mercury next to it, and then the Sun, as *Plato* the Younger did ; others put Mercury next to Mars, then Venus, and then the Sun, as *Porphry* : *Alpetragius* makes Venus to be under Mars, then the Sun, and then Mercury ; others plac'd the Sun next to Mars, then Venus, then Mercury, as *Ptolemy* himself, and so the Sun was number'd, as plac'd, in the Middle, in the Order of the Planets ; (Saturn, Jupiter, and Mars being above it, and Venus, Mercury, and the Moon below it ;) which differ'd very little from the most ancient and true System, in which the Sun is in the Middle of the Planets, which, about that Time, was scarce out of Date, at least not in the Expression. Others, in the mean Time, from the Equality of the mean Motion of those three Cœlestial Bodies, affirm'd, that Venus and Mercury went round the Sun, and that their Orbits were carried by the Sun, as Epicycles ; which Opinion was agreeable to *Cicero* and the *Ægyptian* Philosophers ; unless we rather think, that this Order of those three Bodies was the Remains of the true System of the World, not yet lost among the *Ægyptian* Priests.

But our solar Observer will have no such Things to hinder his Reasonings ; for to him all the Planets will be continually direct, with Periods altogether different, and will appear to return to the same Fix'd Stars, whence their Order will be, without Hesitation, settled from the foresaid Principle : Which, in Course of Time, will be more confirm'd, when (they being easily distin-

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Book VI. of ASTRONOMY. 817

distinguish'd from one another, either by their Magnitude observ'd before, or their different Colour) an inferior one will be easily seen to cover a superior one.

The Spectator will also in a little Time find, that if he compares any Planet with the Fix'd Stars, it will sometimes go slower, and sometimes go swifter, and from the said Rule will conclude that it is sometimes nearer, and sometimes further off, which will appear more plainly from the Change of its apparent Diameter. But when he has observ'd that the Velocity is not increas'd in the same Ratio as its apparent Diameter, but almost in a duplicate Ratio, he will find that the Change of Velocity is not altogether optical, but that half of it is really in the Planet, and that the other half appears, so by Reason of the greatest Nearness.

Tho' he easily has the Order of the Planets rightly establish'd, it will not be so easy for him to compare their different Distances from the Sun one with another: For here is wanting (what we have upon Earth) the Translation of the Observer from one Place to another, or the Diversity of Stations, by which he may compare the different Distances from himself with the Distance of the Stations.

If we remove the Eye from the Sun's Center to its Surface, then all these Things will be plain from the Parallax of the Planets: For the horizontal Parallax of the Earth, to an Eye so plac'd, is equal to the Sun's Semidiameter seen from the Earth, viz. 16', and therefore sensible enough; and the horizontal Parallax of Saturn himself, which is ten times farther distant, will exceed a Minute and a half, whilst that of Mercury arises almost to 50'. Since then these Parallaxes are

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818 *The* ELEMENTS Book VI

sensible enough, the Distances of the Planets may be compar'd with the Sun's Diameter, and therefore also with each other. The Eye being thus plac'd, all the Fix'd Stars and Planets will appear to revolve from East to West in the Space of twenty five of our Days. The Northern Pole of that Revolution will be in that Place which we, Inhabitants of the Earth, refer to the tenth Degree of Pisces; with a Northern Latitude of 83 or 84 Degrees, whence the Arctic Polar Star will be at the second Flexure of the Dragon, and which is not above three Degrees from the Pole itself. The South Pole will be in the tenth Degree of Virgo, 83 or 84 Degrees towards the South, near to a Star of the fourth Magnitude; which, according to the ingenious Dr. *Halley*, is to be seen in the first Oar of the Ship *Argo*.

And tho' the foresaid Observer has no Vicissitude of Night, as being plac'd in the Surface of a most lucid Body, which makes Day, yet the Fix'd Stars and Planets will make unequal Arcs above and below the Horizon, as they declin'd towards the Pole, which is rais'd above the Equator, or to that which is depress'd below it, just as it happens to us, Inhabitants of the Earth.

The Planets, seen from the Sun, appear of different Magnitudes. The Diameter of Saturn subtends an Angle of 18'', that of Jupiter of about 40'', that of Mars of only 8'', of Venus an Angle of 28'', and Mercury 20''. The famous *Hugens* lays down the Diameters of the most remote ones, viz. Jupiter and Saturn as much greater; making that of Jupiter almost 54''; and that of Saturn without his Ring, 27''. If we make the Earth's apparent Diameter from the Sun to be a Mean between the Diameters of the nearest, namely,

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Book VI. of ASTRONOMY. 819

the apparent Diameters of Venus and Mercury, it will be $24''$; if we make it a Mean between the Diameter of the four nearest to the Sun, (for the two superior ones, Jupiter and Saturn, are altogether different, in Magnitude, from the inferior Planets) it will be of about $18''$ or $19''$, such as Mr. *Cassini* lays it down.

Of these six Planets three have Satellites; namely, the Earth has only one, which is the Moon, which will appear to go from it, at most, only ten Minutes; Jupiter has four Moons, the furthest of which will never go further from Jupiter than $9'$; Saturn appears accompanied with five Satellites, (besides the Ring) whose outermost goes about as far from Saturn. The Ways of these Satellites, seen from the Sun, being sometimes more and sometimes less obliquely seen, will appear Ellipses, sometimes broader and sometimes narrower. Sometimes the Planes of the Orbits, produc'd, go thro' the Sun, in which Case the Ellipses are chang'd into Right Lines, which happens twice in every Revolution of that Orbit about the Sun, if the Plane of it remains parallel to itself; whence the Satellite will sometimes appear cover'd by its Primary Planet, and sometimes again to cover it, and at another Time to move in an Ellipsis about its Disc.

The Primary Planets, and their Satellites, have their Surfaces not only not smooth and polish'd, but so uniform as to have several brighter and darker Places, that is, which do more or less reflect the Rays of the Sun, by which they are enlighten'd. These Spots, by the Rotation of the Planets about their proper Axes, describe Circles; therefore, when seen from the Sun, their Ways, on the Plane of the Planetary Disc will be sometimes elliptical, and sometimes streight, (as the before-

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820 *The* ELEMENTS Book VI.

mentioned Satellites Ways) according as the Sun is lifted up above either Side of the Plane of the Circles, or is found in it, as it is at the Time of the *Æquinox* of that Planet. If there are Series or Heaps of those Spots, as in Mars and Jupiter, these will be seen bent into Semi-Ellipses (for the other Half of the Ellipsis lies on the back Side of the Planet) or extended into Right Lines; which also happens to the outward Edge of Saturn's Ring. But some of the Spots are sometimes hid for some Time, and sometimes appear again as long; namely, when a Planet approaches the Pole, which is turn'd from the Sun, or that which is towards it. These things are very observable in Saturn and the Earth, but scarce in the others; because in them the Light of the Sun always reaches from Pole to Pole.

The Eye, plac'd in the Sun, will see no Shadow nor Eclipse, because they are always turn'd from the Sun. But sometimes to an Eye on the Surface of the Sun, when the Planet is in the Horizon, its Satellite will appear in the Penumbra of it, or the Penumbra of the Satellite may appear cast on the Disc of the Primary Planet, which will be known by the Colour, which will be something duller.

Moreover, the Helicentric Places of Planets (and therefore with more Difficulty their Places seen from a Point on the Sun's Surface) will be easily determin'd by what has been said in the 3d Book. Nay, we must determine every Planet's Heliocentric Place, before we can determine its Geometrical one.

Besides, the six Primary Planets, and the ten Satellites (for hitherto we, Inhabitants of the Earth, have seen just so many) which to

Book VI. of ASTRONOMY. 821

a solar Observer will appear to move in a Zodiac scarce sixteen Degrees broad, and not much inclin'd to the Circle of the first Motion and the same Way, and its Orbits almost concentric to the Sun; besides them, I say, there are other Bodies of a different Kind, whose Number is yet uncertain, carried in every excentric Orbits (call'd Comets) which sometimes come near to the Sun, and sometimes go off to immense Distances from it. These Comets will appear to be carried among the Fix'd Stars, not all the same Way, but one one Way, and another another, (tho' every single one, observ'd by itself, will always go the same Road; and for the most Part do not go in the Zodiac, or Way of the Planets, but in Ways very much inclin'd to it, but always in great Circles of the Sphere.

The Comets are not only distinguish'd from the Planets by the great Difference of their Motion, from the Motion of the Planets, but by their Hairiness, which, being seen from the Sun, does not (as it often happens with respect to us) extend itself like a Tail, but appears always to encompass the Head, as we see it, when a Comet is in Opposition with the Sun in respect to the Earth. But such a Periwig is sometimes thicker towards that Part among the Fix'd Stars where the Comet is going, but more extended towards the contrary Parts. Moreover, that Hairiness is greater, and brighter, when the Comet descends to its Perihelium; not only upon optical Accounts, because it is both nearer to the Eye and the Sun which enlightens it; but chiefly because, the Vapour which causes it, is thrown out in greater Quantity, by reason of the Encrease of the Heat which causes it. But the Perewig about a Comet will not even then appear to a

822 *The* ELEMENTS *Book VI.*

solar Observer, to be compar'd to that which at such a Time wou'd appear to a terrestrial Observer.

What we have said is upon Supposition, that the Eye in the Sun is not hinder'd by the Atmosphere about it, from seeing as far, and as freely, as we do in the Night, when the Moon does not shine. For if it had a sharper or duller Sight, it might see other Things further off, or less enlighten'd, or not see the Things which we have mention'd; but if it sees them, it will see them in the foremention'd Manner.

Besides, the foresaid Phænomena seen from far, it will see a great many Changes on the Surface of its own Habitation: For at one Time it will be cover'd with great, dense, and frequent Spots or Islands, which in a little Time will change their Figures, and then be altogether dissipated and vanish; sometimes for a long Time there appear no such Spots (at least not so great or dense) for a long Time. But to an Eye in the Place which we mention, they will not be look'd upon as celestial Phænomena; therefore we shan't say much of 'em in this Place.

PROPOSITION II.

TO describe what Phænomena will appear to an Eye in Mercury.

Besides the Phænomena of the Order of the Fix'd Stars, which agree alike with an Eye plac'd any where within the Orb of Saturn, it is plain that the Eye, posited in Mercury, will see the Sun almost three Times broader than we see it from the Earth, because (by *Prop.* 40, and 42. *B. I.*) Mercury is almost three Times nearer to the Sun than the Earth. Whence also the solar Disc, seen from Mercury, is seven Times greater than the Disc

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Book VI. of ASTRONOMY. 823

as it appears to us, and (by *Prop. 48. B. I.*) Mercury has seven times more Light and Heat, *cæteris paribus*, than the Earth. But these Qualities are much more, or less intense, according to the different Distance of Mercury from the Sun; for its Orbit is much more excentric, than the Orbit of any other Planet.

The accelerating Gravity, towards the Sun, is also seven times greater there than here. The Density of Mercury, and consequently the accelerating Gravity of Bodies, on its Surface towards its Center, may be gather'd from *Prop. 49. B. III.* tho' less accurately than that towards the Sun, because it is only done Analogically. But we shall not speak of such Things here, or in the following Pages, as not belonging to the cœlestial Phenomena, which alone are to be consider'd in this Place.

It has not yet been found by Observation, whether Mercury turns upon its own Axis, and therefore it can't be certainly defin'd, whether to an Eye, plac'd on its Surface, all the Things without will appear to revolve with a contrary Motion; that is, whether it has the Vicissitude of Day and Night, much less what is the Space of its *Nychthemeron*, or natural Day; but one may safely enough conjecture that it does revolve about its Axis, since the other Planets do. But the Year of Mercury is scarce equal to a Quarter of our Year, tho' it is uncertain whether it has different Seasons, because they depend upon the Inclination of the Axis of its Rotation, which is unknown to the Plane of the Orbit, which it describes about the Sun.

The Eye plac'd in Mercury, looking at the Sun, will see the Sun's Spots (when it has any) sometimes in a Right Line crossing his Disc from

824 *The ELEMENTS* Book VI.

East to West, at another Time their Way will appear elliptical and curve, toward one Side or t'other; as was shewn in the *Schol.* of *Prop.* 70. B. IV. and all the Variety of this Phenomenon will happen in one Year, in which the Track or Way of the Spots will appear twice rectilinear. In this Case Mercury will see all the Spots on the Sun's Surface, because, being in the Plane of the Sun's *Æquator*, it will see each of its Poles in the Margin, or Edge of the Disc. But when the Eye is rais'd above the Plane of the solar Equator towards either Pole; that is, when the Spots appear to describe an Ellipse, one Pole and the Spots near it are hid in the Sun's hinder Hemisphere. But in every Case a Spot near the Sun's Limb will appear of a Figure different from that which it had in the Middle of the Disc, as being very obliquely seen.

But the Way of the solar Spots, seen from Mercury, will be always almost streight; because Mercury never much declines from the Plane of the Sun's *Æquator*, and therefore not sensibly from the Planes of the Circles, which the Spots describe by their Rotation.

And, as the Plane of the Orbit of the Earth is most of all inclin'd to the said Plane of the Sun's *Æquator*, the Way of the Spots, seen from the Earth, appears more curve, than if the Sun was seen from any other Planet. This Curvature will be something less seen from Jupiter or Mars, and yet less from Saturn, less still from Venus, and least of all from Mercury, as we just said. Mercury will also see the Way as well of the Satellites as of the Spots of the other Planets, sometimes extended in a Right Line, sometimes bent in a Curve, and some of them sometimes hid for a great while, and sometimes as long visible, according

Book VI. *of* ASTRONOMY. 825

cording as they are near to, or distant from the hidden or seen Pole of the Planet.

The other five Planets are superior to Mercury: Therefore their Phænomena are to an Eye plac'd in Mercury, as the Phænomena of Mars, Jupiter and Saturn, seen from the Earth, concerning which, see the 7th, 8th and 9th Propositions of the first Book. Therefore Venus and the Earth, when they are in Opposition with the Sun, will shine with a full Orb, and reflecting the Sun's Rays very directly, will shine upon Mercury all Night, and poweasfully expel the Darkeness. For since Venus, when it is horned, and shews the least Part of its enlighten'd Hemisphere, to the Inhabitants of the Earth, is so bright as to cast a Shadow, it will appear very bright to Mercury, to which it turns its whole enlighten'd Hemisphere. The Earth also has a great Power of reflecting the Sun's Rays, as it appears from the Light which it casts upon that Part of the Moon's Disc which is turn'd towards it: But being as well more distant from Mercury, which is to be enlighten'd, as from the Sun, from which it receives its Light, it will do much less, in this Case, than Venus, unless it shou'd be more apt to reflect Light than Venus, which is not likely. The superior Planets do yet enlighten Mercury, with a weaker Light, than they do us.

Mercury sees no inferior Planet known to us, and therefore sees no cœlestial Body falcated or horned; and therefore a Spectator there, will want the Argument taken from such Phases of the Planets, to establish the true System of the World: For the Phases of the inferior Planets have clearly shewn, that they moved about the Sun; whence it was natural to think the same of
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the other Motions. But nevertheless, tho' we don't see any Planets inferior to Mercury, it does by no Means follow that there are no such: For Mercury himself is seldom seen in our oblique Sphere, and one, that should be much inferior to it, wou'd never be seen, upon Account of its Nearness to the Sun.

An Observer in Mercury, as well as we upon Earth, wants Mediums to compare the Distances and Diameters of the cœlestial Bodies with the Diameter of his Habitation, or with his own Measures, unless he has very exact Instruments. For the Parallax of the Sun, seen from Mercury (as far as we can guess from Analogy) is less than its Parallax, when seen from the Earth; the apparent Diameter of Mercury, seen from the Sun, being less than the Diameter of the Earth seen from it: And Venus, when in Opposition with the Sun, and therefore nearest to Mercury, is near as far from Mercury as the Sun; and therefore the Parallax of Venus (even when Mercury in its Aphelium sees it opposite to the Sun, and consequently nearest of all to itself) is not greater than the Parallax of the Sun seen from the Earth. And these are the two great Bodies of the World, near to Mercury, which have the most sensible Parallax.

But because Mercury changes its Place, a Spectator in Mercury, by Observations made from different Stations, will gather the Ratio between the Distance of Mercury from the Sun, and the Distance of any other Planet from it, after the same Maner, as we Inhabitants of the Earth do it from the Parallax of the *Magnus Orbis*. But certainly it cannot be so safely determin'd, because the Distance of Mercury from the Sun is not so great, in respect of the Distance of the o-

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Book VI. of ASTRONOMY. 827

ther Planets, as the Distance of our Earth from the Sun. But as it is yet sensible enough in respect of them, being about the five and twentieth Part of the Distance of Saturn from the Sun; the Ratio of all those Distances will conveniently enough be determin'd by Observations made in Mercury, if so be that the Observer knows the Motion of its Habitation about the Sun. But if, trusting to his Senses, he thinks his Mercury to be immoveable, he won't be able to do even this, any more than our Astronomers Inhabitants of the Earth, as long as they look'd upon the Earth as immoveable; which is evident from the Distances of the Planetary Orbits, assign'd by the Followers of *Ptolemy*, so vastly different from one another, and yet agreeing well enough with the Phænomena. For in the System, whereby the Observer's Place is suppos'd immoveable, if an Epicycle be suppos'd for any one of the superior Planets to run thro' in one Revolution about the Sun, and sufficient to explain its greatest observ'd Distance from the Place in its Excentric near the Quadrature with the Sun, the Semidiameter of the Excentric, which is the Defe-
 ferent of the Epicycle, may (without contradicting the Phænomena) be taken of any Magnitude, so that it be such that the Planet, when nearest, may be further from the Earth, than what wou'd make its Distance liable to be defin'd from the Parallax: Nay, nothing hinders Mars from being plac'd above Jupiter, or Saturn below Mars, unless it be the Rule given in the foregoing *Prop.* and that depends upon Sagacity, and not Demonstration.

The Places of the Sun, and all the Planets, seen from Mercury, are found the same Way, as when seen from the Earth, and defin'd by the
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Help of the Tables in *B. III.* For since out of them the Heliocentric Places, as well of Mercury, as of any other Planet, are readily defin'd, the Hermocentrical Place of this other will be defin'd by an Operation, like the Operation of the 40th *Prop. B. III.* which is also true of any other Planet, and therefore I shall not repeat it in the following Propositions.

The Phænomena of the Comets seen from Mercury, or any other of the Planets, are almost the same as their Phænomena seen from the Earth, of which we spoke in the foregoing Book; for they differ in nothing but their Distance.

PROPOSITION III.

TO describe the Phænomena, as seen by an Eye in Venus.

In Venus the Sun's Diameter appears almost as little again as it does in Mercury, and once and a half greater than to us on Earth. Whence the Sun's Disc, seen from Venus, is more than twice greater than seen from hence by us, and also the Light, Heat, and accelerating Gravity towards the Sun, are encreas'd in the same Proportion.

Venus's Year (namely, its Period about the Sun) is equal to seven and a half of our Months: But the Day in Venus is almost an Hour less than our Day.

Venus sees four superior Planets, whose Phænomena therefore are like those of Mars, Jupiter and Saturn, seen from the Earth, or all of them seen from Mercury. But our Earth seen in Opposition with the Sun, and for the whole Night, since (by *Prop. 9. B. I.*) it shines with a full Orb, will appear very bright; and this Light will be

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Book VI. of ASTRONOMY. 829

increas'd by that of our Moon, shining also at that Time with a full Orb. The Moon will appear always to accompany the Earth, as it really does, and sometimes will be above half a Degree distant from the Earth.

Venus has only one inferior Planet, namely, Mercury, which is never farther distant from the Sun than about 38 Degrees, and which will appear in the same Manner to Venus, as Venus and Mercury do to the Inhabitants of the Earth, concerning which, see *Book I. Prop. 4, 5, and 6.*

An Observer in Venus will not want Means to compare the Distances of celestial Bodies with his own Measures; for he will compare the Diameter of his Venus, with the Measures which he commonly makes use of, as we do our Earth's Diameter with our Measures, by *Schol. Prop. 17. B. II.* and the Parallax of the Earth, in Opposition to the Sun, is four Times greater than the Parallax of the Sun seen from the Earth, and therefore not altogether insensible: Therefore he knows the Ratio of the Diameter of his Venus to the Diameter of the Earth, opposite to the Sun, and retrograde from Venus; that is, the Difference of the Distances of Venus and the Earth from the Sun; but the Parallax of the other Planets (even when nearest to Venus) is almost insensible. If the Observer in Venus knows that Venus moves about the Sun, he will from thence gather the Distances of the Planets from the Sun, as the Inhabitants of the Earth do: And from the Ratio between two Quantities, and their Difference being given, the Quantities themselves are also given, and consequently those others having given Ratio's to one of the said Quantities. But if he does not know that his Habitation moves, he will indeed (by the
Rule

830 *The* ELEMENTS *Book VI.*

Rule given in *Prop. 1.*) settle, by Conjectures, the Order of the superior Planets, but not their Distances, as has been said in the foregoing Proposition concerning Mercurial Astronomers. He will have the same Reason of doubting of the Order of Mercury and the Sun, as the Ancients (Inhabitants of the Earth) had of doubting of the Order of Mercury, Venus, and the Sun; some saying that Mercury is above the Sun, in respect of Venus, at rest in the Center; others saying that it is below it; and, lastly, others saying that Mercury moves about the Sun as the Center of its Motion. But at last, it will be out of Doubt that Mercury moves about the Sun, when the Sight can be so far help'd as to make its Phases visible. For before that, one may (without contradicting the Phenomena) err so much in the Estimate of the Distances of the Planets, that if an Epicycle be allow'd to Mercury, whose Diameter, seen from Venus, is equal to two Signs and a half, and its Center be plac'd in a Line joining the Sun and Venus, or produc'd beyond the Sun, the Deferent of the Epicycle may be plac'd beyond the Sun's Orbit, or any where short of it, & so be that it exceeds the Limits of a Diurnal Parallax. Which is plain from hence, that in the System of the Earth immoveable, (without Contradiction to the Phenomena then known) the Deferents of the Epicycles of Mercury and Venus are plac'd far short of the Sun, by almost all the Astronomers who are the Followers of *Ptolemy*; tho' now there is no Body but what may of himself discover that those Planets go round the Sun.

Having once determin'd the Distances of the Planetary Orbs, from the Sun, in the true System of Venus moving about the Sun, if another Astronomer

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Book VI. of ASTRONOMY. 831

Astronomer will have Venus (his Habitation) to be in the Center of the World, and immoveable, he may suppose it two Ways, according to the Laws of Optics. First, that Venus being in the Center, the Sun, together with all the Planets, and their Orbits, which they describe about the Sun, will move the same Way, in Right Lines, parallel to each other; such an Observer in Venus will have the *Tychonic* System, namely, one in which the Sun is carried in an Orbit equal, similar, and contrary to that which Venus does really describe about the Sun, and all the Planets with their Satellites (except that they are carried by the Sun about Venus) will perform the same Motions about the Sun which they do in the true System, the Primary Planets in Epicycles, and the Satellites in Epicyclo-Epicycles; and the Orbit of the Earth will intersect the Sun's Heaven, just as the Orbit of Mars in the Scheme of *Tycho* on Earth. But the Orbit of Jupiter does not intersect, but contain the Orbit of the Sun in such a System on Mars. Secondly, an Astronomer may rightly establish a System of Venus immoveable in the Center, after he has promoted his Venus to the central Place, and at the same time made the Sun, and all the Planets, go in Right Lines parallel to each other as before, but not the Orbits of the Planets, which he keeps unchang'd, and the same which they describ'd about the Sun (whilst it remain'd in the Center) because our Astronomer makes them move in such Orbit about himself; and he will, on his Venus, have the *Ptolemaick* System, in which the Sun is carried about Venus after the Manner explain'd in the *Tychonic* System, and every Planet is mov'd in an Epicycle equal, similar, and contrarily posited to the true Orbit of Venus about the

832 *The* ELEMENTS Book VI.

the Sun; but the Center of the Epicycle is carried in the same Orbit, and after the same Manner, that the Planet itself was mov'd, before the Sun was thrust out of the central Place to put Venus there, and the Epicycle of the Earth descends into the Heaven of the Sun, after the same Manner that the Epicycle of Mars does in such a Sort of a System upon the Earth, which also will be true in any other Planet besides Mars. Thus wou'd the *Ptolemaic* System be made consistent with itself, and the Motion of the Center of the Epicycle be taken away from the Sun, and the Motion of the Planet in the Epicycle be ascrib'd to it, as well in the inferior, as in the superior, Planets; and universally this System wou'd agree with all the Phænomena, as well as the *true one*, or as the *Tychonic one*. These two Systems (namely the *Ptolemaic* and the *Tychonic*) interchange the Deferents and the Epicycles: For the *Tychonic* makes the Deferent the same, and equal for all the Planets, similar, and contrarily posited to the Orbit of the Spectator's Planet about the Sun, but with different Epicycles, similar and similarly posited to the true Orbits of the respective Planets about the Sun; for in this System the Sun has no Epicycle, as in the true one it has no Orbit, for in such a System the Epicycle, and in our true one the Orbit is only look'd upon as a Point, which Figures are similar and similarly posited. But the *Ptolemaic* System gives all the Planets Epicycles equal to the Orbit of the foresaid Spectator's Planet about the Sun, similar and contrarily posited to it, but different Deferents, namely, the same as their respective Planets Orbits about the Sun in the true System. But because the System asserted by the *Ptolemaics* is not made out of the true System,

Book VI. of ASTRONOMY. 833

stem, but built by Degrees on the Phenomena without the Help of the true one ; therefore in it the Proportions of the Deferent Orbs (as the Followers of it make their System) is unknown, and only the Ratio between the Deferent and Epicycle of each Planet is settled from its Motions ; whence it happens that in Saturn, Jupiter and Mars, the Semidiameter of the Deferent (from *Ptolemy's* own Numbers) is to the Semidiameter of the Epicycle, as the Distance of a Planet from the Sun to the Distance of the Earth from it. But as in the *Tychonic* System all Planets have the same Deferent, if from the Motion of any one be found the true Ratio between its Epicycle and Deferent, there will be found (*ex æquo*) the Ratio between the Epicycles of them all, that is, between the Orbits of all the Planets ; and nothing else is done by the true System. The just System of *Ptolemy* (that is, the *Ptolemaic* System, built on the foregoing Principles) supposes the Epicycles of the inferior Planets equal to the Sun's Orbit : But as it appear'd monstrous to make an Epicycle greater than its Deferent, therefore the *Ptolemaics* chang'd the Deferent of each of the inferior Planets into an Epicycle, but yet in such Manner that both (which they cou'd not hinder) the Deferent and the Epicycle shou'd retain their Nature ; namely, that the Motion of the Epicycle in the Deferent shou'd be join'd to the Sun, and the Planet's Motion in the Epicycle shou'd be off from the Sun. By this Change they left their genuine above-describ'd System, and, as it were, fell into the *Tychonic* ; and the *Ptolemaic* System of the inferior Planets differs from the *Tychonic* only in this, that, without knowing the true Semidiameter of the Deferent, it lays down only

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834 The ELEMENTS Book VI.

the true Ratio between the Semidiameter of the Deferent and the Semidiameter of the Epicycle ; and That according to *Ptolemy* is the same with the Ratio of the Distance of the Earth from the Sun to the Distance of the said inferior Planet in the true System. And the foresaid two Systems are made by retaining the Planetary Orbit of the true System ; but if you change them you may make an infinite Number of Systems, each of which will agree with the Phænomena, nay, find the Place of every Planet to be the same in respect of the Earth (that is, a right Line being drawn the same in Magnitude and Position) as that in the true System ; and none of these Systems wou'd be found to be false without the Help of Natural Philosophy.

We have more than a bare Suspicion to incline us to believe that Venus has a Satellite, from the Observations of that curious Astronomer Mr. *Cassini*, made by Chance in the two Years 1672 and 1686 ; in the last of which, on the 28th Day of *August*, at a Quarter after Four in the Morning, through a 34 Foot Telescope, he, for a Quarter of an Hour, saw as it were a Satellite of Venus distant from Venus about $\frac{1}{3}$ of the Diameter of Venus, of the same Phasis as Venus, (as it ought to be, since their situation in respect of the Sun enlightening, and the Earth seeing, was the same) but of no regular Figure, and of a Diameter which was about a fourth Part of Venus's Diameter. That Mr. *Cassini* did not see that Satellite at any other Time, tho' he much endeavour'd it, might be owing to this, that the Surface of that Satellite is less apt to reflect the Sun's Light, as we find the same Thing true in respect of the Spots of our Moon. For if our whole Moon shou'd as weakly reflect the Light

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Book VI. of ASTRONOMY. 835

as its darkeſt Spots do, it wou'd ſcarce be viſible at Venus, and appear of an irregularly terminated Figure : Yet we muſt confeſs, that ſuch a Satellite of Venus (if it has one) is of a very different Nature from its Primary Planet, which does ſo powerfully and ſtrongly reflect the Light of the Sun, that *Hugens* with a Teſcope of 60 Foot cou'd ſee nothing like a Spot in it ; which is alſo the Reaſon, why the Poſition of the Axis of its diurnal Rotation is not defin'd. But this Difference between Venus and its Satellite does perhaps ariſe from hence, that it is not the ſolid Globe of Venus, but an Atmosphere about it very denſe and full of Vapours which reflects the Sun's Light to us, ſuch an one as which is wanting to its Satellite ; as it happens in our Moon, tho' the Earth is encompaſs'd with an Atmosphere denſe enough and full of Clouds, which wou'd hinder an Obſerver in Venus from perceiving the Diſtinction of Seas and Land, but he wou'd ſee its Surface almoſt ſmooth and equally enlighten'd, ſuch as Venus appears to us. If Venus has its Moon, it is almoſt in all Reſpects like our Earth, from which it differs leſs in the Quantity of its Day and Night, and Degree of Heat, than any other Planet.

PROPOSITION IV.

TO deſcribe the Phænomena to an Eye poſited in Mars.

The Sun's Diameter appears one and a Half leſs from Mars than from the Earth, and therefore it gives twice leſs Light and Heat in the Region of Mars that it does to us on Earth. But theſe Things, by reaſon of the great Excentricity of the Orbit of Mars, are ſenſibly chang'd, tho' not ſo much as in Mercury.

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836 *The* ELEMENTS Book VI.

Mars's Year is almost equal to two of our Years; but the natural Day but a little greater than ours: And the artificial Day, during which the Sun shines above the Horizon, (except the Twilight before the Sun's Rising, and that after its Setting, according to the Height of the constant Atmosphere) is almost always and every where equal to the Night, and therefore very little Difference of Summer and Winter is there felt; and that because the Axis of Mars's diurnal Revolution is almost perpendicular to the Plane of its Orbit. In the mean time the Places, at different Distances from the middle Circle between the Poles, have very different Degrees of Heat, upon account of the different Inclination of the Sun's Rays to the Horizon, as it is on the Earth at the Time of the *Æquinox*: And from hence I suspect that the *Fascia* or Belts arise, which run parallel in the Circle that is in the Middle between the Poles. For since the same Degree of Heat always continues in the same Climate, it is likely that the Spots in Mars (like the Clouds and Snow on Earth) owing their Rise to Heat and Cold, are extended according to the Climate, and make *Fascia* parallel to the Circles of Mars's diurnal Rotation; which is also true of Jupiter, who like Mars has a perpetual *Æquinox*.

The Observer in Mars sees two superior Planets, *viz.* Jupiter and Saturn, and two inferior Planets, namely, the Earth and Venus; whose Phænomena are like the above-described Phænomena of the superior and inferior Planets. He will never see Mercury, (as never being above half a Sign from the Sun) unless by Chance like a Spot in the Sun as it goes over his Disc; at least if Mars has an Atmosphere about it as large

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Book VI. of ASTRONOMY. 837

large and as dense as that of our Earth, as it seems likely by the Observation of the Fix'd Stars which grow duller, and seem almost extinguish'd, before they appear to touch Mars's Globe. He will see Venus as far distant from the Sun as Mercury appears to us, and the Earth as far as we see Venus: And as the Earth, seen from Mars, appears near, and almost in Conjunction with the Sun, he will see the same Thing at the Earth, as the ingenious Mr. *Cassini* saw once or twice at Venus; namely, the lower Part falcatated or horned, and its Satellite by it of the same Phasis, which at most will not appear above a third Part of a Degree from it.

An Astronomer in Mars is almost destitute of Means to compare the cœlestial Bodies with his Mars, and their Distances with known Measures; by reason of the Distance of the nearest Planets, *viz.* Jupiter and the Earth. For when the Earth is nearest to it, and the lowest of all the cœlestial Bodies, its Parallax will not be sensible, not amounting to above 12 Seconds, much the same as the Sun's Parallax with us. But if he is sensible of the Motion of his Mars, about the Sun, he may compare the Distances of the cœlestial Bodies, one with another: If he is not sensible of it, he can't even know that; as has been amply shewn in the two foregoing Propositions.

PROPOSITION V.

TO describe the cœlestial Phenomena, as they appear to an Eye in Jupiter.

The Diameter of the Sun, seen from Jupiter, will appear a little more than five Times less than it does, when seen from the Earth; and therefore its Light and Heat will be twenty seven

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838 *The* ELEMENTS Book VI.

Times less in Jupiter than with us. Jupiter's Year is almost equal to twelve of our Years: But on the other Hand, the natural Day will be less than ours, scarce arising to ten of our Hours: (And the Motion of Jupiter, about its Axis, is perform'd in the least Period of all the diurnal Rotations which the Astronomers have hitherto observ'd in any of the celestial Bodies:) Whence, in Jupiter, the Year consists of above ten thousand Days, Jupiter's Day is divided into an artificial Day and Night, almost equally over the whole Surface of Jupiter; because the Axis, about which Jupiter revolves, is almost perpendicular to the Plane of the Orbit, which he describes about the Sun; whence the same Things will obtain as in Mars, concerning which we spoke in the foregoing Proposition.

Tho' there are four Primary Planets below Jupiter, the Eye in Jupiter will see none of them, if we suppose it not more sharp-sighted than we, and believe that it has such an Atmosphere as the Earth: For Mars himself, who goes farthest of all from the Sun, will not be above 18 Degrees distant from it; but as Mars is both small, and weakly reflects the Sun's Light, it will scarce be seen at that Distance from the Sun. It is true indeed, that the Sun's Light, at Jupiter, is not so bright as at the Earth, and therefore, that the neighbouring Stars are not so much darken'd by it; but also the Light of Mars, the Earth and Venus, is diminish'd in the same Ratio, by reason of the encreas'd Distance of the Eye: Whence, at the same Distance from the Sun, when seen from Jupiter, they are likewise extinguish'd by the Light of the Sun. But the Planets below Jupiter may be seen like Spots upon

Book VI. of ASTRONOMY. 839

upon the Sun's Disc; but that will only happen by Chance: For an Astronomer in Jupiter being not more certain of their Existence, than we are of the Existence of Planets below Mercury, will not reckon upon their Motions, so as expect to see them at a due Time; an Eye therefore in Jupiter will see only Saturn, and that above it.

But then such a Spectator, in a Circle in the Middle, between the Poles of its first Motion, will see his four Satellites plac'd near him, and strongly reflecting the Sun's Light, unless so much as they appear by the Sun's Parallax to decline towards one Pole or the other, when the Eye is plac'd towards either Pole; so that the Number of the Planets will appear to be Six, to an Observer in Jupiter. For there is no Doubt but that looking upon his Habitation, the Globe of Jupiter, as unmov'd (which by reason of its immense Bigness, that he easily finds his Globe to have above the Planets near him, he may much more reasonably do, than the Inhabitants of the Earth concerning the Earth) he will reckon his four Attendants as Planets, to which Number he will add the Sun and Saturn, just as we reckon our Moon among the other Six: And indeed he will not err much in their Order, if he looks upon that as most remote, whose Period about Jupiter takes up the longest Time.

Moreover, he will easily compare the Distances of the four inferior Planets with his own Measures; namely, with the Diameter of Jupiter, as we do ours with the Diameter of the Earth; and it will be easier for him to compare the Distance of the four nearest Planets with the Diameter of Jupiter, than it is for us to compare the Distance of the Moon with the Diameter of the Earth: For the horizontal Parallax of the remotest of them,

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seen from Jupiter, is above twice greater than the horizontal Parallax of the Moon, seen from the Earth, and therefore very sensible. And tho' the Globe of Jupiter be of an immense Bigness, in respect of our Earth, the Parallax of the Sun, seen from Jupiter, will be scarce sensible, as not arising to 20 Seconds; neither will the Parallax of Saturn (even when opposite to the Sun, and nearest to Jupiter) be much greater: It will therefore be difficult for the Jovial Astronomer, to compare the Distances of the Sun and Saturn with his own Measures. If he knows the Motion of his Jupiter about the Sun, he will easily find the Ratio between the Distances of Jupiter and Saturn from the Sun, otherwise not; as it has been explain'd in the 3d *Prop.*

Therefore the Jovial Observer, relying upon his Senses, will distinguish two Sorts of Planets; *viz.* four near him, and two very remote, that is, the Sun and Saturn of a small, and the other four of a much greater apparent Diameter. For, of the remotest, the Sun appears about 6' broad, but Saturn, even when nearest, has scarce a Diameter of half a Minute. Of the others, the fourth from Jupiter will appear to be of about the same Bigness, as the Moon appears to the Inhabitants of the Earth, that is, of a Diameter five Times as big, and Disc five and twenty Times as big as the Sun, to an Inhabitant of Jupiter; and if the remaining Satellites are not much less than the outermost, they will yet appear bigger, especially if they are not less than our Earth, as that accurate Cosmographer, Mr. *Hugens*, affirms, that they are not. Besides, the four nearest and apparently greatest, will be distinguish'd from the lesser and more remote, because in the nearest Planets, the Squares of the Peri-

Book VI. *of* ASTRONOMY. 841

Periodical Times are, as the Cube of their Distances from the Center of Jupiter; but this will by no means hold, if you compare any of the great ones with any of the little ones. Altho' those, that are nearest, appear to be biggest, the Sun will appear vastly brighter than them; nay, from their Phases, which depend upon their Aspects to the Sun, they will appear as so many Moons. Whence the Spectator in Jupiter will have four Sorts of Months denominated from the four Moons: There will be more than 2407 of the least Months in a Year; of the Months immediately greater, the Number will be about as little again; of the third in Order, the Number will be about subduple in a Jovial Year, or about subquadruple of the least; but of the greatest Months there are about 254 in a Year. And therefore tho' the Notation of the Time be something more intricate on Jupiter, because the Year contains a great Number of Days, yet by those four Kinds of Months it becomes ready enough: For in the least Month there will be only four Days and a Quarter; but in the greatest something more than forty Days.

Besides, these Moons, when they are in Opposition with the Sun in respect of Jupiter, running into Jupiter's Shadow, are eclips'd, and again, when they are in Conjunction with the Sun, as they throw their Shadows on Jupiter, they will make the Appearance of an Eclipse of the Sun to an Eye posited on that Part of Jupiter where the Shadow falls (which is a small Part enough of Jupiter) as our Moon does. But because the Orbits of those Moons about Jupiter are in a Plane (all of them indeed nearly in the same Plane, except that the second from Jupiter does a little deflect from it) inclin'd to the Plane of Jupiter's

Jupiter's Way about the Sun, the Eclipses of these Moons are central and very lasting, when the Sun is in either Node of the Moons. But when the Sun is not in it, the Eclipses may also be total, tho' not central; because the Shadow of Jupiter is almost ten times broader than any of the Satellites, and the apparent Diameter of each Moon is five times greater than the Sun's apparent Diameter. And this great Inequality of the Diameters, and the small Inclination of the Plane of the Orbit of the Satellites, to the Plane of the Orbit of Jupiter about the Sun, is the Cause why the Eclipses, as well of the Moons as of the Sun, happen at every Revolution, tho' the Sun is very far distant from the Nodes. And, indeed, the lowest of them (tho' at the same time the Sun, seen from Jupiter, is most of all distant from the Nodes) will nevertheless fall into Jupiter's Shadow, and also, on the other Hand, their Shadows will fall on Jupiter; but, in that Case, the most remote Satellite, for the sixth Part of a Jovial Year, will miss the Shadow, when it is in Opposition with the Sun, and Jupiter also will not be any where darken'd by its Shadow, when in Conjunction with the Sun: But an Eye, plac'd in Jupiter, will, in intermediate Times and Places, see the Phænomena of partial Eclipses. Moreover, an Eclipse of the Moon is made here by a Moon, whose Phasis is sometimes quite different, nay, contrary to the before describ'd Eclipse of a Moon running into Jupiter's Shadow: For in this the Eastern Limb of the Moon is first eclips'd, and the Western one does last emerge out of the Shadow; but, in some of them, first the whole Western Limb is eclips'd, and the Eastern one does last of all recover his Light, in others just the contrary.

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Book VI. of ASTRONOMY. 843

Tho' the Shadow of Jupiter is far extended beyond its Satellites, it does not reach so far as any other Planet, which also is true of all the other Primary Planets. For Saturn alone, even tho' it were infinite, cou'd be immers'd in it; but the Shadow of Jupiter cannot be extended so far as Saturn, unless Jupiter's Diameter shou'd be nearly half the Sun's Diameter; and it is scarce the ninth Part of it.

If Jupiter's Surface has a great Part cover'd with a Fluid, and we suppose that Navigation cou'd be practis'd on those Seas; it wou'd be very convenient, as well by Reason of the very short Nights, (and those upon Account of the Number of the Moons, scarce without a Moon, and for the most Part enlightened by several of them, and those very bright,) as because by the Help of the Moons the Course may be directed: But also from so many Eclipses the Longitude of a Place will be easily defin'd, and accurate Hydrographical Tables may be constructed; which is very convenient in so vast a Globe, whose Surface being an hundred times (nay, according to *Hugens*, four hundred times) bigger than the Surface of the Earth, wou'd render its Measure very laborious. But the reciprocal Tide arising from the four Moons (for the Sun's Forces are of no sensible Effect for raising a Tide at this vast Distance) is various and uncertain, except to one who is us'd to Calculations.

PROPOSITION VI.

TO shew the chief Cælestial Phenomena, which wou'd appear to an Observer plac'd in Saturn.

The Diameter of the Sun, seen from Saturn, is almost ten times less than the Earth, and his Disc, Light, and Heat at least ninety times less there

there than here. Saturn's Year is almost equal to thirty of our Years : But it is uncertain what Proportion a Day in Saturn bears to our Day ; for, the Period of Saturn's Revolution, about his Axis, is not yet known. *Hugens*, indeed, from the Distance and Period of his inmost Satellite, and a Comparison of them with the Distance and Period of the inmost of the Jovial Satellites, thinks it very likely, that the Days are not longer there than in Jupiter. He is of that Opinion, because he looks upon it as an undoubted Truth, that the Rotation of the Sun, and of a Primary Planet, about its Axis, is the Cause of the Rotation of the Primary and Secondary Planets, (that, indeed, it is made towards the same Parts, is certain) or, at least, that the Rotation of a central Body, and the Planets about it, depend upon the same Cause ; therefore, conversly, having observed the Satellites rotated about Saturn, he did not at all doubt, but that Saturn revolv'd about its Axis, and nearly in the same Time as Jupiter, because the Periods of the Satellites differ but very little. But, if we consider, that the Cause of the Rotation of the Primary Planets about the Sun, or of the Satellites about a Primary Planet, is not any Vortex, or any thing like it (on which Opinion the foresaid Argument is grounded) but that the Planets are carried by a Motion, compounded of a projectile Motion along a Right Line, and of Gravity towards a central Body (about which they are moved in a free Space, and almost void of Matter, and the Saturnian Satellites gravitate after the same Manner towards Saturn, whether it revolves, or is at Rest ; it is plain, that from the Revolution of the Satellites, one cannot rightly deduce the Rotation of Saturn about its Axis,

much

Book VI. of ASTRONOMY. 845

much less the Period of that Rotation. It is true indeed, that it is probable, that Saturn revolves about his Axis, namely, that it may sometimes turn one, and sometimes another, Part of its Surface to the Sun, which is the Fountain of Light and Heat in his System: But it seems impossible to gather from hence the Period of the Rotation, these Things being, by the Allwise and Almighty Creator, fitted to the Nature of the Things contained in Saturn, which is altogether unknown to us. If the Axis of Saturn be perpendicular to the Plane of Saturn's Ring, and of the Orbit of the Satellites (as *Hugens* will have it) then the Situation of the Poles, and of the Æquator, is the same in Saturn, as it is in the Earth; therefore the same Polar Star will shine upon it, and the Constellations, in similar Latitudes, rise and fall after the same Manner. And these Things are thus, if the Situation of the Ring be the same as *Hugens* has laid it down in the *System of Saturn*, which also we have suppos'd in the Fourth Book, when we spoke of its Phænomena: But, if it be, by more accurate Observations, found to be different (as he hints in his Theory of the World) any one may gather the Measures of the foresaid Phænomena, from what has been laid down before. The Days at Saturn are very unequal, and the Differences of the Summer and Winter are very great, for they depend upon the Quantity of the Inclination of the Plane of his Æquator, to the Plane of the Orbit of Saturn about the Sun, which *Hugens* afterwards suppos'd 31 Degrees, almost a third Part greater than in the Earth, where those Differences are very sensible. At Saturn, in the Latitude of 59 Degrees, the longest Day has no Night, neither

846 *The* ELEMENTS Book VI.

ther has the longest Night and Day. And the two frigid Zones extended about the Poles (each of which is 62 Degrees broad) are ten times greater than the whole Surface of our Earth.

Of all the Primary Planets, the Eye, posited in Saturn, will be able to see only Jupiter, always accompanying the Sun, and never above 37 Degrees distant from it. Reckoning those two amongst his own five Companions, or Moons, a Number of seven Planets is made up to an Observer in Saturn, which he distinguishes into five Moons, and two other Planets, (as wou'd be done in Jupiter by the foregoing *Prop.*) as well for the same Reasons as also upon this Account, that the five nearest are mov'd in the same Plane, inclin'd to the Plane of the Sun's apparent Way about Saturn, in an Angle of 31 Degrees; so that the Zodiac to Saturn is more than the third Part of Heaven in Breadth, if the Deviation of all the Planets from the Sun's Way be comprehended in it. The Phases of these Moons are similar to those of the Jovial Moons; except it be, that in Summer and Winter, when the Sun declines very much to the Æquator, each Moon, oppos'd to the Sun, does not shine with a full Orb, neither is it quite dark when in Conjunction with the Sun. For, in the Middle of Summer, or of Winter, a Moon, in Conjunction with the Sun, is as far distant from it, as our Moon is two Days, or more, before or after the New Moon, and, in Opposition of the Sun, it is as far from that direct Opposition which makes a full Orb, as our Moon is two Days before or after the Full: Those Moons therefore will then have the same Phases, as our Moon has two Days before or after the Syzygy. Whence, at those Seasons of the Year of Saturn, there will be no Eclipses

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Book VI. of ASTRONOMY. 847

Eclipses of the Luminaries. But at the Equinoctial Times, Saturn will never be without Eclipses, and those of all the Kinds mention'd in the foregoing Propositions.

Since the Year in Saturn is longer than that in Jupiter, there are also several Sorts of Months to distinguish it into Parts. Of the least, there are above 5700 in a Year, of those immediately greater 3932, of the third Months above 2352, more than 674 of the fourth, and 135 of the greatest.

An Inhabitant of Saturn will easily compare the Distances of his Moons with his own Measures; because the Parallax of the fifth and last Moon is greater than the Parallax of our Moon. But the Sun's Parallax does not exceed nine Seconds; therefore it is insensible, in Comparison with the Sun's Parallax seen from the Earth.

The Ring which encompasses Saturn, of which we spoke in the fourth Book, is a wonderful Phenomenon to an Eye in Saturn; for it is the only Thing of that Kind which is seen from far, and which has long since excited the Curiosity of all Philosophers. This Ring gave Occasion to the learned and ingenious Dr. *Halley*, when he enquir'd into the Causes of the Variation of the declining magnetic Needle, to suppose the Globe of the Earth to be divided into Crusts and concentrical Nucleus's (See *Philosophical Transactions*, N^o. 195.) after *Kepler*, who did the same in the 4th Book, Pag. 586. of his *Epit. to the Copernican Astronomy*, when, from the magnetic Direction, he investigated the Causes of the Inequalities of the Planets. What if this Ring shou'd be the remaining Part of an outward Crust, fallen on the Nucleus within it, and the Ruins of it still to be seen? For, if Saturn was
ever

848 *The* ELEMENTS *Book VI.*

ever of the same Diameter, as now his Ring is; it was of the same Magnitude, when seen from the Sun, as now Jupiter's apparent Diameter, seen from it.

Since the outward Edge of Saturn's Ring stands above Saturn, at an Height which is four Times and a Half the Semidiameter of Saturn, it will not be visible at a Distance from Saturn's *Æquator* (in whose Plane it is) greater than 64 Degrees: Therefore a Spectator, having a greater Latitude, will never see the Ring; and there is a Zone about each Pole, about 53 Degrees broad, altogether depriv'd of the Sight of the Ring. And, as the Eye goes nearer to Saturn's Pole, the first Satellite is hid, then the second, and so on, till the Eye, at last, being within one Degree of the Pole, loses Sight of the fifth also, unless it be by Refraction; neither will it, in Winter, see the Sun, any Moon, or any Planet, or any celestial Body known to the Inhabitants of the Earth except the Fix'd Stars, and perhaps some Comet.

The Eye in the very *Æquator* of Saturn, or a Zone near it, will see no Stars in the *Æquator*, or very near it; and therefore will never see any of the Moons, the Ring being in the Way: Therefore, at the Time of the *Æquinox*, one that lives under the *Æquator* will not see the Sun; and one, that lives in another Place, will not see the Ring; because, in that Case, no Face of the Ring is shin'd upon. How broad that Zone is, is a difficult Thing to determine, by Observations made from the Earth, because the Thickness of the Ring is small; *Hugens* supposes it to be 600 *German Miles*. It will be easy to determine it accurately from the Surface of Saturn, because it is rais'd a little more than

Book VI. of ASTRONOMY. 849

than a Semidiameter above it. If the Eye be suppos'd plac'd out of the *Æquator*, the Space of the Heaven, which contains Stars cover'd by the Ring will, be of a different Magnitude and Figure, according to the different Situation of the Eye.

During half a Saturnian Year, one Face of the Ring is enlighten'd by the Sun; whence it happens that those who inhabit the Hemisphere towards which it is turn'd, that is, who have Summer, see that Part of the Ring which is above the Horizon, shining with a faint Light by Day (much such as we see the Moon shine with, when the Sun also shines at the same Time) if Saturn has an Atmosphere, and that posited towards that Part where the Pole is depress'd; but its Light is stronger by Night, as also that of our Moon in the Sun's Absence: And from the Spots may be found whether it is at rest, or moves, and if it moves, the Velocity of its Motion. After the Sun is set, the Eastern Part of this lucid Arch has a Shadow (namely, the Shadow of Saturn) cast upon it, which, as the Night comes on, rises by Degrees, till in the Middle of the Night it is got to its upper Part or Vertex, from whence it moves by Degrees towards the Western Part, as the Sun tends to its Rising. This Arch will always shew the Meridian Line, for a Plane rais'd upon the Horizon, going thro' the Vertex of the Arch, is in the Meridian. If the Distance of the Eye from the *Æquator* be less than 52 Degrees, it will see the Arch of the Ring as much concave as convex, like an Arch in a Building, rising on each Side from the Horizon; below which, namely, between the lucid Arch and the Horizon, he will see the Heaven and the Stars. But if the

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850 *The* ELEMENTS Book VI.

Eye is farther distant than 52 Degrees, but less than 64, it will not at all see the concave Part, but a lucid Body rising as it were from the Ground, and contiguous to the Horison, like the Morning Twilight.

During the other half of Saturn's Year, namely, when the Sun declines from the *Æqua-*tor towards the invisible Pole, that is, in Winter Time, the Eye will not at all see the Ring, because it is not enlighten'd on that Side which is towards the Eye; but the Ring will cover from the Eye Part of Heaven, and the cœlestial Bodies contained in it, as has been said before, and so by this Means it will become sensible. But the Shadow of the Ring becomes more and more extended towards the Pole, so that the Eye being plac'd any where within the foresaid Space, the Sun, when it has attain'd a certain Declination, will appear to be cover'd and eclips'd at Noon, and then immediately again it will become visible. The next Day the same Thing happens again, but the Eclipse begins sooner and ends later; and this Darkness in the Middle of the Day will daily encrease in Duration, (or perhaps come to last a whole Day) till the Middle of Winter, from which Time they will again begin to decrease, till at last they are quite gone (or do not happen at all in the Day Time) when the Sun, returning from the Tropick is come back to the same Degree of Declination, which it was in, when the fore-mention'd Darkness began to appear. And this will be so, if the Eye be in a greater Latitude than 25 or 26 Degrees: But if the Spectator is in a lesser Latitude, when the Meridian Darkness is the most lasting, the Sun will suddenly appear just at Noon, and then immediately
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Book VI. of ASTRONOMY. 851

disappear again, and the next Day there will again be such an Appearance of Light, but something more lasting; and so on, the Meridian Light becoming daily more and more lasting till the Middle of the Winter: And then again by Degrees it will become of less and less Continuance, till it wholly vanishes, when the Sun is return'd from the Winter Tropic to a Declination, equal to that at which the Mid-Day-Light began first to appear; at which Time, on a sudden, the longest Meridian Darknes will again begin, and last a shorter and shorter Time, as the Sun comes towards the *Æquator*. And hence it follows, that there is a much greater Difference between Summer and Winter on Saturn's Globe than on the Earth, as well upon account of the Duration of each, and the Sun's great Declination from the *Æquator*, as upon account of the Meridian Darknes in Winter, by reason of the Interposition of the Ring which hides the Sun.

PROPOSITION VII.

TO describe the Phænomena, when the Eye is suppos'd in one of the Comets.

We refer the Comets to the Primary Planets, because it is in the Excentricity of their Orbits that they chiefly differ from them. An Eye, plac'd in a Comet, will find itself encompass'd with a very great and dense Atmosphere, which encreases in its Descent from the Aphelion, especially when it is immers'd in the Region of the lesser Planets. This Atmosphere is so troubled and thick that, towards its Bottom, it is almost like a Chaos. It is not known whether a Comet revolves about itself, but it is probable that, like all the other great Bodies of the

852 *The* ELEMENTS Book VI.

World, it turns all its Faces towards the Sun, that a plentiful Vapour may be rais'd from it. But whether this Rotation is about a given Axis, is difficult to determine. If the Nucleus be turn'd about, the whole Atmosphere (especially that Part of it which is extended towards the Regions opposite to the Sun) does not revolve along with it, as the Atmosphere of the Earth does: But that Vapour, which, going out of the Comet, makes the Tail, is not so much to be look'd upon, as the Atmosphere of the Comet join'd with it (as the denser Atmosphere of the Earth is join'd with it) and making Part of it; as a separate Body, all the Parts of which perform their Motions in a free Space, like the Comet itself.

As the Way of every Comet about the Sun is a very excentrical Ellipse, it is not likely that they are made to serve the same Purposes in the Universe as Planets, which are carried in Orbits nearly concentric to the Sun, and which seem design'd for the Production of Things which are almost always to remain in the same State; which Comets are by no Means fit for, by reason of the very different Degrees of that which they sustain. If therefore we may give Way to Conjecture, it is to be believ'd that the Comets serve for the renewing of the Fluid of the Sun and Planets. For since the Sun continually loses a great deal of its Fluid, sent out under the Form of Light; and the Fluid of the Planets is daily chang'd into Solids (scarce becoming fluid again) as we see it yearly happen in our Earth; it is plain either that the Sun's Forces will by Degrees be lost, and the Fluid of the Planets be consum'd (which is very unlikely) or else that they are to be renew'd at stated Times. But there

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Book VI. of ASTRONOMY. 853

there is no Way that seems so fit for it, as to look upon Comets (which pass thro' vast Spaces in the Æther) to be small Bodies which are design'd in Time to come to make Light, projected thro' those Spaces, and as it were scatter'd and lost in them, which likewise by Attraction (a Quality natural and belonging to all Bodies) draw to themselves, and as it were treasure up the moist Vapours; that having rais'd 'em up in Tails they may mix them with the Atmospheres of Planets (which was also the Opinion of *Hippocrates of Chios*, taken notice of by *Aristotle*, in the *First Book of Meteors*, Chap. 6.) and themselves renew the Sun, the Fountain of Light, by falling into it. Hence will the Primary Planets be augmented; whence the Satellite's Orbit (that is, if such a Planet has a Satellite) and Period will be contracted; of which Phenomenon we spoke above in its Place; and other Things will happen, which we mention'd, when we spoke of Comets in the foregoing Book.

There may also sometimes be another Effect or Use of a Comet. Namely, if a Comet passes near a Planet (its Orbit and Motion carrying it that Way) it will so attract it that its Orbit will be chang'd (the Comet's Orbit being also chang'd by the mutual Action;) whence the Planet's Period will also be chang'd. But the Comet may also by its Attraction so disturb the Satellite, as to make it leave its Primary Planet, and itself become a Primary Planet about the Sun, and continually move round it. Besides, it may produce much greater Changes in the Globe of the Planet it self, not only by attracting the Fluid, if it has any, but also by other Qualities, as if, for Example, so vast a Body goes from

854 *The* ELEMENTS Book VI.

the Sun's Neighbourhood, and being red-hot is carried near our Earth. But we have said enough of this Matter, especially since it does not belong to our present Purpose.

The Years of Comets are different, according to the Magnitude of the greater Axes of the Orbits which they describe about the Sun; namely, (by the 40th and 42d *Propositions* of the 1st *Book*) the Year of each Comet is to our Year in a sesquiplicate Ratio of the greater Axis of the Orbit of the Comet, to twice the mean Distance of the Sun from the Earth. The Sun's apparent Diameter, his Disc, and also his Light and Heat, as well as the apparent Magnitude of the Planets, will be very much chang'd in a Comet. Besides, the Observer in a Comet will find the Motions of the Planets to, be very different; accordingly, when immers'd in their Regions, he has them sometimes all superior to him, sometimes some inferior, and some superior; or, being rais'd out of those Regions, he sees all the Planets below him; all which Things may, from what has been said before, be defin'd in a Comet whose Orbit is given. As, for Example: If a Comet, in its Aphelium, ascends to a Distance from the Sun six Times greater than the Distance of Saturn, which will happen, if the Period of the Comet be equal to about 150 of our Years (supposing that in the Perihelium it comes extremely near, as it happens in some of them) then all the Planets, upon Account of their Nearness to the Sun, will then become invisible to an Eye plac'd in a Comet; for Saturn himself will scarce be 10 Degrees distant from the Sun, and the Sun's Diameter will be equal to half a Minute. But, to an Eye in a Comet whose Period is much greater, the Planets are
invisible

Book VI. of ASTRONOMY. 855

invisible all the while that the Comet is in the upper Part of its Orbit, which is for the greatest Part of its Year; for the Velocity of the Comet towards the Aphelium is very small, in respect of that which it has when it is at, or near, the Perihelium. If the Period of the Comet be too great; that is, if the Distance of the Comet from the Sun in its Aphelium does often contain the Distance of Saturn, perhaps the Order of the Fix'd Stars, seen from such a Comet, will be very different in different Places of its Journey: But yet, I hardly think this Difference to be very sensible, but rather, that the Orbits, even of the outermost Comets, are very small, when compar'd to the Distance of the Sun from the Fix'd Stars.

COROLLARY.

Hence it appears, that the System of the circumfolar Planets, seen from a Fix'd Star, wou'd become almost invisible, and confounded with the Sun in the same lucid Point, whose Diameter wou'd become insensible, if it be seen distinctly, and free from that Hairiness, or those Rays, which the Fix'd Stars appear to have to the naked Eye. For, since all the Planets cannot be seen from a Comet in its Aphelium, by reason of their Nearness to the Sun; how much more wou'd they become invisible from so great a Distance as that of the Fix'd Stars? For, supposing the Parallax of the *Magnus Orbis* (or the Semidiameter of the *Magnus Orbis* seen from a Fix'd Star) to be equal to the Parallax of the Sun seen from the Earth (neither have we any Observations which shew it to be greater) the Sun's Diameter will, at a Fix'd Star, scarce subtend an Angle of five Thirds, and Saturn will never be above two first Minutes from the Sun. Besides, the Light which the

856 *The* ELEMENTS Book VI.

Planets borrow, by reflecting the Sun's Rays, will be altogether unable to affect such a Sight as ours, at that immense Distance; and tho' it were strong enough to affect it, the Sun's Species would appear to extend itself into a Semidiameter, equal to that of the System, as it happens to all lucid Bodies, and especially to the Fix'd Stars seen from the Earth; whence they wou'd all, as it were, fall in with the lucid Body. Therefore, on the other Hand, the Planets which revolve about such Suns (namely, the Fix'd Stars) do altogether escape our Sight. There is no Doubt, but that, to an Eye plac'd in any Fix'd Star, or Planet revolving about it, the Order of the other Fix'd Stars, and the Figures of the Constellations, wou'd be quite different from those that we see; especially since our Sun wou'd in that Case appear as one of the Fix'd Stars.

Now this immense Distance of the Fix'd Stars or Suns from each other hinders those vast Bodies, and the Planets (very small, if compar'd with them) which are carried about those Suns, from sensibly exerting their attractive Forces on each other, and disturbing each other's Motions. The indefinite Number of those Systems, included in no Space, is the Reason why they don't run into one, but, being separated from one another, will for ever stand in the Universe, as Marks of the Power and Wisdom of their Almighty Creator.

PROPOSITION VIII.

TO describe the chief Phenomena of the Fix'd Stars, of the Sun, and of all the Planets, except the Earth, as they wou'd appear to an Eye in the Moon.

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Book VI. of ASTRONOMY. 857

Since the Moon (as was said in *Prop. 56. B. IV.*) turns round it self from West to East, about an Axis nearly perpendicular to the Plane of the Ecliptic, and always parallel to itself, equally, and in the same Time that she revolves about the Earth; that is, in a periodical Month; it is plain, that, to an Eye plac'd in its Surface, the Sun, the fix'd Stars, and all the other Bodies (that is, all the Planets except the Earth) will seem nearly to revolve, on the Poles of the Ecliptic, from East to West. And those Poles are remarkable by fix'd Stars; for the northern One has a Star of the fourth Magnitude, after the third Flexure of the Dragon, about three Degrees distant from the Pole; and the southern Pole has near it four Stars of the Fifth *Xipbias*, one of which is nearer the Pole than our Arctic Polar Star, but yet more remarkable by a nebulous Star near it. And then also, as it happens with us, the Stars are all carried at the same Time by the common Motion, except that their proper Motions (whether true or apparent) do a little alter the Case. And first, the Sun's apparent Revolution about the Moon, which is at rest, as to Sense, is perform'd in a longer Time, because the Moon is, at the same Time, together with the Earth, carried about the Sun *in consequentia*; so that the Moon's natural Day is of the same Duration as our synodical Month, which almost exceeds, by two and a Half of our Days, the before describ'd Revolution of the said lunar Meridian, in respect to the same fix'd Star. Whence to an Observer in the Moon, the Sun only rises twelve Times in a Year, but the fix'd Stars thirteen Times; or more accurately, in nineteen Years, the Sun rises 200 and 35 Times, but the fix'd Stars, 200 and 54 Times:

856 *The* ELEMENTS Book VI.

Planets borrow, by reflecting the Sun's Rays, will be altogether unable to affect such a Sight as ours, at that immense Distance; and tho' it were strong enough to affect it, the Sun's Species would appear to extend itself into a Semidiameter, equal to that of the System, as it happens to all lucid Bodies, and especially to the Fix'd Stars seen from the Earth; whence they wou'd all, as it were, fall in with the lucid Body. Therefore, on the other Hand, the Planets which revolve about such Suns (namely, the Fix'd Stars) do altogether escape our Sight. There is no Doubt, but that, to an Eye plac'd in any Fix'd Star, or Planet revolving about it, the Order of the other Fix'd Stars, and the Figures of the Constellations, wou'd be quite different from those that we see; especially since our Sun wou'd in that Case appear as one of the Fix'd Stars.

Now this immense Distance of the Fix'd Stars or Suns from each other hinders those vast Bodies, and the Planets (very small, if compar'd with them) which are carried about those Suns, from sensibly exerting their attractive Forces on each other, and disturbing each other's Motions. The indefinite Number of those Systems, included in no Space, is the Reason why they don't run into one, but, being separated from one another, will for ever stand in the Universe, as Marks of the Power and Wisdom of their Almighty Creator.

PROPOSITION VIII.

TO describe the chief Phenomena of the Fix'd Stars, of the Sun, and of all the Planets, except the Earth, as they wou'd appear to an Eye in the Moon.

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Book VI. of ASTRONOMY. 857

Since the Moon (as was said in *Prop. 56. B.IV.*) turns round it self from West to East, about an Axis nearly perpendicular to the Plane of the Ecliptic, and always parallel to itself, equally, and in the same Time that she revolves about the Earth; that is, in a periodical Month; it is plain, that, to an Eye plac'd in its Surface, the Sun, the fix'd Stars, and all the other Bodies (that is, all the Planets except the Earth) will seem nearly to revolve, on the Poles of the Ecliptic, from East to West. And those Poles are remarkable by fix'd Stars; for the northern One has a Star of the fourth Magnitude, after the third Flexure of the Dragon, about three Degrees distant from the Pole; and the southern Pole has near it four Stars of the Fish *Xipbias*, one of which is nearer the Pole than our Arctic Polar Star, but yet more remarkable by a nebulous Star near it. And then also, as it happens with us, the Stars are all carried at the same Time by the common Motion, except that their proper Motions (whether true or apparent) do a little alter the Case. And first, the Sun's apparent Revolution about the Moon, which is at rest, as to Sense, is perform'd in a longer Time, because the Moon is, at the same Time, together with the Earth, carried about the Sun *in consequentia*; so that the Moon's natural Day is of the same Duration as our synodical Month, which almost exceeds, by two and a Half of our Days, the before describ'd Revolution of the said lunar Meridian, in respect to the same fix'd Star. Whence to an Observer in the Moon, the Sun only rises twelve Times in a Year, but the fix'd Stars thirteen Times; or more accurately, in nineteen Years, the Sun rises 200 and 35 Times, but the fix'd Stars, 200 and 54 Times:

858 *The* ELEMENTS Book VI.

Times : And the rising Sun is almost a whole Sign forwarder than before.

This natural Day of the Inhabitants of the Moon is equally divided into Light and Darknefs, because the Axis, about which the Moon revolves, is almost perpendicular to the Plane of the Moon's Way about the Sun ; and therefore there is no Twilight at the Moon before the Sun's Rising, or after its Setting ; because such a Thing can't happen, unless there was an Atmosphere about the Moon, (as has been shewn in the 8th *Prop.* of the 2d *Book.* Whence, to the Moon's Inhabitants, the Sun seems to move in a Circle between the Poles of the first Motion.

Altho' the Sun's Way, seen from the Moon among the Fix'd Stars, be nearly the same as its Way seen from the Earth, and the Places of the Planets are nearly the same seen from the Moon as from the Earth ; yet, when the Moon is not in the Nodes, a nice Observer will see the Sun deviate from the Ecliptic towards the contrary Side, and extend its Light beyond the Pole of the Moon, which is under that towards which it declines, leaving the other Pole in Darknefs. But all these Things become right again in half a lunar Day, when the Moon returns to its Node and the Plane of the Ecliptic, in order to go to the other Side of the Ecliptic, and to undergo the same Phænomena towards the other Pole, for the remaining Part of the Day.

Besides all the Inequalities in the Planets Motions seen from the Earth, which must obtain in the Moon as well as the Earth, because it is carried along with it ; there are others peculiar to the Moon, the chief of which are these. When it is Day to the Inhabitants of the Middle
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Book VI. of ASTRONOMY. 859

of that Hemisphere of the Moon which is turn'd from the Earth, the Sun appears to go slowly among the Fix'd Stars; and very slowly at the Point of Noon; but the next Day it will be found to have gone faster among the Fix'd Stars in the Night-Time. And on the contrary, to an Observer in the Middle of that Hemisphere of the Moon, which is towards the Earth, the Sun will appear, by Day, to move swiftly under the Fix'd Stars towards the East, and very swift at Noon: But at Night the Sun goes very slowly under the Fix'd Stars, and slowest of all at Midnight. The same Things are also true of Venus and Mercury, because they are always near the Sun; and also of Mars, when it is in Conjunction with the Sun, tho' those Things are less remarkable then, upon Account of its great Distance from the Moon at that Time; but they are insensible in Jupiter and Saturn, when their Position is towards the Sun. If any one of the superior Planets, seen from the Moon, be in Opposition to the Sun, in which Case it is retrograde, it will appear to an Inhabitant in the Middle of the Hemisphere, turn'd from the Earth, to be more retrograde than ordinary, by Night, but less by Day; and least of all, or most of all, just at Noon, or just at Midnight. So likewise an Observer, living in that Part of the Moon, which the Earth seems to hang over, will see a Planet in Opposition to the Sun, to be the least retrograde of all at Midnight, and most of all at Noon. And these Things in Mars are twice more sensible than in the Sun: But in Jupiter and Saturn these Differences almost vanish. Likewise, if any Planet, which from the Earth appears Stationary, appears to be in Opposition to the Earth, in respect to the Moon, it will be retro-

trograde ; but direct, when in Conjunction with the Earth, which will be very sensible in Venus.

An Astronomer, plac'd in the Moon, and ignorant of the Motion of his Habitation about the Earth, or the Motion of the Earth about the Sun, for making good the Theory of every Planet, will, to all the Epicycles invented by a Terrestrial Astronomer, super-add a new Epicycle, or Epicyclo-Epicycle, to explain the above-mention'd Inequalities. As for *Example* : To the Excentric of the Sun, describ'd about the Earth, made Use of by an Inhabitant of the Earth, (for an Inhabitant of the Earth, that admits an Excentric, does not want an Epicycle for explaining the Sun's Motion) will add, over and above an Epicycle, and will suppose that the Sun is carried round in it, in the Space of one of the lunar Days, whilst at the same Time the Center of the Epicycle is carried in the Excentric by the annual Motion : Thus will the Sun's diurnal Inequality, observ'd by a lunar Astronomer, be explain'd, if, whilst it is Night, with the lunar Inhabitant, which is directly under the Earth, he supposes the Sun to be in the lower Part of the Epicycle, and in the upper Part of it whilst it is Day with him. So also to the Excentric of Mars about the Earth and its Epicycle, contriv'd by a terrestrial Astronomer, by which the Motion of Mars, seen from the Earth (suppos'd at rest) is explain'd, he will super-add another Epicycle carried in the Circumference of the first, which Mars will be imagin'd to go thro' in the Space of *Nycthemeron*, or natural Day. And this is the only Way that our lunar Astronomer can settle his Astronomy, if he be ignorant of the Motion of his Habitation, and as-

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Book VI. of ASTRONOMY. 861

fers it is at rest: Namely, by transferring the Circle which is really describ'd by himself and his Habitation in the Heavens, and supposing the Motion to be made in it, in the same Time, and in an inverted Order; and retaining its true Magnitude, if the true Distance is known; but if it is not (and there are no Means to find it out in the Supposition of the Moon at rest) at least the Epicycle is suppos'd of such a Magnitude, that it may appear from the Observer's Place, of the same Magnitude, as the true Orbit of the said Place wou'd appear at the same Distance. If the Inhabitant of the Moon's Hemisphere, which the Earth hangs over, be suppos'd to know the Motion of his Habitation about the Earth, and of the Earth about the Sun, and capable of making Observations as accurately as we make them on Earth; he will be able, exactly enough, to compare the Magnitudes and Distances of the Sun, and the Planets of the solar System, with such Magnitudes and Distances as are familiar to him. For he may measure the Diameter of his Habitation, after the same Manner as we do that of our Earth. Moreover, he will easily compare the Distance of the Earth, with the Moon's Diameter; for the Moon's Diameter subtends an Angle sensible enough at the Earth, namely, an Angle of half a Degree: And he will make use of the Diameter of his Orbit, as a Scale, to find the Distances of the other Bodies of the World. For that subtends an Angle at the Sun, the third Part of a Degree, supposing that the Sun's Parallax, seen from the Earth, is only $10''$; but arises to almost half a Degree at Mars, when it is in Opposition with the Sun, and at Venus in Conjunction with the Sun, and, being retrograde, exceeds a Fifth and a Half of

862 *The* ELEMENTS Book VI.

of a Degree. Moreover, an Inhabitant of the Moon will, after the same Manner, find out the Ratio's between the Distances of the Primary Planets from the Sun, as an Inhabitant of the Earth does; and knowing the Distances, he will easily find their Magnitudes from their apparent Diameters.

PROPOSITION IX.

TO describe the chief Phænomena in the Earth, seen by an Eye in the Moon.

There are in the Moon two Hemispheres, altogether different, nay, even contrary, in respect of the Phænomena, of which we are to speak in this Proposition. For they constantly see the Earth in one Hemisphere of the Moon, and never in the other; unless that, near the Limits of them, there is a Tract, to whose Inhabitants the Earth sometimes rises a little above the Horizon, and sometimes going back, as it were, subsides again under it. In the Hemisphere, from which the Earth is seen, it appears as it were nail'd up, in the same Point of the Heavens, (except that it appears to move from thence a little towards one Side or the t'other, and then to return by a libratory Motion) whilst, in the mean Time, in the Space of a natural Day, the Sun, and the other cœlestial Bodies, move towards it from the East, to go afterwards from it towards the West. And a Star coming to the Earth, hanging in the Æther, is not extinguish'd suddenly, but, by little and little, is first cover'd with a Mist, and at last, getting behind the Earth, ceases to be seen by a lunar Observer; which is occasion'd by the Atmosphere about the Earth, by which the Rays, going from a Star to the Eye, are almost choak'd up. Moreover, to some of
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Book VI. of ASTRONOMY. 863

the Inhabitants of the said lunar Hemisphere, to Earth appears continually to be fix'd directly over their Heads; and that is, to such of them as live in the very Middle of the said Hemisphere: But to some of them it appears to decline towards the North, to others towards the South, to some towards the East, and to others towards the West, more or less, according as their Habitation is nearer to, or farther from, the said Middle, or some Side or other of it; till at last, to the Inhabitants of the foresaid Limits, which divide the Moon's two Hemispheres, the Earth will appear to be in the Horizon, one Half of it standing up like a flaming Mountain: But they that dwell in a Circle of the Moon, which goes thro' the Middle of the said Hemisphere and the Poles, have always the Earth in the Meridian.

They that observe the Earth at Rest, in its Place, see its Orb fifteen Times greater than we see the Moon, but all the while turning about its Axis, and appearing to carry its Spots, (which they will, in a rude Manner, distribute into some Figures, after the Manner that the common People suppose a Thing like a Man's Face in the Orb of the Moon) from East to West, and then return to its first Situation, in about the twenty-ninth Part of a lunar natural Day. *Kepler*, from that Rotation of it, supposes, in his *Lunar Astronomy*, that the lunar Astronomers call the Earth *Volva*, (Whirler) and the Moon, suppos'd fix'd, *Vesta*, as never to be mov'd from its Place, and that they look upon our Globe, which they see hanging in the Æther, as a cœlestial Body, which therefore they must not call by the Name of *Earth*: Therefore he calls the Inhabitants of that Hemisphere, under this *Volva*, *Subvolvæ*,
and

and *Subvolvani*, and the others which are depriv'd of the Sight of it, *Privolvæ*; which Names I shall also make use of hereafter. Besides, the said Revolution of the *Volva*, there is also, as it were, another nodding Motion of it, by which sometimes it shews the *Lunarians* its Northern, and sometimes its Southern Pole, and that in the Space of one Day. For when by the First Motion, or Motion of the *Primum Mobile*, *Sagittarius's* Bow comes towards the *Volva*, the *Lunarians* (especially those which are at the North) will see the Northern Pole of the *Volva*, and the Spots, and brighter Parts about it, which, for some Revolutions of the *Volva*, will not retire behind it, when at the same Time those, that are about the Southern Pole, do not offer themselves to Sight: After half a *Nychthemeron* is over, and that *Castor* is come to the *Volva*, the Northern Spots of the *Volva* will be hid, and the Southern ones will come in View.

The *Lunarians* see this *Volva* encrease, or decrease, in the same Manner, with the same Phases, and for the same Reasons, as our Moon; but the same Period of Phases, which we call a Month, they call a natural Day. They, that see the *Volva* in their Meridian, have the *New-Volva* or New-Earth at Noon, the First Quarter at Night, the *Full-Volva* at Midnight, and the last Quarter in the Morning. They that dwell under any other Meridian, at a Distance from the foresaid Meridian (which is the principal One, and, upon Account of its Situation, may, according to *Kepler*, be suppos'd to be call'd, *Circulus Medivolvanus*, by the *Lunarians*) towards the East or West, have the Moment of each Phasis at a different Hour, according to the Distance of their Meridian from the Medivolvani Circle,

Book VI. of ASTRONOMY. 865

Circle, and the Number of Hours into which they divide their Day; therefore the Phases of the *Volva* denote the Hours, which also is done by the Distance of the Sun from the *Volva* observ'd by Day. Whence the *Volva* serves for a Dial to all the *Subvolvani*: For by the Diversity of its Phases, by the Approach of the Sun or Fix'd Stars towards it, by the Name of the Pole that it shews, and lastly, by the swift Motion of the Spots; (for the remarkable Parts of the *Volva*, our Mountains, Lakes, Islands, Seas, &c. even tho' they are not immediately shin'd upon by the Sun, become visible, especially before and after the New-*Volva*, as the Moon's Parts to us before and after the New-Moon, tho', *cæteris paribus*, not so distinctly, because the Moon, being a less Body, must reflect less Light to the Earth, than the Earth, which is a greater Body, does to the Moon;) it divides the Day and Night (which are indeed very long) into a sufficient Number of Parts. This *Volva* also does shew the Parts of the Year distinctly enough in the following Manner: Since in Summer-time to the Inhabitants of *Europe* the Arctic Pole points towards the Sun, and the whole Arctic frozen Zone is enlighten'd by the Sun for several Days, and that our Earth is the *Volva* of the Lunarians, whose Arctic Pole, and a great Region about it, is enlighten'd, during some Part of every *Nychthemeron*; it is plain that when a great Region of the *Volva*, lying about its Arctic Pole, appears enlighten'd to a Lunarian, the Sun will be seen from the Moon at the Constellation of *Gemini*, and that will be the Time of the Year which an Inhabitant of the Northern Hemisphere of the Earth calls Summer, and in that Case the Lunarians will clearly

see the dry Land and Seas about the North Pole, which are yet undiscover'd by us. After a Quarter of a Year our Lunar Observer will see the Light terminated at each Pole of the *Volva*; a Quarter of a Year after that, he will see the Regions about the South Pole of the Earth, which are altogether unknown to us, enlighten'd by the Sun; and after another Quarter the Light will again reach each Pole. He will also distinguish the Seasons of the Year according to the different brightness of *Scandinavia* and *Scythia*, for Example, as being greater, *cæteris paribus*, in Winter, when it is cover'd with Snow, than in Summer: And this Difference of Brightness will be more visible in that Sea which is near the Pole of the *Volva*, because in our Summer it looks like a Spot, and in Winter it is as bright as the most shining Parts.

The Lunarians have Eclipses of the Sun and of the *Volva*, as the Inhabitants of the Earth see Eclipses of the Sun and Moon, and at the same Time, but with some Difference. For as to us an Eclipse of the Moon happens at the Time of the Full-Moon, so to them an Eclipse of the *Volva* happens at the Time of the Full-*Volva*; and as the Sun appears to us to be in an Eclipse at the Time of the New-Moon, it appears so to them at the Time of the New-*Volva*: But tho' to the Inhabitants of the Earth the whole Moon appears Eclips'd, the Lunarians never have a total Eclipse of the *Volva*; for they see a certain reddish Spot pass from East to West over the Disc of the *Volva*, making more haste than its native Spots, and sometimes going over the Center of the Disc, being four Hours in going over; but oftner going beside it, when the Appearance is less lasting.

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Book VI. of ASTRONOMY. 867

Their solar Eclipse is caus'd by the *Volva*, just as our solar Eclipse is by the Moon; and when we have an Eclipse of the Moon, the Sun is totally Eclips'd to the Lunarians that inhabit the darken'd Part of the Moon: But it often happens that an Inhabitant of the Moon (especially one that dwells near the Limits of the *Subvolvæ* and *Privolvæ*) sees a partial Eclipse of the Sun, when the Moon appears to us with a full Orb, the Earth's *Penumbra* not sensibly extinguishing the Moon's Light. And it likewise happens, that to a certain Tract of the Earth the Sun appears to suffer a partial Eclipse, when at the same Time a Lunarian does not perceive the *Volva* to be Eclips'd at all: But when we have a total Eclipse of the Moon, the Sun is wholly Eclips'd to all the Lunarians. Since the Diameter of the *Volva* is four times greater than the Sun's Diameter, the Sun will be fully hidden by the *Volva*, and that for a long Time, which cannot happen to us by the Interposition of the Moon; nay, the Sun's Eclipse may be total and lasting when it is not Central. The Inhabitant of the Lunar Hemisphere, over which the *Volva* hangs, sees frequenter Eclipses of the Luminaries than the Inhabitants of a given Hemisphere of our Earth; for the *Subvolvans* alone see all the Eclipses, and the *Privolvans* none, since a great Part of the Eclipses seen by the Inhabitants of the Earth, belong to the opposite Hemisphere. The before-describ'd total Eclipse of the Sun, tho' it happens often to the Lunarians, cannot but be very remarkable; for that is the only Time that the *Subvolvans* are altogether depriv'd of Light: For it has been shewn before, that the *Medivolvans*, or those who are in that Part of the Moon which is directly under the Earth, have the first Quar-

ter of the *Volva* at Sun-set, and the last Quarter at Sun-rise ; whence the *Volva*, shining all Night, is to them fifteen Times greater than our Moon, and powerfully dispels the Darkeness. But in the total Eclipse of the Sun, both the Sun and the *Volva* are extinguish'd at the same Time, and all is dark. But this Inconveniency has its Remedies ; for the Sun's Rays, being refracted by the Atmosphere which is about the *Volva*, make the Body of the Sun appear enlarg'd, and as it were of a greater Diameter, but at the Edges, (which only are seen) of an Iron-colour, to those Inhabitants of the Moon which receive those refracted Rays ; for which Reason there is hardly a total Darkeness, except when the Sun's Eclipse is nearly central, and that only for a little Time about the Middle of the Eclipse. But if it happens that a very high Mountain, standing almost out of the Atmosphere of the *Volva*, is in the Edge of it ; he, on whom the Shadow of that Mountain falls, will wholly lose the Sun's Light, and not enjoy the Benefit of the Refraction of the solar Rays, made at the Atmosphere of the *Volva* : And he, that from a great Distance sees the Shadow of the *Volva* receiv'd on the Moon's Disc, will perceive a very black Spot in that Place. On the other Hand, there is here no Diminution of the Darkeness in a solar Eclipse, upon account of the Illumination of the Spectator's Atmosphere, because he has none.

Moreover, at the Time of the Sun's Eclipse, the Light that ends in a Point extended along the Ecliptic from each Side of the Sun (of which we spoke in the *Schol.* of *Prop.* 8. B II.) will be clearly seen by a Lunarian, if there be at that Time a Heap of Corpuscles in the Shape of a Lens to produce it ; nay, that Light may also

Book VI. of ASTRONOMY. 869

also be seen when there is no Eclipse, if the Sun be cover'd from the Eye by the Interposition of some opaque Body ; for the Eye in the Moon is not affected by the Rays which are reflected from the ambient enlighten'd Atmosphere, because the Moon has no Atmosphere about it ; which is likely from observing that it is so evenly circumscrib'd, and not vanishing at the Edges as if it had Down upon them ; and because the Stars near its Limb are not at all darken'd, but vivid and shining till they are wholly hidden. By reason of that Want of an Atmosphere, the Lunarians will have no Twilight before the Rise of the Sun (except the shining of the Solar Atmosphere) or after its Setting ; because the Cause which occasions it upon Earth ceases here : Moreover, the celestial Bodies will have no Refractions, and therefore no Change of Place upon that account.

We have hitherto suppos'd that to the Lunarians the *Volva* is unmov'd in the Heaven, because for the most part it is really so : But it has a certain libratory Motion, by which it appears to any Lunarian sometimes to move a little towards one Side, then to return to the same Place, and afterwards to move to the other Side. But this libratory Motion of the *Volva* is easily divided into two : For either the *Volva* appears to librate towards the South and North upon the same Circle of Latitude ; or towards the East and West upon the Ecliptic. Let us suppose the Sun and the *Volva* to be in a central Conjunction, when the Sun is seen in the Ecliptic it self, or that a right Line, joining the Centers of the Sun and *Volva*, goes thro' the Moon ; that is, that to the Inhabitants of the Earth the Moon is in Opposition to the Sun, when in one

870 *The* ELEMENTS Book VI.

of the Nodes; as for *Example*, the descending Node; in which Case the Sun is centrally Eclips'd to the Lunarians, as has been before explain'd. After the Eclipse, the Sun is by the Common Motion carried to the West, whilst at the same time the *Volva* will seem to decline towards the North, and this Declination will be greatest of all after a quarter of the Lunar *Nychthemeron*, namely 5 Degrees, when the Polar Region of the *Privolvans* becomes visible to the *Volva* five Degrees beyond the North Pole, and an equal Region of the *Subvolvans* at the South Pole is reduc'd to the State of the *Privolvans*, from which time the *Volva* returning back upon its Steps, comes again to the Ecliptic, in a Space something less than half of the Lunar *Nychthemeron*. Afterwards it will run out as far towards the South, and make the Lunarians of the South Pole from *Privolvans* to become *Subvolvans*, and so contrarily towards the North; and before the whole *Nychthemeron* be over, it will return a second time to the Ecliptic. This whole Libration of the *Volva* to the North, and to the South, is perform'd in a space of Time not only less than the *Nychthemeron*, but also less than that in which the Fix'd Stars, which have left any Lunar Meridian, are by the first Motion brought to it again; and the *Volva* being the second time return'd to the Ecliptic, will be found in Conjunction with more Westerly Fix'd Stars than those with which it was in Conjunction at the time of the Sun's Eclipse, from which we begun to reckon. After another whole Libration of this Kind is finish'd, it will be in Conjunction with Fix'd Stars that are yet more Westerly, and so on; so that it will be

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Book VI. of ASTRONOMY. 871

nineteen Years before the *Volva* be found again in the Ecliptic, in Conjunction with the same Fix'd Stars, just as it is about to decline Northward. From the different Magnitude of the Period of the Libration of the *Volva* in Latitude, and of the *Nychthemeron*, it happens, that when half a *Nychthemeron* is elaps'd after the Sun's central Eclipse, the Luminaries will have no Eclipse of the *Volva*: For the Sun being come to the Opposition with the *Volva* (that is, in the *Plenivolvium*, or Full-*Volva*, in which alone such an Eclipse may happen) the *Volva* having just pass'd the Ecliptic (for it was come to it a little before the Sun was in Opposition with the *Volva*) will be got so far Southward by its Libration, as to escape the Moon's Shadow. An Inhabitant of the *Volva* will, indeed, see the Sun's Eclipse at that Time, if he lives towards the North Pole of it, but not total; in which Case alone, the *Volva* appears to a Lunarian to have an Eclipse. Likewise, in the following *Novivolvium*, or New-*Volva*, tho' centrally eclipsed, the Moon will escape the Shadow of the *Volva*.

Those Lunarians, which are nice Observers, will sometimes see the *Volva* bigger, and sometimes lesser, and that whole Variety is gone thro' nearly in the Space of a lunar natural Day; but, more accurately, in the Space that the Fix'd Stars take up to return to the same Meridian. Let us suppose, that to a Lunarian the *Volva* appears to have the least Diameter when it is seen in the said Circle of Latitude on which it has its North and South Libration. The Diameter of the *Volva* being, by Degrees, encreas'd, the *Volva* itself will deviate from the said Circle towards the West, during almost a Quarter of the natural Day, and return to it the next Quar-

872 *The* ELEMENTS Book VI.

ter, and then the Diameter of the Volva will appear the greatest; and then from that Place it will again deviate towards the East, and its Diameter will grow less, and it will perform one entire Libration by returning to the Ecliptic (at which time its Diameter will again be least of all) in one Revolution of the Fix'd Stars. This Deviation, when greatest, will be about 5 Degrees; and just so much will the Circle, which divides the *Subvolvans* from the *Provolvans*, be carried sometimes in the West leaving the East, sometimes in the East leaving the West, as by the former Libration it was carried to the North, or to the South. And the Volva, after having perform'd one whole Libration in Longitude of this Kind, returning to its first Circle of Latitude, namely the mean one, between the Extremes of its Libration, (during which Time, from appearing least, it gradually encreases till it appears biggest, and returns again to its first Appearance) will appear in Conjunction with more Easterly Fix'd Stars than those with which it was in Conjunction before, when it was in the same Circle of Latitude: And again after the next Libration, to Stars still more Easterly; and so on, till after almost nine Years the Volva appears both least and in Conjunction with the same Fix'd Stars, and in its proper Circle of Latitude, ready to go from it by librating towards the West. But neither of these Librations is found to be always equal to itself, but the Libration in Latitude is the greatest on those Days which are near to the Eclipse of either of the Luminaries; and least on those Days which are the most remote from the Eclipses: And the Libration in Longitude is greatest of all, when the Volva in the Sun's

Book VI. of ASTRONOMY. 873

Sun's Syzygy appears greatest or least; but least of all, when the Volva in the Sun's Quadrature appears greatest or least.

PROPOSITION X.

TO determine the principal Phænomena that wou'd appear to an Eye, plac'd in some one of the Satellites of Jupiter or Saturn.

The Phænomena of the Sun and of the other Primary Planets, except that to which the Satellite belongs, when seen from a Satellite of Jupiter or Saturn, will be like or similar to their Phænomena seen from our Moon, if those Satellites are like it turn'd about their own Axes; namely, each of them upon an Axis almost perpendicular to the Plane of the Orbit of his Primary Planet about the Sun, and in a Period equal to the Period of the said Satellites about its Primary Planet, as it is very likely. In the mean time we must observe, that tho' from Jupiter we can see no Primary Planet besides Saturn, and from Saturn none but Jupiter; yet from some of the Satellites of Jupiter or Saturn they may all be seen. For we said that Mars and the Planet below Mars cou'd not be seen from Jupiter for this Reason, *viz.* Because we suppos'd Jupiter to have an Atmosphere, which, being enlighten'd by the Sun, hinder'd such Planets as were nearer to the Sun than 20 Degrees from being seen. But if we suppose the Satellites of those Primary Planets to be like the Satellite of the Earth, they will also be without an Atmosphere, as our Moon is: Therefore an Eye in one of 'em will see the Planets, when they are near the Sun, unless the Sun's shining Atmosphere hinders it; but even Mercury, seen from Saturn, will sometimes appear 3 Degrees distant from the Sun.

And

874 *The* ELEMENTS Book VI.

And, as the Sun's apparent Diameter from Saturn is about $90''$, it is plain, that Mercury goes farther from the Sun than the Limits of the Atmosphere; and then, that it may be seen by an Eye that is not hinder'd by an encompassing Atmosphere, that is, by an Eye placed in a Satellite of Saturn, to which Mercury may yet (tho' but a little) appear farther distant from the Sun, than to an Eye in Saturn itself. Moreover, the Phænomena of Jupiter seen from Jupiter's Satellite, or of Saturn seen from a Satellite of Saturn, will be the same as the Phænomena of our Earth seen from the Moon, and differ in nothing but the Measure of their Digressions and Librations: As, for Example, To the *Subvolvans* of any Satellite of Jupiter, the *Volva* does not appear to librate from the Ecliptic, towards the North, or South, above 3 Degrees; tho' the Libration in Latitude of the *Volva* seen from one of Saturn's Satellites, is more than 30 Degrees. But in the latter, as well as in the former Satellites, there is no Libration in Longitude, or, if there is any, it is not sensible, because the Orbits of the Satellites of Jupiter and Saturn are nearly circular.

The Phænomena which appear to an Observer in any of the said Satellites, very different from the Phænomena which appear in our Moon, are such as appear from their Companions. For, if the *Volva* has any Satellite nearer to itself than that in which the Eye is, that Satellite will never appear in Opposition to the *Volva*, but always near it, librated like a Pendulum, sometimes towards the West, and sometimes towards the East of the *Volva*, describing the Arc of a great Circle, which the *Volva* divides into two equal Parts; and, if there are
more

more such Satellites, one will run out into a lesser Arc on each Side, and the other into a greater; and these Satellites will never set to the middle *Subvolvans*, and never rise to the middle *Privolvans*. But those which have the *Volva* a little below the Horizon to the East or West, will see these Satellites rise in the East or West, and go towards the Meridian, then go back again, and set in the same Place. If the *Volva* has a Satellite more remote from it than that in which the Eye is, it will appear to the *Privolvans* once in their natural Day, to the middle *Privolvans*, and middle *Subvolvans*, it rises and sets very slowly, and when it comes to the Meridian, it appears to the *Subvolvans* to move faster towards the West, and to the *Privolvans* to move slower towards it; from which it is easy to guess what Appearance the Inhabitants of the Satellites between both will have. In the mean Time, any one of those Companions, or Satellites of the *Volva*, make the Sun to be eclips'd, and it self is also eclips'd, either when it runs into the Shadow of the *Volva*, or into the Shadow of the Satellite where the Observer is, or of any other of the Satellites. But any one may easily find the various Symptoms and Times of these Eclipses, and several other Things relating to this Matter.

COROLLARY.

From the ten foregoing Propositions, it follows, that Astronomy is the most simple of all to an Eye placed in the Sun. For, if the Eye be placed in its Center, the true System does not differ from the apparent: If it be rais'd up to its Surface, the Observer may also compare the Distances of the six Primary Planets from his Habitation with his own Measures, and
thence

876 *The* ELEMENTS Book VI.

thence determine both their Magnitudes, and that of their Satellites. But he will falsely imagine that all Things are carried from East to West; but if he supposes that his Habitation moves about the same Axis towards the West or contrarywise, this one and simple Motion will reduce him from the apparent to the true System.

If the Eye be suppos'd in any one of the six Primary Planets, the Astronomy is more complex and difficult to be settled; because (contrary to the Observer's Senses) a double Motion is to be attributed to his Habitation, in order to find out the true System; and that the Proportion that the Planetary Orbits bear to each other may be known. Mercury is the most unfit of any for Astronomy, because he has no Satellite from which he may certainly conclude the Opacity of any cœlestial Body; he has no inferior Planet, by which he may from the Phases perceive that some Planet revolves about the Sun. Mars and Venus want Satellites, but they have inferior Planets; and therefore they are not so unfit as Mercury; but less apt than the Earth, which has both Conveniencies. Tho' Jupiter has a great many Satellites, yet he does not see any inferior Planet, unless he can distinguish Planets nearer the Sun than we can: Saturn, on the contrary, sees no superior Planet, and but one inferior one. Therefore no one of the Primary Planets (be the Cases as much varied as possible) has so many Helps for finding out the genuine solar System, and from it knowing the Proportion of the Planetary Orbits, as the Earth.

But

Book VI. of ASTRONOMY. 877

But the System being discover'd, the Magnitudes and Distances of the Planets will be very hard to be found out, by the Inhabitants of any of the Primary Planets, and will not exactly be compar'd with their common Measures. Jupiter seems the fittest for that Work, then Venus, then Saturn, next Mercury, and Mars is the unfittest of any: What Place we must give to the Earth, in this Case, is uncertain, because the Angle at the Sun, which the Earth's Diameter subtends, as has been hitherto found, rather by Analogy, than Observation. Perhaps the Magnitude of the solar System will, in Time, be first discover'd by Observations of Comets: For if, by any of the Methods given, in the second or third Book, you can find the Distance of a Comet passing near the Earth, whose Orbit is given in Position and Magnitude (from the fifth Book,) you will find the Magnitude of the Orbit of any Planet, or Comet, whose Period is known.

Astronomy is more intricate and difficult to an Observer in one of the Secondary Planets. For in Order to establish it, so as it may agree with the Phænomena, a *Primum Mobile* must be introduc'd, to cary all cœlestial Bodies from East to West, in the Space of a natural Day: But that will not be enough; for the Volva is not carried by that first Motion, altho' it be a cœlestial Body; and in one of Jupiter's, or Saturn's Satellites, some of the cœlestial Bodies (inferior to the Volva) are carried round by the *Primum Mobile*, and others not. Moreover, for demonstrating the second Motions, there must be Deferents, Epicycles, and Epicyclo-Epicycles, and the Astronomer will often be obliged to change them all; whence his System
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will be very complex, and the Calculation very intricate. But he will, with Ease, practise Geography and Navigation: For besides all the Helps he may receive from Geometry, and the same Means of finding the Latitude as we have, he may compute the Longitude from the Meridian, which is under the unmov'd *Volva*, and will immediately observe the Distance of the Meridian of any Place in the Hemisphere of the *Subvolvans*, from the first Meridian; which can't so well be done in a Primary Planet, because it has nothing unmov'd in the Heavens to look at, besides the Poles of the first Motion. These Things may very nicely be perform'd in one of Jupiter's Satellites, because to them the *Volva* remains, as it were, unmov'd; less accurately of all in our Moon; to which the *Volva* does not only librate in Latitude (as it happens to the Inhabitants of the Saturnian Moons) but also in Longitude; so that for finding the Longitude of a lunar Place, by the Observation of the *Volva*, there must be had a Theory of the Libration of the *Volva*. Moreover, by Observations made in several Places of his Habitation at the same Time, an Astronomer, in a Satellite, may find the Distance of the *Volva*, and of the other Satellites of it, which wou'd scarce be known any other Way, (except, perhaps, from the different apparent Magnitude of those Moons) and thence deduce the Motion of the Moon about the *Volva*; and this wou'd be the first Step towards finding out the true System. For the Variety of Phases, which gave Occasion to the Inhabitants of the Earth, to find out that Venus and Mercury move about the Sun, can be of no Service here.

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Book VI. of ASTRONOMY. 879

But at last, if the Inhabitants of any Satellite knows that this Habitation moves about the *Volva*, and that the *Volva* revolves about the Sun, he is situated in the best Place of any, for improving Astronomy: For, besides an accurate Geography, very necessary to Astronomy, there being no Atmosphere, no Mists or Clouds do ever hinder Observations, and he may observe the celestial Bodies by Day as well as by Night, and all the Planets, even those that are near the Sun; therefore, if there are any Planets below Mercury, which we can't see, he will see them, and encrease Astronomy so much the more; Besides, he will observe the celestial Bodies without Refraction, (because it is occasion'd by an ambient Atmosphere) and this alone does so much disturb the Observations of the Inhabitants of the Earth, that the taking off small Angles, is a Matter almost despair'd of. Moreover, the *Volva* rotated in its Place, serves as a Dial for all the *Subvolvans* to look at, and by Help of it, the Moments of Observations are easily known. The Earth's Companion, the Moon, is the most convenient of all the Satellites for this Use, for its apparent Astronomy, (such as the first Beginnings of Astronomy must be every where, unless the true System be divinely reveal'd) will be least compounded or perplex'd, because the Earth has no other Companion; whence an Observer there will sooner come to the Knowledge of the true System. And tho' the Diameter of the Orbit of the outmost Satellite of Jupiter or Saturn, subtends an Angle at the Sun, almost equal to that which the Diameter of the Moon's Orbit subtends, and so the Distance of the Sun from Jupiter, or Saturn may, by Observations made in their outer-

880 *The* ELEMENTS Book VI.

outermost Satellites, be almost exactly defin'd, as its Distance from the Earth, by Observations made in the Moon; yet, since the Distance of the Sun is known, by knowing the Distance of any Primary Planet, an Inhabitant of one of Jupiter's Satellites will, in order to this Enquiry, choose the next Primary Planet to itself, namely, Mars, when it is in Conjunction with the Sun, and retrograde; as we do Venus, when it is similarly posited: But the Angle, which the Diameter of the Moon's Orbit subtends at Venus, when it is near the Earth, is almost three Times greater than the Angle which the Diameter of the Orbit of the outermost of Jupiter's Satellites subtends at Mars, when it is nearest to Jupiter.



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A
SYNOPSIS
OF THE
Astronomy of COMETS.

By EDMUND HALLEY, L. L. D. *Savilian* Professor of Geometry at Oxford.

THE ancient Egyptians and Chaldeans (if we may credit *Diodorus Siculus*) by a long Course of Observations, were said to be able to predict the Apparitions of Comets. But since they are also said, by the Help of the same Arts, to have prognosticated Earthquakes and Tempests, 'tis past all Doubt, that their Knowledge, in these Matters, was the Result rather of meer Astrological Calculation, than of any Astronomical Theories of the Cœlestial Motions. And the Greeks, who were the Conquerors of both those People, scarce found any other Sort of Learning amongst them, than
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882 *A SYNOPSIS of the*

this. So that 'tis to the Greeks themselves as the Inventors (and especially the great *Hipparchus*) that we owe the Astronomy we have, and which is now improv'd to such a Height. But yet, amongst the Greeks, the Opinion of *Aristotle* (who wou'd have Comets to be nothing else, but Sublunary Vapours, or Airy Meteors) prevail'd so far, that this most difficult Part of the Astronomical Science lay altogether neglected; for no Body thought it worth while to take notice of, or write about, the wandering uncertain Motions of what they esteemed Vapours floating in the *Æther*; whence it came to pass, that nothing certain, concerning the Motion of Comets, can be found transmitted from them to us.

But *Seneca* the Philosopher, having consider'd the Phænomena of two remarkable Comets of his Time, made no Scruple to place them amongst the celestial Bodies; believing them to be Stars of equal Duration with the World, tho' he owns their Motions to be govern'd by Laws not as then known or found out. And at last (which was no untrue or vain Prediction) he foretells, that there should be Ages sometime hereafter, to whom Time and Diligence shou'd unfold all these Mysteries, and who shou'd wonder how 'twas possible the Ancients cou'd be ignorant of them, after some lucky Interpreter of Nature had shewn, *in what Parts of the Heavens the Comets wander'd, what Sort of Beings, and how great they were.* Yet almost all the Astronomers differ from this Opinion of *Seneca*; neither did *Seneca* himself think fit to set down those Phænomena of the Motion, by which he was enabled to maintain his Opinion; nor the Times of those Appearances, which might be

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Astronomy of COMETS. 883

of Use to Posterity, in order to the determining these Things. And indeed, upon the turning over very many Histories of Comets, I find nothing at all that can be of Service in this Affair, before *A. D.* 1337. at which Time *Nicephorus Gregoras*, a *Constantinopolitan* Historian and Astronomer, did pretty accurately describe the Path of a Comet amongst the Fix'd Stars, but was too lax, as to the Account of the Time; so that this most doubtful and uncertain Comet only deserves to be inserted in our Catalogue, for the sake of its appearing near four-hundred Years ago.

The next of our Comets was in the Year 1472, which being the swiftest of all, and nearest to the Earth, was observ'd by *Regiomontanus*. This Comet (so fearful upon the account both of the Magnitude of its Body, and the Tail) mov'd forty Degrees of a great Circle in the Heavens, in the Space of one Day; and was the first, of which any proper Observations are come down to us. But all those that consider'd Comets, until the Time of *Tycho Brabe* (that great Restorer of Astronomy) believ'd them to be below the Moon, and so took but little Notice of them, reckoning them no other than Vapours.

But in the Year 1577, (*Tycho* seriously pursuing the Study of the Stars, and having gotten large Instruments for the performing celestial Mensurations, with far greater Care and Certainty than the Ancients cou'd ever hope for) there appear'd a very remarkable Comet; to the Observation of which *Tycho* vigorously applied himself; and found by many just and faithful Trials, that it had no Diurnal Parallax that was perceptible: And consequently was not only no Aerial Vapour, but also much higher than

the Moon; nay, might be plac'd amongst the Orbs of the Planets, for any Thing that appear'd to the contrary; the cavilling Opposition, made by some of the School-men in the mean time, being to no Purpose.

Tycho was succeeded by the most sagacious *Kepler*. He, having the Advantage of *Tycho's* Labours and Observations, found out the true Physical System of the World, and vastly improv'd the Science of Astronomy.

For he demonstrated that all the Planets perform their Revolutions in *Elliptic Orbits*, whose Plains pass thro' the Center of the Sun, observing this Law, that the Areas of the *Elliptic Sectors*, taken at the Center of the Sun (which he proved to be in the common Focus of these Ellipses) are always proportional to the Times in which the correspondent *Elliptical Arcs* are describ'd. He discover'd also that the Distances of the Planets from the Sun are in the sesquialtera Ratio of the Periodical Times, or (which is all one) that the Cubes of the Distances are as the Squares of the Times. This great Astronomer had the Opportunity of observing two Comets, one of which was a very remarkable one. And from the Observations of these (which afforded sufficient Indications of an Annual Parallax) he concluded, That the Comets mov'd freely thro' the Planetary Orbs, with a Motion not much different from a Rectilinear one; but of what Kind he cou'd not precisely determine. Next, *Hevelius* (a noble Emulator of *Tycho Brahe*) following in *Kepler's* Steps, embrac'd the same Hypothesis of the Rectilinear Motion of the Comets, himself accurately observing many of them. Yet he complain'd that his Calculations did not perfectly agree to what he observed in the Heavens: And was aware, that the Path of

a Comet

Astronomy of COMETS. 885

a Comet was bent into a Curve Line concave towards the Sun. At length, came that prodigious Comet of the Year 1680. which descending (as it were from an infinite Distance) perpendicularly towards the Sun, arose from him again with as great a Velocity.

This Comet, (which was seen for four Months continually) by the very remarkable and peculiar Curvity of its Orb (above all others) gave the fittest Occasion for investigating the Theory of its Motion. And the Royal Observatories, at Paris and Greenwich, having been for some Time founded, and committed to the Care of most excellent Astronomers, the apparent Motion of this Comet was most accurately (perhaps as far as human Skill cou'd go) observ'd by Mr. Cassini and Flamsteed.

Not long after, that great Geometrician the Illustrious Newton, writing his *Mathematical Principles of Natural Philosophy*, demonstrated not only that what Kepler had found, did necessarily obtain in the Planetary System; but also, that all the Phænomena of Comets wou'd naturally follow from the same Principles; which he abundantly illustrated by the Example of the aforesaid Comet of the Year 1680. shewing, at the same Time, a Method of delineating the Orbits of Comets Geometrically; therein solving (not without meriting the highest admiration of all Men) a Problem, whose Intricacy render'd it scarce accessible to any but himself. This Comet he prov'd to move round the Sun in a Parabolical Orb, and to describe Area's (taken at the Center of the Sun) proportional to the Times.

886 A SYNOPSIS of the

Wherefore (following the Steps of so great a Man) I have attempted to bring the same Method to arithmetical Calculation; and that with all the Success I cou'd wish. For, having collected all the Observations of Comets I cou'd, I have fram'd this following Table, the Result of a prodigious deal of Calculation; Which, tho' but small in Bulk, will be no unacceptable Present to Astronomers. For these Numbers are capable of representing all that has been yet observ'd about the Motion of Comets, by the Help only of the annex'd General Table; in the making of which, I spar'd no Labour, that it might come forth perfect, as a Thing consecrated to Posterity, and to last as long as Astronomy itself.

Not long after, that great Geometician the illustrious Newton, writing his Mathematical Principles of Natural Philosophy, demonstrated not only that what Kepler had found, did necessarily obtain in the Planetary System; but also, that the Phenomena of Comets would naturally follow from the same Principles, which he abundantly illustrated by the Example of the famous Comet of the Year 1680, showing the same Time, a Method of delineating the Orbit of Comets Geometrically; therein showing (not without meeting the highest admiration of all Men) a Problem, whose Intimacy render'd it scarce accessible to any but himself. This Comet he prov'd to move round the Sun in the Parabolic Orb, and to describe it's Area in the Center of the Sun, proportional to the Time.

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1684	28.15.	065.48.40	1128.52.	0	90015	9.511883	6.14.33	86.25.50	Direct
1686	20.34.49	31.21.40	1117.00.30	32900	9.511883	8.16.57	3.7.0	Retrog.	
1698	27.44.15	11.46.	01800.51.15	69129	9.839660				

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The Astronomical Elements of the Motions in a Parabolic Orb, of all the Comets that have been hitherto duly observ'd.

Com. An.	Nodus Ascend.		Inclin. Orbitæ.		Perihelion.		Diffan. Periheli. à Sole.	Log. Diff. Periheli. à Sole.	Temp. equat. Periheli.		Perihelion à Nodu.		
	gr.	'	gr.	'	gr.	'			d.	h.	gr.	'	
1337	1124.21.	0	32.11.	0	7.59.	0	40665	9.609236	June 2.	6.25	46.22.	0	Retrog.
1472	VS11.46.20	0	5.20.	0	5.33.30	0	54273	9.734584	Feb. 28.	2.22.23	123.47.10	0	Retrog.
1531	119.25.	0	17.56.	0	1.39.	0	56700	9.753583	Aug. 24.	20.18½	107.46.	0	Retrog.
1532	1120.27.	0	32.36.	0	21.7.	0	50910	9.766803	Oct. 19.	22.12	30.40.	0	Direct
1556	125.42.	0	32.630	0	VS 8.50.	0	66390	9.666424	Apr. 21.	20.3	103.8.	0	Direct
1577	125.52.	0	74.32.45	0	9.22.	0	18342	9.263447	Oct. 26.	18.45	103.30.	0	Retrog.
1580	118.57.20	0	64.40.	0	19.5.50	0	59628	9.775450	Nov. 28.	15.00	90.8.30	0	Direct
1585	7.42.30	0	6.4.	0	8.51.	0	109358	0.038850	Sept. 27.	19.20	28.51.30	0	Direct
1590	115.30.49	29.40.40	11.6.54.30	0	6.54.30	0	57661	9.760882	Jan. 29.	3.45	51.23.50	0	Retrog.
1596	112.12.30	55.12.	11.18.16.	0	11.18.16.	0	51293	9.710058	July 31.	19.55	83.56.30	0	Retrog.
1607	120.21.	0	17.2.	0	2.16.	0	58680	9.768490	Oct. 16.	3.50	108.05.	0	Retrog.
1618	116.1.	0	37.34.	0	2.14.	0	37975	9.579498	Oct. 29.	12.23	71.47.	0	Direct
1652	1128.10.	0	79.28.	0	12.18.40	0	84750	9.928140	Nov. 21.	5.40	59.51.20	0	Direct
1661	112.30.30	32.35.50	12.5.58.40	0	12.5.58.40	0	44851	9.651772	Jan. 16.	23.41	33.28.10	0	Direct
1664	1121.14.	0	21.18.30	0	10.41.25	0	102575½	0.011044	Nov. 24.	11.52	49.27.25	0	Retrog.
1665	1118.02.	0	76.5.	0	11.1.54.30	0	10649	9.027109	Apr. 14.	5.15½	156.7.30	0	Retrog.
1672	VS27.30.30	83.22.10	11.6.59.30	0	11.6.59.30	0	69739	9.843476	Feb. 20.	8.37	109.29.	0	Direct
1677	1126.49.10	79.03.15	11.17.37.5	0	11.17.37.5	0	28059	9.448072	Apr. 26.	00.37½	99.12.5	0	Retrog.
1680	VS 2.2.	0	60.56.	0	2.2.39.30	0	60612½	7.787106	Dec. 8.	00.6	9.22.30	0	Direct
1682	112.16.30	17.56.	2.52.45	0	2.52.45	0	3828	7.765877	Sept. 4.	07.39	108.23.45	0	Retrog.
1683	1123.23.	0	83.11.	0	11.25.49.30	0	56020	9.748343	July 3.	2.50	87.53.30	0	Retrog.
1684	128.15.	0	65.48.40	0	11.28.52.	0	96015	9.982339	May 29.	10.16	29.23.00	0	Direct
1686	120.34.40	31.21.40	11.17.00.30	0	11.17.00.30	0	32500	9.511883	Sept. 6.	14.33	86.25.50	0	Direct
1698	127.44.15	11.46.	VS00.51.15	0	VS00.51.15	0	69129	9.839660	Oct. 8.	16.57	3.7.	0	Retrog.

This Table needs little Explication, since 'tis plain enough from the Titles, what the Numbers mean. Only it may be observ'd, that the Perihelium Distances are estimated in such Parts, as the Middle Distance of the Earth from the Sun contains 100000.

*A General Table for Calculating the Motions
of Comets in a Parabolical Orbit.*

Med. mot.	Angul. à perihelio.	Logar. pro dist. à Sole.	Med. mot.	Ang. à perihelio.	Logar. pro dist. à Sole.
0	gr. ' "		0	gr. ' "	
1	1.31.40	0.000077	41	53.29.44	0.098300
2	3. 3.15	0.000309	42	54.27.32	0.102019
3	4.34.43	0.000694	43	55.24.21	0.105752
4	6. 6. 0	0.001231	44	56.20.12	0.109490
5	7.37	0.001921	45	57.15. 6	0.113240
6	9. 7.43	0.002759	46	58. 9. 3	0.116995
7	10.38. 2	0.003745	47	59. 2. 4	0.120756
8	12. 7.54	0.004876	48	59.54.11	0.124518
9	13.37.17	0.006151	49	60.45.25	0.128278
10	15. 6. 7	0.007564	50	61.35.45	0.132035
11	16.34.20	0.009115	51	62.25.14	0.135792
12	18. 1.54	0.010798	52	63.13.52	0.139544
13	19.28.47	0.012609	53	64. 1.40	0.143291
14	20.54.54	0.014550	54	64.48.38	0.147029
15	22.20.14	0.016607	55	65.34.50	0.150762
16	23.44.44	0.018783	56	66.20.13	0.154482
17	25. 8.22	0.021072	57	67.04.50	0.158192
18	26.31. 8	0.023470	58	67.48.42	0.161890
19	27.52.55	0.025969	59	68.31.50	0.165578
20	29.13.47	0.028570	60	69.14.16	0.169254
21	30.33.40	0.031263	61	69.55.58	0.172914
22	31.52.32	0.034045	62	70.36.56	0.176557
23	33.10.23	0.036916	63	71.17.16	0.180188
24	34.27.12	0.039864	64	71.56.56	0.183803
25	35.42.59	0.042892	65	72.35.57	0.187404
26	36.57.4	0.045989	66	73.14.15	0.190978
27	38.11.20	0.049154	67	73.51.59	0.194540
28	39.23.54	0.052382	68	74.29. 6	0.198085
29	40.35.23	0.055668	69	75.05.38	0.201614
30	41.45.47	0.059009	70	75.41.35	0.205122
31	42.56.06	0.062400	71	76.16.56	0.208612
32	44. 3.20	0.065838	72	76.51.43	0.212080
33	45.10.29	0.069319	73	77.25.57	0.215529
34	46.16.35	0.072839	74	77.59.41	0.218963
35	47.21.36	0.076396	75	78.32.54	0.222378
36	48.25.33	0.079984	76	79. 5.35	0.225769
37	49.28.27	0.083600	77	79.37.45	0.229142
38	50.30.19	0.087244	78	80. 9.23	0.232488
39	51.31. 8	0.090910	79	80.40.34	0.235809
40	52.30.56	0.094596	80	81.11.16	0.239127

The General Table continued.

Med. mot.	Ang. à perihelio.	Logar. pro diff. à Sole.	Med. mot.	Ang. à perihelio	Logar. pro diff. à Sole.
o	gr. ' "		o	gr. ' "	
81	81.41.31	0.242416	142	102.32.41	0.407380
82	82.11.19	0.245684	144	103.00.31	0.411784
83	82.40.40	0.248933	146	103.27.47	0.416132
84	83. 9.34	0.252159	148	103.54.31	0.420430
85	83.38. 4	0.255366	150	104.20.43	0.424676
86	84. 6. 8	0.258552	152	104.46.22	0.428866
87	84.33.49	0.261720	154	105.11.33	0.433012
88	85. 1. 5	0.264865	156	105.36.16	0.437110
89	85.27.58	0.267989	158	106.00.32	0.441164
90	85.54.27	0.271092	160	106.24.23	0.445178
91	86.20.34	0.274176	162	106.47.47	0.449144
92	86.46.20	0.277239	164	107.10.44	0.453060
93	87.11.43	0.280284	166	107.33.17	0.456936
94	87.36.45	0.283306	168	107.55.27	0.460772
95	88.01.27	0.286308	170	108.17.14	0.464208
96	88.25.49	0.289293	172	108.38.37	0.468318
97	88.48.40	0.292252	174	108.59.39	0.472030
98	89.13.32	0.295201	176	109.20.20	0.475705
99	89.36.54	0.298122	178	109.40.40	0.479340
100	90.00.00	0.301030	180	110.00.40	0.482937
102	90.45.14	0.306782	182	110.20.20	0.486498
104	91.29.18	0.312469	184	110.39.41	0.490022
106	92.12.14	0.318060	186	110.58.44	0.493512
108	92.54. 4	0.322587	188	111.17.28	0.496965
110	93.34.52	0.329042	190	111.35.55	0.500384
112	94.14.40	0.334424	192	111.54.05	0.503769
114	94.53.30	0.339736	194	112.11.58	0.507121
116	95.31.22	0.344979	196	112.29.34	0.510441
118	96. 8.22	0.350153	198	112.46.55	0.513729
120	96.44.30	0.355262	200	113. 4.00	0.516984
122	97.19.48	0.360306	204	113.37.25	0.524406
124	97.54.19	0.365284	208	114. 9.52	0.529705
126	98.28.00	0.370200	212	114.41.23	0.535886
128	99.00.57	0.375052	216	115.12.02	0.541958
130	99.33.11	0.379842	220	115.41.5	0.547922
132	100. 4.43	0.384576	224	116.10.52	0.553782
134	100.35.45	0.389252	228	116.39. 7	0.559538
136	101. 5.22	0.393868	232	117.16.38	0.565199
128	101.35.22	0.398428	236	117.33.27	0.570762
140	102. 4.19	0.402930	240	117.59.35	0.576233

Med.

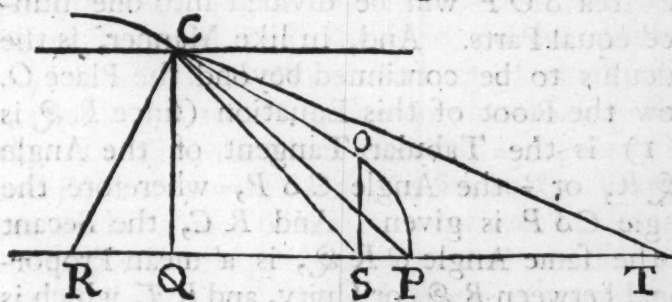
The General Table continued.

Med. mot.	Ang. à perihelio.	Logar. pro dist. à Sole.	Med. mot.	Ang. à perihelio	Logar. pro dist. à Sole.
°	gr.	"	°	gr.	"
244	118.25.5	0.581616	620	137.33.13	0.882575
245	118.49.57	0.586912	640	138.3.58	0.892649
252	119.14.14	0.592122	660	138.33.21	0.902401
256	119.37.56	0.597252	680	139.1.29	0.911866
260	120.1.6	0.602301	700	139.28.25	0.921012
264	120.23.44	0.607274	720	139.54.16	0.929907
268	120.45.52	0.612174	740	140.19.5	0.938549
272	121.7.30	0.616998	760	140.42.56	0.946951
276	121.28.39	0.621750	780	141.5.55	0.955124
280	121.49.22	0.626438	800	141.28.3	0.963082
284	122.9.38	0.631056	820	141.49.24	0.970836
288	122.29.28	0.635608	840	142.10.00	0.978397
292	122.48.54	0.640098	860	142.29.56	0.985771
296	123.7.57	0.644525	880	142.49.10	0.992970
300	123.26.36	0.648893	900	143.7.48	0.100000
310	124.11.40	0.659559	920	143.25.51	1.006871
320	124.54.36	0.669880	940	143.43.21	1.013586
330	125.35.34	0.679876	960	144.00.18	1.020155
340	126.14.44	0.689568	980	144.16.46	1.026583
350	126.52.12	0.698970	1000	144.32.46	1.022876
360	127.28.6	0.708104	1500	149.26.8	1.158188
370	128.2.33	0.716976	2000	152.26.15	1.246058
380	128.35.38	0.725606	2500	154.32.20	1.313703
390	129.7.27	0.734006	3000	156.7.27	1.368678
400	129.38.4	0.742186	3500	157.22.49	1.414974
410	130.7.34	0.750160	4000	158.24.36	1.454950
420	130.36.2	0.757936	4500	159.16.36	1.490125
430	131.3.30	0.765516	5000	160.1.12	1.521521
440	131.30.2	0.772918	5500	160.40.5	1.549874
450	131.55.41	0.780148	6000	161.14.24	1.575718
460	132.20.3	0.787216	6500	161.45.00	1.599460
470	132.44.32	0.794122	7000	162.12.34	1.621417
480	133.7.50	0.800882	7500	162.37.34	1.641838
490	133.30.25	0.807494	8000	163.00.23	1.660922
500	133.52.20	0.813969	8500	163.21.20	1.678834
520	134.34.18	0.826522	9000	163.40.42	1.695708
540	135.14.0	0.838600	9500	163.58.38	1.711662
560	135.51.28	0.850187	10000	164.15.20	1.726784
580	136.27.6	0.861369	50000	170.52.0	2.197960
600	137.00.57	0.872155	100000	172.45.44	2.399655

The

The Construction and Use of the general Table.

As the Planets move in Elliptic Orbs, so do the Comets in Parabolic ones, having the Sun in their common Focus, and describe equal Areas in equal Times. Now since all Parabola's are similar to one another, therefore if any determinate Part of the Area of a given Parabola, be divided into any Number of Parts at Liberty, there will be a like Division made in all Parabola's under the same Angles, and the Distances will be proportional: Consequently this one Table of ours will serve for all Comets. Now the Manner of the Calculation of this Table is thus: In the *Fig.*



Let S be the Sun, POC the Orbit of a Comet, P the Perihelion, O the Place where the Comet is 90 gr. distant from the Perihelion, C any other Place. Draw the right Lines CP , CS , and make ST , SR , equal to CS ; and having drawn the right Lines CR , CT (whereof the one is a Tangent, and the other a Perpendicular to the Curve) let fall CQ perpendicular to the Axis PSR .

Now, any Area, as $COPS$, being given, 'tis requir'd to find the Angle CSP , and the Distance CS . From the Nature of the Parabola

RQ

892 A SYNOPSIS of the

RQ is ever $= \frac{1}{2}$ the Parameter or *Latus rectum* of the Axis, and consequently if the Parameter be put $= 2$, then $RQ = 1$. Let $CQ = z$; then PQ shall $= \frac{1}{2} z^2$, and the Parabolic Segment $COP = \frac{11}{12} z^2 z$. But the Triangle CSP will $= \frac{1}{4} z$, and so the Mixtilineal Area $COPS = \frac{1}{12} z^3 + \frac{1}{2} z = a$, whence $z^3 + 3z = a$. Wherefore, resolving this Cubical Equation, z or the Ordinate CQ will be known. Now, let the Area OPS be propos'd to be divided into one hundred Parts; this Area is $\frac{1}{12}$ of the Square of the Parameter, and consequently $12a$ is $=$ that Square $= 4$. If therefore the Roots of these Equations $z^3 + 3z = 0,04 : 0,08 : 0,12 : 0,16, \&c.$ be successively extracted, there will be obtain'd so many z or Ordinates CQ respectively, and the Area SOP will be divided into one hundred equal Parts. And, in like Manner, is the Calculus to be continued beyond the Place O . Now the Root of this Equation (since RQ is $= 1$) is the Tabular Tangent of the Angle CQR , or $\frac{1}{2}$ the Angle CSP , wherefore the Angle CSP is given. And RC , the Secant of the same Angle CRQ , is a mean Proportional between RQ , or Unity, and RT , which is the Double of SC , as is plain from the Conics. But if SP be put $= 1$, and so the *Latus rectum* $= 4$ (as in our Table) then RT will be the Distance sought, viz. the Double of SC in the former Parabola. After this Manner, therefore, I compos'd the foregoing Table, which serves to represent the Motions of all our Comets; of which hitherto there has been none observed, but come within the Laws of the Parabola.

It remains now, to give the Rules for the Calculation, and to shew the Way of determining the Place of a Comet seen, by these Numbers.

Astronomy of COMETS. 893

bers. *The Velocity of a Comet, moving in a Parabola, is every where to the Velocity of a Planet describing a Circle about the Sun, at the same Distance from the Sun, as $\sqrt{2}$ to 1. as appears from Cor. 7. Prop. 16. Lib. 1. of the Princip. Phil. Nat. Math.* If therefore a Comet, in its Perihelium, were suppos'd to be as far distant from the Sun as the Earth is, then the Diurnal Area, which the Comet wou'd describe, wou'd be to the Diurnal Area of the Earth, as $\sqrt{2}$ to 1. And consequently, the Time of the Annual Revolution is to the Time in which such a Comet wou'd describe the Quadrant of its Orbit from the Perihelium, as 3.14159, &c. (that is the Area of the Circle) to $\sqrt{\frac{1}{2}}$. Therefore the Comet wou'd describe that Quadrant in 109 Days, 14 Hours, 46 Minutes; and so the Parabolic Area (analogous to the Area *POS*) being divided into 100 Parts, to each Day there wou'd be allotted 0.912280 of those Parts, the Log. of which, viz. 9.960128, is to be kept for continual Use. But then the Times in which Comets, at a greater or less Distance, wou'd describe similar Quadrants, are as the Times of the Revolutions in Circles, that is, in the sesquiplicate Ratio of the Distances: Whence the Diurnal Areas estimated in Centesimal Parts of the Quadrant (which Parts we put for Measures of the mean Motion, like Degrees) are in each, in the subsesquialtera Proportion of the Distance from the Sun in the Perihelion.

These Things being necessarily premis'd, let it be propos'd to compute the apparent Place of any one of the foremention'd Comets for any given Time. Therefore,

1. *Let the Sun's Place be had, and the Log. of its Distance from the Earth.*

2. *Let*

894 . A SYNOPSIS of the

2. Let the Difference between the Time of the Perihelion and the Time given be gotten, in Days and decimal Parts of Days. To the Log. of this Number, let there be added the constant Log. 9.960128, and the Complement Arithmetical of three Halves of the Log. of the Perihelion Distance of the Comet from the Sun: The Sum will be the Log. of the mean Motion, to be sought in the first Column of the general Table.

3. With the mean Motion, let there be taken the correspondent Angle from the Perihelion in the Table, and the Log. for the Distance from the Sun: Then in Comets that are direct, add, and in retrograde ones subtract; if the Time be after the Perihelion, the Angle thus found, to or from the Perihelion: or in direct Comets, subtract; and in retrograde ones add; if the Time be before the Perihelion, the foresaid Angle to or from the Place of the Perihelion; and so we shall have the Place of the Comet in its Orbit. And to the Log. for the Distance found, let there be added the Log. of the Distance in the Perihelion, and the Sum will be the Log. of the true Distance of the Comet from the Sun.

4. The Place of the Node, together with the Place of the Comet in its Orbit, being given, let the Distance of the Comet from the Node be found; then the Inclination of the Plane being given, there will be given also (from the common Rules of Trigonometry) the Comet's Place reduced to the Ecliptic, the Inclination or Heliocentric Latitude, and the Log. of the Curtate Distance.

5. From these Things given (by the very same Rules that we find the Planet's Places, from the Sun's Place and Distance given) we may obtain the Apparent or Geocentric Place of the Comet, together with the apparent Latitude. And this it may be worth while to illustrate by an Example or two.

EXAMPLE I.

Astronomy of COMETS. 895

EXAMPLE I.

Let it be requir'd to find the Place of the Comet of the Year 1664, March 1^d, 7^h, 00', P. M. London. That is, 96^d, 19^h, 8', after the Perihelion, which happen'd November 24°, 11^h, 52'.

Log. Dist. Perihel.	0. 011044
Log. Sefquialt.	0. 016566
Comp. Arith.	9. 983434
	9. 960128
Log. Temp.	1. 985862
Log. Med. Mot.	1. 929424
Medius Motus.	85. 001.
Perihel. $\angle$	10. 41. 25
Ang. Corresp.	83. 38. 05—
Comet in Orb. $\oslash$	17. 3. 20
Ascend. Nod. Π	21. 14. 00
Com. à Nodo	34. 10. 40
Red. ad Eclip.	32. 19. 05
Com. Helioc. $\oslash$	18. 54. 55
Incl. Bor.	11. 46. 50
Log. pro dist.	0. 255369
Log. Perihel.	0. 011044
Co-fin. Incl.	9. 990754
Log. dist. Curt.	0. 257167
Log. dist. $\odot$	9. 997918
$\odot \times$	21. 44. 45
Com. Visus Υ	29. 18. 30
Lat. Visa	8. 36. 15

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896 *ASYNOPSIS of the*

EXAMPLE II.

Let it be requir'd to find the Place of the Comet of the Year 1683, July 23^o, 13^h, 35', P. M. London: Or, 13^h, 40' Equat. Time. That is, 21^d, 10^h, 50', after the Perihelion.

Log. Dist. Perihel.	9. 748343
Log. Sefquialt.	9. 622514
Comp. Arith.	0. 377486
	9. 968128
Log. Temp.	1. 310723
Log. Med. Mot.	1. 648337
Medius Motus	44. 498

Perihel. Π	25. 29. 30
Ang. Corresp.	56. 47. 20—
Comet. in Orb. ∇	28. 42. 10
Nod. Descend. $\times$	23. 23. 00
Com. à Nodo	35. 19. 10
Red. ad Eclip.	4 48. 30
Com. Helioc. $\times$	28. 11. 30
Incl. Bor.	35. 2. 00

Log. pro dist.	0. 111336
Log. Perihel.	9. 748343
Co-fin. Incl.	9. 913187
Log. dist. Curt.	9. 772866
Log. dist. $\odot$	0. 006104
$\odot$ Locus $\mathcal{A}$	10. 41. 25
Com. Visus $\ominus$	5. 11. 50
Lat. Bor.	28. 52. 00

At the Instant of Time, specified in the first Example, 'twas observ'd (at *London*) that the Comet applied to the second Star of *Aries*; so that it was found to be 9' more Northerly, and

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Astronomy of COMETS. 897

and 3' to the East, according to Dr. *Hook's* Observation. But at that of the second Example, I my self (near *London*, with the same Instruments whereby I formerly observ'd the Southern Constellations) found the Place of the Comet to be 5° , $11\frac{1}{2}'$; and 28° , $52'$ North Latitude, which agreed exactly with the Observation made at *Greenwich*, almost at the very same Moment.

As for the Comet of the Year 1680, which came almost to the very Sun itself (being in its Perihelion, not above one Third of the Semidiameter of the Sun distant from the Surface of it,) since the *Latus Rectum* of its Orb is so very small, it could hardly be contained within the Limits of the general Table, because of the excessive Velocity of the mean Motion. Wherefore, in this Comet, the best Way will be (after the mean Motion is found) to get from thence (by the

Help of the foregoing Equation $z^3 + 3z = \frac{4}{100}$ of the mean Motion) the Tangent of half the Angle from the Perihelion, together with the Log. for the Distance from the Sun. Which being found, we are to proceed by the same Rules, as in the rest.

After this Manner therefore, the Astronomical Reader may examine these Numbers, which I have calculated with all imaginable Care, from the Observations I could meet with. And I have not thought fit to make them publick before they have been by my self duly examin'd, and made as accurate as 'twas possible, not without the Labour of many Years. I have publish'd this Specimen of *Cometical Astronomy*, as a *Prodromus* of a future Work I have in Design, lest, happening to be prevented by Death,

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898 *ASYNOPSIS of the*

these Papers might chance to be lost, which every Man would not be capable to retrieve, by Reason of the great Difficulty of the Calculation.

Now, it may not be amiss to put the Reader in Mind that our five first Comets, (the third and fourth observ'd by *Peter Apian*, the fifth by *Paulus Fabricius*) as also the tenth, seen by *Mæstlin*, if I mistake not, in the Year 1596, are not so certain as the rest; for the Observations were made neither with sufficient Instruments, nor due Care, and, upon that Account, are disagreeing with themselves, and can by no Means be reconcil'd with a regular Computus. The Comet, which appear'd in the Year 1684. was only taken Notice of by *Blanchinus*, who observed it at *Rome*: And the last, which appear'd in the Year 1698. was seen only by the *Parisian* Observers, who determin'd its Course after a very uncommon Manner. This Comet was a very obscure one, and altho' it mov'd swift, and came near enough to our Earth; yet we, who are wont not to be incurious in these Matters, saw nothtng of it. For Want of Observations I have also left out of the foregoing Catalogue, those two remarkable Comets which have appear'd in this our Age, one in *November* in the Year 1689, the other in *February* in the Year 1702. For they directing their Courses towards the Southern Parts of the World, and being scarce conspicuous any where in *Europe*, met with no Observers proper for the Purpose. But if any one shall bring from *India*, or the Southern Parts, an accurate Series of Observations, I will willingly fall to Work again, and undergo the Fatigue of representing their Orbits in Numbers, as I have done the rest.

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Astronomy of COMETS. 899

By comparing together the Elements of the Motions of these Comets, 'tis apparent, their Orbits are dispos'd in no Manner of Order; nor can they, as the Planets are, be comprehended within a Zodiac, moving indifferently every Way, as well retrograde as direct; from whence it is clear, they are not carry'd about, or mov'd, in a Vortical System. Moreover, the Distances in their Perihelia are sometimes greater, sometimes less; which makes me suspect, there may be a far greater Number of them, which may move in Regions more remote from the Sun; and being therefore very obscure, and wanting Tails, may pass by us unseen.

Hitherto I have consider'd the Orbits of Comets as exactly parabolic; upon which Supposition it wou'd follow, that Comets, being impell'd towards the Sun by a centripetal Force, would descend as from Spaces infinitely distant; and, by their so falling, acquire such a Velocity, as that they may again fly off into the remotest Parts of the Universe, moving upwards with a perpetual Tendency, so as never to return again to the Sun. But since they appear frequently enough, and since none of them can be found to move with an hyperbolic Motion, or a Motion swifter than what a Comet might acquire by its Gravity to the Sun, 'tis highly probable they rather move in very excentric elliptic Orbits, and make their Returns after long Periods of Time: For so their Number will be determinate, and, perhaps, not so very great. Besides, the Space between the Sun and the Fix'd Stars is so immense, that there is Room enough for a Comet to revolve, tho' the Period of its Revolution be vastly long. Now, the *Latus Rectum* of an Ellipsis is to the *Latus Rectum* of a Parabola,

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900 *A SYNOPSIS of the*

which has the same Distance in its Perihelium ; as the Distance in the Aphelium, in the Ellipsis, is to the whole Axis of the Ellipsis. And the Velocities are in a subduplicate Ratio of the same : Wherefore, in very excentric Orbits, the Ratio comes very near to a Ratio of Equality ; and the very small Difference which happens, on Account of the greater Velocity in the Parabola, is easily compensated in determining the Situation of the Orbit. The Principal Use therefore of this Table of the Elements of their Motions, and that which indeed induced me to construct it, is, that whenever a new Comet shall appear, we may be able to know, by comparing together the Elements, whether it be any of those which has appear'd before, and consequently to determine its Period, and the Axis of its Orbit, and to foretel its Return. And, indeed, there are many Things which make me believe, that the Comet which *Apian* observ'd in the Year 1531, was the same with that which *Kepler* and *Longomontanus* more accurately describ'd in the Year 1607 ; and which I my self have seen return, and observ'd in the Year 1682. All the Elements agree, and nothing seems to contradict this my Opinion, besides the Inequality of the Periodic Revolutions. Which Inequality is not so great neither, as that it may not be owing to physical Causes. For the Motion of Saturn is so disturbed by the rest of the Planets, especially Jupiter, that the Periodic Time of that Planet is uncertain for some whole Days together. How much more therefore will a Comet be subject to such like Errors, which rises almost four Times higher than Saturn, and whose Velocity, tho' encreased but a very little, would be sufficient to change its Orbit, from an Elliptical

Astronomy of COMETS. 901

tical to a Parabolical one. And I am the more confirmed in my Opinion of its being the same; for, that in the Year 1456, in the Summer-Time, a Comet was seen passing retrograde between the Earth and the Sun, much after the same Manner: Which, tho' nobody made Observations upon it, yet, from its Period, and the Manner of its Transit, I cannot think different from those I have just now mention'd. And since looking over the Histories of Comets, I find, at an equal Interval of Time, a Comet to have been seen about *Easter* in the Year 1305, which is another double Period of 151 Years before the former. Hence, I think, I may venture to foretel, that it will return again in the Year 1758. And, if it should then so return, we shall have no Reason to doubt but the rest may return also: Therefore Astronomers have a large Field wherein to exercise themselves for many Ages, before they will be able to know the Number of these many and great Bodies revolving about the common Center of the Sun, and to reduce their Motions to certain Rules. I thought, indeed, that the Comet, which appear'd in the Year 1532, might be the same with that observ'd by *Hevelius* in the Year 1661. But *Apian's* Observations, which are the only ones we have concerning the first of these Comets, are too rude and inaccurate for any thing of Certainty to be drawn from them, in so nice a Matter. But, as far as Probability from the Equality of Periods, and similar Appearance of Comets, may be urged as an Argument, the late wondrous Comet of 1682. seems to have been the same, which was seen in the Time of our King *Henry I.* Anno 1106, which began to appear in the West about the Middle of *February*, and continued for many Days after, with such a Tail as was

962 *A SYNOPSIS of the*

seen in that of 1687. And again, in the Consulate of *Lampadius* and *Orestes*, about the Year of Christ 531, such another Comet appeared in the West, of which *Malela*, perhaps an Eye-Witness, relates that it was μέγας καὶ φοβερός, a great and fearful Star; that it appeared in the West, and emitted upwards from it a long white Beam; and was seen for 20 Days. It were to be wish'd the Historian had told us what Time of the Year it was seen; but, 'tis however plain, that the Interval, between this and that of 1106, is nearly equal to that between 1106 and 1687, viz. about 575 Years. And, if we reckon backward such another Period, we shall come to the 44th Year before Christ, in which *Julius Cæsar* was murder'd, and in which there appear'd a very remarkable Comet, mentioned by almost all the Historians of those Times, and by *Pliny* in his *Natural History*, lib. 11. c. 24. who recites the Words of *Augustus Cæsar* on this Occasion, which lead us to the very Time of its Appearance, and its Situation in the Heavens. These Words being very much to our Purpose, it may not be amiss to recite them. *In ipsis Ludorum meorum diebus, sydus crinitum per septem dies, in regione Cæli quæ sub Septentrionibus, est conspectum. Id oriebatur circa undecimam horam diei, clarumq; & omnibus terris conspicuum fuit.* Now these *Ludi* were dedicated *Veneri genetrici*, (for, from *Venus* the *Cæsars* would be thought to be descended, and began with the Birth-day of *Augustus*, viz. Sept. 23. (as may be collected from a Fragment of an old Roman Calendar extant in *Gruter*, pag. 135.) and continued for 7 Days, during which the Comet appeared. Nor are we to suppose, that it was seen only those 7 Days, but, possibly, both before and after. Nor, are we to interpret the Words
sub

Astronomy of COMETS. 963

sub Septentrionibus, as if the Comet had appeared in the North, but that it was seen under the *Septem triones*, or brighter Stars of *Ursa major*. And as to its Rising, *Hora undecima diei*, it can no ways be understood, unless the Word *diei* be left out, as it is by *Suetonius*; for it must have been very far from the Sun, either to rise at five in the Afternoon, or at eleven at Night; in which Cases it must have appear'd for a long Time, and its Tail have been so little Remarkable, that it cou'd by no Means be call'd, *clarum & omnibus Terris conspicuum Sydus*. But supposing this Comet to have traced the same Path with that of the Year 1680, the ascending Part of the Orb will exactly represent all that *Augustus* hath said concerning it; and is yet an additional Argument to that drawn from the Equality of the Period. Thus 'tis not improbable, but this Comet may have four Times visited us at Intervals of about 575 Years: Whence the transverse Diameter of its elliptic Orb will be found $\sqrt[3]{575 \times 575}$ Times greater than the annual Orb; or 138 Times greater than the mean Distance of the Sun; which Distance, tho' immensely great, bears no Proportion to that of the Fix'd Stars.

I have lately found out a ready Method to compute the Motion of Comets in these elliptic Orbs, of which, perhaps shortly, we may exhibit a Specimen, giving this Comet for an Example. In the mean Time, those that desire to know how to construct Geometrically the Orb of a Comet, by three accurate Observations given, may find it at the End of the third Book of Sir *Isaac Newton's* Principles of Natural Philosophy, entituled, *De Syemate Mundi*, in the Words of its renowned Inventor. Which have since been

962 *A SYNOPSIS of the*

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Astronomy of COMETS. 963

Sub Septentrionibus, as if the Comet had appear'd in the North, but that it was seen under the *Septem triones*, or brighter Stars of *Ursa major*. And as to its Rising, *Hora undecima diei*, it can no ways be understood, unless the Word *diei* be left out, as it is by *Suetonius*; for it must have been very far from the Sun, either to rise at five in the Afternoon, or at eleven at Night; in which Cases it must have appear'd for a long Time, and its Tail have been so little Remarkable, that it cou'd by no Means be call'd, *clarum & omnibus Terris conspicuum Sydus*. But supposing this Comet to have traced the same Path with that of the Year 1680, the ascending Part of the Orb will exactly represent all that *Augustus* hath said concerning it; and is yet an additional Argument to that drawn from the Equality of the Period. Thus 'tis not improbable, but this Comet may have four Times visited us at Intervals of about 575 Years: Whence the transverse Diameter of its elliptic Orb will be found $\sqrt[3]{575 \times 575}$ Times greater than the annual Orb; or 138 Times greater than the mean Distance of the Sun; which Distance, tho' immensely great, bears no Proportion to that of the Fix'd Stars.

I have lately found out a ready Method to compute the Motion of Comets in these elliptic Orbs, of which, perhaps shortly, we may exhibit a Specimen, giving this Comet for an Example. In the mean Time, those that desire to know how to construct Geometrically the Orb of a Comet, by three accurate Observations given, may find it at the End of the third Book of Sir *Isaac Newton's* Principles of Natural Philosophy, entituled, *De Syemate Mundi*, in the Words of its renowned Inventor. Which have since been

904 *ASYNOPSIS of the*

more fully explained by my very worthy Colleague, Dr. Gregory, in his learned Work of *Astronomia Physica & Geometrica*.

One Thing more, perhaps, it may not be improper or unpleasant to advertise the Astronomical Reader; That some of these Comets have their Nodes so very near the annual Orb of the Earth, that if it shall so happen, that the Earth be found in the Parts of her Orb, next the Node of such a Comet, whilst the Comet passes by; as the apparent Motion of the Comet will be incredibly swift, so its Parallax will become very sensible; and the Proportion thereof, to that of the Sun, will be given. Wherefore such Transits of Comets do afford us the very best Means, tho' they seldom happen, to determine the Distance of the Sun and Earth: Which hitherto has only been attempted by Mars, in his Opposition to the Sun; or else Venus in Perigæo, whose Parallaxes, tho' triple to that of the Sun, are scarce any Ways to be perceived by our Instruments; whence we are still in great Uncertainty in that Affair. This Use of Comets was the ingenious Thought of that excellent Geometrician Mr. *Nicolas Fatio*. Now the Comet of 1472, had a Parallax above twenty Times greater than the Sun's. And if the Comet of 1618, had come down, about the Middle of March, to his descending Node; or if that of 1684, had arrived a little sooner at its ascending Node, they would have been yet much nearer the Earth, and consequently have had more notable Parallaxes. But hitherto none has threaten'd the Earth with a nearer Appulse, than that of 1680. For by Calculation I find, that *Novemb. 11^o, 1^h, 6'*, *P. M.* that Comet was not above the Semidiameter of the Sun, to the Northwards of the
Way

Astronomy of COMETS. 905

Way of the Earth. . At which Time, had the Earth been there, the Comet would have had a Parallax equal to that of the Moon, as I take it. This is spoken to Astronomers: But what might be the Consequences of so near an Appulse; or of a Contact, or lastly, of a Shock of the celestial Bodies, (which is by no Means impossible to come to pass,) I leave to be discuss'd by the Studious of physical Matters.





A N
I N D E X
OF THE
S E C T I O N S.

B O O K I.

Of the System of the World.

- Sect. I. **O**f the Order, Distances and Periods of
the Primary Planets, revolving a-
bout the Sun, and the principal Phænomena
thence arising. Page 1.
- II. Of the Direction of the Forces, which retain
the Primary Planets in their Orbits. 12
- III. Of the Order, Distances, and Periods of the
Secondary Planets revolving about their Pri-
mary ones, and their Phænomena ; together
with the Direction of the Forces, whereby
they are kept in their Orbits. 25.
- IV. Of the Periods of the Primary Planets a-
bout the Sun, and Secondary about their re-
spective Primary, compar'd together, and of
their Distances likewise compar'd together,
with their mutual Relation to one another,
the Reason and Causes of it. 42.
- V. Of

The INDEX.

- Sect. V.** *Of the Motion of the Primary Planets about their proper Axes, and of the Phenomena thence arising.* 58.
- VI.** *Of the Orbits of the Planets and Comets, and their Figures, together with the Law of Centripetal Forces, requisite to make them move in such Orbits.* 69.
- VII.** *The Planets and Comets are kept in their Orbits by Gravity, which is propagated thro' the whole solar System according to the same Law.* 96.
- VIII.** *Of the Motion of Bodies attracting one another, and their Affections; which are all applied to the System of the Sun and Primary Planets.* 105.
- IX.** *Of the Motion of a System of Bodies, which revolves about another Body; all which is applied to the System of the Sun and Planets, Primary and Secondary.* 117.
- X.** *Of the Causes and Reason of the Motions of the Planets, assign'd by Philosophers.* 133.
- XI.** *Of other Systems of the World, and Forces necessary to preserve them.* 183.

BOOK II.

Of the first Motion. 199.

- Sect. I.** *Of the Generation of the Circles of the Sphere, and of the Terms used in Astronomy, depending thereupon.* 201.
- II.** *Of the Division of Time, and several other Things depending thereon.* 237.
- III.** *Of Spheres and other Instruments invented to represent the First Motion of the Heavens, and of their Use.* 253.
- IV.** *Of the determining by Observation, the Situation*

The INDEX.

- uation and Respect that the Circles of the Sphere have to one another.* 277.
- V. *Of the Fix'd Stars, and the determining their Places by Observation, and some other Things belonging thereto.* 288.
- VI. *Of the Resolution of the more considerable Problems of the first Motion by Calculation.* 317.
- VII. *Of the Parallax of the Stars.* 336.
- VIII. *Of the Refraction of the Stars.* 358.
- IX. *Of the Tables of the First Motions, and of the Fix'd Stars, depending upon the Second Book.* 368.

B O O K III.

- Of the Theory of the Primary Planets. 375.
- Sect. I. *Some General Things belonging to the Theory of all the Planets* 378.
- II. *Of the determining the Orbit of the Earth and the Theory of the Earth seen from the Sun, or of the Sun seen from the Earth.* 401.
- III. *Of the determining the Orbits of the rest of the Primary Planets, and composing the Theory of them, seen both from the Sun and the Earth.* 423.
- IV. *Of the greatest Elongation of the Planets from the Sun, their Direction, Station, and Retrogradation.* 449.
- V. *Of the Tables of the Primary Planets, and the Use of them.* 462.
- VI. *Of the Magnitude of the Orbits of the Planets.* 474.
- VII. *Of the Magnitude and Density of Primary Planets.* 481.
- VIII. *Of the Figure of the Sun and Planets.* 489.
- IX. *Of the Distances of the Fix'd Stars.* 498.

BOOK

The INDEX.

B o o k IV.

Concerning the Theory of the Secondary Planets. 465.

- Sect. I. *Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is concentric to its Primary Planet.* 467.
- II. *Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is excentric to its Primary Planet.* 471.
- III. *Of the Errors which the Sun produces in the Motion of a Satellite, when the Plane of the Orbit describ'd by it, is inclin'd to the Plane of the Orbs describ'd about the Sun by the Primary.* 509.
- IV. *Of the Errors which the Sun produces in the Motion of a Satellite, when the Primary Planet is mov'd in an Excentric Orbit about the Sun.* 521.
- V. *Of the Motion of the Moon seen from the Earth.* 531.
- VI. *Of the Moon's Tables and their Use.* 543.
- VII. *Of the Moon's Eclipse.* 575.
- VIII. *Of the Sun's Eclipse.* 601.
- IX. *Of the Motions of Satellites revolving about other Primary Planets besides the Earth.* 640.
- X. *Of the Motions of Satellites about their own Centers respectively.* 649.
- XI. *Of the Magnitude and Density of the Satellites.* 654.
- XII. *Of the Figure of the Satellites, and of the Primary Planets arising from their mutual Gravity.* 662.
- XIII. *Of Saturn's Ring and its Phases.* 678.

BOOK

THE INDEX.

BOOK V.

Of Comets.

Sect. I. General Propositions concerning Comets.

II. To determine very nearly the Way and apparent Place of a Comet. 694. 723.

III. To determine the true Trajectory of a Comet. 741.

IV. Of the Way of finding a Comet's Heliocentric or Geocentric Place in its Trajectory, before determin'd, for a given Time; and of the Tables necessary for readily defining the Comets Motion. 795.

BOOK VI.

Of Comparative Astronomy.

In which is shewn what will be the most remarkable Phænomena of the World; the Eye being suppos'd to be plac'd in the Sun, or any Planet or Satellite of a Planet, or in a Comet; where the Reason of the several Systems of the World and of the Theories of several Philosophers is explain'd, and the Nature of them discover'd. 810.

A Synopsis of the Astronomy of Comets.

By Dr. E. Halley. 881.

A Table of the Astronomical Elements of the Motions in a Parabolic Orb of the Motions that have been hitherto duly observ'd. 887.

A General Table for calculating the Motions of Comets in a Parabolical Orbit. 888.

FINIS.